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Exploring the Relations between Cattell–Horn–Carroll (CHC) Cognitive Abilities and Mathematics Achievement

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Summary: As standardized measures of cognitive abilities and academic achievement continue to evolve, so do the relations between the constructs represented in these measures. A large, nationally representative sample of school-aged children and youth between 6 and 19 years of age (N = 4,194) was used to systematically evaluate the relations between cognitive abilities and components of academic achievement in mathematics. The cognitive abilities of interest were those identified from the Cattell–Horn–Carroll model of intelligence. Specific areas of mathematics achievement included math calculation skills and math problem solving. Results suggest that fluid reasoning (Gf), comprehension-knowledge (Gc), and processing speed (Gs) have the strongest and most consistent relations with mathematics achievement throughout the school years.

The development of mathematics skills is complex. It requires the mastery of multiple sub-skills (Locuniak & Jordan, 2008; VanDerHeyden & Burns, 2009) and the use of numerous cognitive abilities (Floyd, Evans, & McGrew, 2003; Proctor, Floyd, & Shaver, 2005; Taub, Floyd, Keith, & McGrew, 2008). Some research has reported that fluid reasoning, crystallized intelligence, and processing speed have significant relations with mathematics achievement (Taub et al., 2008). Other studies identified memory, attention, processing speed, and language skills as the primary cognitive abilities contributing to mathematics achievement (Geary, 2011; Geary, Hoard, Nugent, & Bailey, 2011). In some cases, the differences between studies may be related to different components of cognitive abilities being examined, whereas in other studies, differences could be due to the lack of a common nomenclature being used to identify the cognitive abilities under investigation. Regardless, there appears to be variability in the specific cognitive abilities relevant to different components of math skill acquisition (Floyd et al., 2003; Geary, Hoard, & Bailey, 2012). For example, short-term memory, working memory, and visual–spatial thinking demonstrate significant relations with mathematical calculation performance, whereas fluid reasoning demonstrates a significant relation with mathematical reasoning (Gelbart, 2007). Thus, a systematic evaluation of the cognitive abilities that contribute to mathematics achievement can be useful in determining which cognitive abilities are the primary contributors to various mathematics skills.

In addition to variation depending on the specific area of mathematics interest, there are also different developmental trajectories in the relations that are observed between specific cognitive abilities and areas of academic achievement. For example, previous research suggests that processing speed has a strong relation with math calculation skills from age 7 to 15, inclusively, and only a moderate relation from age 16 to 19, inclusively (Floyd et al., 2003). Although many of these relations and their developmental trajectories have been established in previous research (e.g., Evans, Floyd, McGrew, & Leforgee, 2002; Floyd, McGrew, & Evans, 2008; Floyd et al., 2003; McGrew, 1993; McGrew & Hessler, 1995; McGrew & Knopik, 1993), the measures used in these prior studies have been revised and re-normed, which brings into question whether these relations have remained the same. Thus, the purpose of this study is to examine the relations between specific cognitive abilities on different areas of mathematics achievement and to produce developmental trajectories of the relations between specific cognitive abilities and areas of mathematics achievement across the school age years.

Measuring cognitive abilities

The Cattell–Horn–Carroll (CHC) Theory of Human Cognitive Abilities provides a taxonomy describing all validated cognitive abilities (Schneider & McGrew, 2012). This hierarchical model of intelligence includes three strata, with psychometric *g*, which represents general intelligence, located at the highest level (stratum III). The two other strata represent broad abilities (stratum II) and narrow abilities (stratum I). It should be noted that this theory is viewed as an evolving rendition of known cognitive abilities, rather than being static or necessarily complete (McGrew, 2009). For example, short-term memory (Gsm) has been renamed (and the narrow abilities clarified) as short-term working memory (Gwm) in the latest summary of the CHC model (McGrew, LaForte, & Schrank, 2014). Although some of the abilities are better validated than others (Schneider & McGrew, 2012), those most often identified as relevant to the purpose of assessing cognitive abilities are fluid reasoning (Gf), long-term storage and retrieval (Glr), processing speed (Gs), comprehension-knowledge (Gc), visual processing (Gv), auditory processing (Ga), and Gwm (McGrew et al., 2014; Schneider & McGrew, 2012).

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Dr. Kevin McGrew is a coauthor of the Woodcock–Johnson IV battery and discloses that he has a financial interest in the WJ IV.

CHC abilities and mathematics schievement

Findings from extant CHC studies (e.g., McGrew & Hessler, 1995; Taub et al., 2008) consistently indicated strong relations between certain CHC cognitive abilities and mathematics achievement. However, the relations between CHC cognitive abilities and mathematics achievement appear to vary from one age group to another and depending on the specific mathematical skill. For example, Taub et al. (2008) reported that although some CHC broad cognitive abilities (e.g., Gf) demonstrate significant relations with mathematics achievement across four age groups (5 to 6, 7 to 8, 9 to 13, and 14 to 19), processing speed (Gs) only demonstrates a significant relation with mathematics achievement at the earlier age levels and comprehension knowledge (Gc) only demonstrates a significant relation with mathematics achievement for older school-age children and adolescents (9 to 13 and 14 to 19). Other studies, however, have provided more specific information about the developmental trajectories of broad CHC abilities that are used in response to specific mathematics tasks (e.g., Floyd et al., 2003).

Developmental trajectories

Floyd et al. (2003) examined the relations between broad CHC abilities and mathematics achievement across the school age population ranging from 6 to 19 years old using the Woodcock–Johnson III Tests of Cognitive Abilities (WJ III COG) and Woodcock–Johnson III Tests of Achievement (WJ III ACH). Two math clusters were used to examine the relations between broad CHC abilities and mathematics achievement: math calculation skills and math reasoning. The WJ III COG included seven CHC cognitive clusters (e.g., Gc, Glr, Gv, Ga, Gf, Gs, and Gwm). Floyd and colleagues found that Gc has the strongest relations with math calculation skills and math reasoning, with a moderate relation being observed for math calculation skills after age 9. Glr showed moderate relations between math calculation skills and math reasoning only at the younger ages (6 through 8 years). Gv, however, did not have significant relations with the two math clusters. Ga only showed a moderate relation with the math calculation skills cluster during the early school years. The relation between Gf and math calculation skills was moderate, whereas relation with math reasoning was moderate to strong, across all age groups. Gs showed a moderate to strong relation with math calculation skills, but the relation with math reasoning was moderate only until the age of 14. Gwm showed a moderate relation with math calculation skills after the age of 7, whereas the relation between Gwm and math reasoning was moderate until the age of 17.

CURRENT STUDY

Considerable empirical evidence has accumulated during the last two decades demonstrating the relationship between cognitive abilities and academic achievement (e.g., Floyd et al., 2003; Proctor et al., 2005; Taub et al., 2008). However, theories of cognitive abilities, as well as the assessment tools used to measure the constructs represented in these theories, continue to evolve (Schneider & Flanagan, 2015). The assumption that previously reported relations between cognitive abilities and areas of academic achievement remain constant within this evolution may lead to incorrect inferences being drawn from assessment results. Thus, we seek to determine if the previously reported relations between specific cognitive abilities and academic achievement in mathematics remain unchanged, despite numerous changes being made to various test batteries.

This study focuses specifically on the Woodcock Johnson Tests of Cognitive Abilities, Fourth Edition (WJ IV COG; Schrank, McGrew, & Mather, 2014a) and the Woodcock Johnson Tests of Academic Achievement, Fourth Edition (WJ IV ACH; Schrank, McGrew, & Mather, 2014b), given that much of the existing literature has focused on using this measure when examining the relations between cognitive abilities and academic achievement (see McGrew & Wendling, 2010, for a review). The benefit of using the WJ IV COG is that all broad cognitive abilities represented in the CHC model can be assessed, which allows for a systematic examination of the relations between cognitive abilities and areas of academic achievement (Newton & McGrew, 2010). The specific research questions to be answered are as follows:

- What are the relations between broad WJ IV COG clusters and the WJ IV mathematics achievement clusters?
- What are the developmental trajectories of the relations between the broad WJ IV COG clusters and the WJ IV mathematics achievement clusters?

METHOD

Sample

The normative samples for the WJ IV COG and the WJ IV ACH were used to examine the relations between broad CHC abilities and areas of academic achievement in mathematics.[1] The WJ IV COG and WJ IV ACH batteries are conormed. The complete norming sample included data from 7416 people ranging from ages 2 to over 90 (McGrew et al., 2014). The norming sample is representative of the US population across 46 states and the District of Columbia (McGrew et al., 2014). The sample used for this study only includes the school-age participants from the norming sample, which ranges from 6 to 19 years of age, inclusively. Therefore, the total sample size for this study was 4194. However, the sample was divided into individual age groups for ages 6 to 19, inclusively (see Table 1).

Measures

CHC clusters

The WJ IV COG includes two batteries of tests—a standard battery of 10 tests and an extended battery of eight additional tests. Broad CHC cluster scores (Gc, Gf, Gwm, Gs, Glr, Ga, and Gv) generated from pairs of tests from these batteries

[1] Standardization data from the *Woodcock-Johnson*™ *IV* (*WJ IV*™). Copyright © 2014 by The Riverside Publishing Company. All rights reserved. Used with permission of the publisher.

were used for this study. The individual tests associated with each broad CHC cluster are listed in Table 2. Strong, comprehensive reliability and validity information supports the use of the WJ IV COG for the purpose of assessing cognitive abilities (see Reynolds & Niileksela, 2015, for a review). For example, the median CHC-cluster reliability coefficients range from .88 to .98 throughout the school age years (i.e., ages 6 to 19, inclusively).

Mathematics achievement clusters

The WJ IV ACH is comprised of a standard battery of 11 tests and an extended battery of nine tests. Mathematics achievement cluster scores are calculated from pairs of tests included in the standard or extended batteries. The individual tests and their corresponding mathematics achievement clusters are listed in Table 2. The reliability and validity evidence for the WJ IV ACH is extensive (McGrew et al., 2014), and Reynolds and Niileksela (2015) 'consider it one of the premier individually administered measures of human intelligence' (p. 390). For example, across the entire norming sample, the median reliability coefficients for the WJ IV ACH mathematics clusters math calculation skills and math problem solving are .97 and .95, respectively. The CHC-cluster reliability coefficients for each age level throughout the school years (i.e., ages 6 to 19, inclusively) range from .91 to .97 (for a comprehensive review of the WJ IV ACH, see Reynolds & Niileksela, 2015 and Villarreal, 2015).

Data analysis

A data analysis plan was developed to systematically evaluate the relative contributions of broad CHC abilities in predicting mathematics achievement. The methods used in this study are similar to those used in Cormier, McGrew, Bulut, and Funamoto (2016), which was an evaluation the relations between CHC abilities and academic achievement in reading. The first step in the data analysis plan for the current study was to determine if the broad CHC clusters demonstrate meaningful relations with mathematics achievement, beyond the variance that is accounted for by general intellectual ability (GIA). This procedure will be referred to as the hierarchical regression models. Second, a series of multiple regression analyses were completed to examine the relations between seven WJ IV broad CHC cluster scores and two WJ IV ACH mathematics clusters by age group.

Table 2. Cognitive and achievement clusters and individual tests

Domains	Broad clusters	Tests associated the cluster
Cognitive	Fluid reasoning (*Gf*)	Number series
		Concept formation
	Comprehension knowledge (*Gc*)	Oral vocabulary
		General information
	Short-term working memory (*Gwm*)	Verbal attention
		Numbers reversed
	Long-term storage and retrieval (*Glr*)	Story recall
		Visual–auditory learning
	Processing speed (*Gs*)	Letter-pattern matching
		Pair cancellation
	Visual processing (*Gv*)	Visualization
		Picture recognition
	Auditory processing (*Ga*)	Phonological processing
		Nonword repetition
	General intellectual ability (GIA)	Oral vocabulary
		Number series
		Verbal attention
		Letter-pattern matching
		Phonological processing
		Story recall
		Visualization
Achievement	Math calculation skills	Calculation
		Math facts fluency
	Math problem solving	Applied problems
		Number matrices

Hierarchical regression models

In order to demonstrate the relations between the CHC broad clusters and mathematics achievement beyond the contribution of *g*, a series of hierarchical regression analyses were conducted. It should be noted that although the GIA was used to account for *g*, this score is an estimate of *g*. However, the individual tests included in this composite demonstrate 'high loadings on the general intelligence (g) factor' (p. 8, McGrew et al., 2014). For each of the mathematics

Table 1. Sample demographics by age groups, gender, and race

Age	*N*	Gender: Male	Gender: Female	Race: White	Race: Black	Race: Indian	Race: Asian or Pacific Islander	Race: Other or mixed
6	293	49.1%	50.9%	81.2%	13.0%	1.7%	2.4%	1.7%
7	308	49.7%	50.3%	80.8%	12.3%	.3%	4.5%	1.9%
8	335	50.1%	49.9%	77.9%	12.5%	.6%	6.6%	2.4%
9	306	49.0%	51.0%	77.1%	14.4%	.7%	3.6%	4.2%
10	314	50.0%	50.0%	81.2%	11.1%	.6%	4.8%	2.2%
11	329	50.5%	49.5%	75.7%	14.0%	.9%	6.4%	3.0%
12	317	50.2%	49.8%	79.5%	12.3%	.9%	5.0%	2.2%
13	307	46.9%	53.1%	74.9%	15.6%	1.0%	5.9%	2.6%
14	299	49.8%	50.2%	81.3%	11.4%	.7%	5.4%	1.3%
15	277	52.0%	48.0%	80.9%	13.0%	.4%	3.2%	2.5%
16	284	50.0%	50.0%	76.1%	16.9%	0.0%	5.6%	1.4%
17	254	46.5%	53.5%	78.7%	16.9%	1.2%	1.6%	1.6%
18	276	46.7%	53.3%	70.7%	22.8%	1.1%	3.6%	1.8%
19	295	47.5%	52.5%	76.6%	17.6%	.3%	3.7%	1.7%

achievement clusters, there were two nested regression models. First, a model with the GIA cluster from the WJ IV as the sole predictor of mathematics achievement was produced and will be referred to as the *reduced model*. Second, a model with GIA and the seven broad CHC clusters as the predictors were generated and will be referred to as the *full model*. It is possible to make a direct comparison between the two models because the reduced model is nested within the full model. This comparison can be made by examining the change in R-squared (R^2), which represents the additional variability that explained by the full model compared with the reduced model (see Tabachnick & Fidell, 2013, for details on this procedure). If the change in R^2 is statistically significant, then it can be inferred that the broad CHC clusters explain variance in mathematics achievement clusters, beyond the variance that is explained by the GIA. Individual models were produced for each of the age groups included in the sample. To account for the number of statistical tests that were computed, the alpha value for each of the tests was set at .001 to determine statistical significance.

Broad CHC ability regression models

The regression models for these analyses included all seven broad CHC cluster scores (i.e., Gc, Gf, Gwm, Gs, Ga, Glr, and Gv) as predictors. Separate regression analyses were conducted by using the following WJ IV ACH mathematics clusters as criterion variables: (a) math calculation skills and (b) math problem solving. Age-based standard scores ($M = 100$, $SD = 15$) were used for all analyses. The standardized regression coefficients from each regression model were then interpreted to determine the relations between the predictors and the outcome variables. This method for presenting and evaluating the results of the regression models is similar to those used in the prior studies to allow comparisons with previous cognitive-achievement regression research that also used the WJ batteries as their measure of psychological and educational constructs (Evans et al., 2002; Floyd et al., 2003; McGrew, 1993; McGrew & Hessler, 1995; McGrew & Knopik, 1993).

RESULTS

Hierarchical regression models

Figure 1 shows the results from the hierarchical regression models for the two mathematics achievement clusters (i.e., math calculation skills and math problem solving). The degree to which the broad CHC abilities explain additional variance in specific areas of mathematics varies according to age and the type of mathematics skill. The R^2 change ranged from .05 to .10 for math calculation skills and from .04 to .13 for math problem solving across 14 age groups. All of the R^2change values in Figure 1 were statistically significant, suggesting that broad CHC abilities can explain additional variance in math calculation skills and math problem solving above and beyond the variance accounted for by the GIA score. The results from the two mathematics achievement clusters were similar across all age groups except for 6 years of age in which the R^2 change was the highest for math problem solving and much lower for math calculation skills.

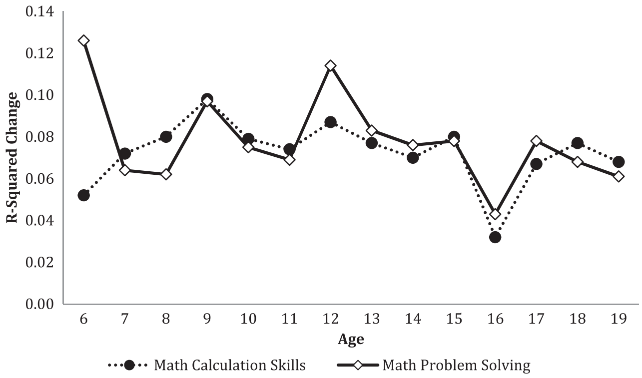

Figure 1. R-squared change by age for the reduced model with the general intellectual ability (GIA) cluster score compared with the full model with the GIA and broad CHC abilities. The R-squared change was significant for all data points

Broad CHC ability regression models

The individual standardized regression coefficients for each model, by age groups, are summarized visually in Figures 2–4. Each of the standardized regression coefficients indicates the proportion of a standard deviation unit

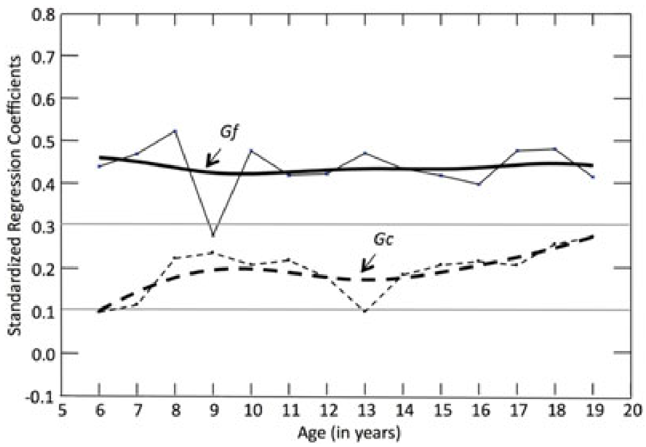

Figure 2. Math calculation skills and the relations to the fluid reasoning (Gf) and comprehension-knowledge (Gc) clusters [Colour figure can be viewed at wileyonlinelibrary.com]

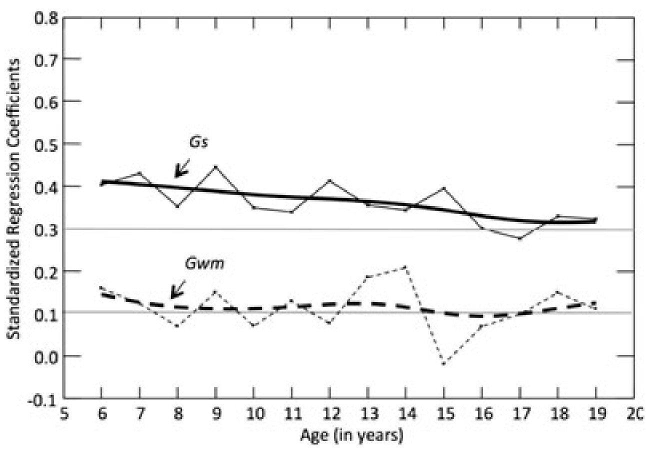

Figure 3. Math calculation skills and the relations to the processing speed (Gs) and short-term working memory (Gwm) clusters

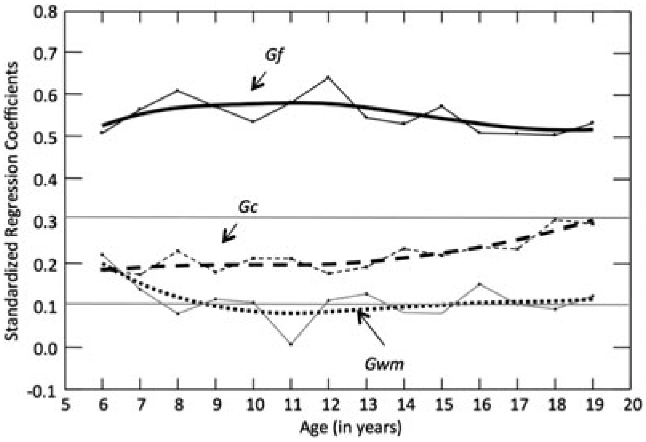

Figure 4. Math problem solving and the relations to the fluid reasoning (Gf), comprehension knowledge (Gc), and short-term working memory (Gwm) clusters

of change in the criterion variable as a function of one standard deviation change in the predictor variable. Smoothed curves were produced from the individual data points to represent the developmental trajectories of the relations between each broad CHC cluster and mathematics achievement throughout the school years. A distance weighted least squares smoother with a tension of .50 was used to produce relatively smooth, continuous curves when linking the individual data points. According to McGrew and Wrightson (1997), smoothed curves 'are more likely better estimates of the population parameters' (p.194). Figures include only the models that produced standardized regression coefficients consistently at or above .10 because standardized regression values below .10 were reported to have no practical significance (Evans et al., 2002; Floyd et al., 2003; McGrew, 1993; McGrew & Hessler, 1995). As seen in similar studies, each figure includes two parallel lines corresponding to standardized regression coefficients of .10 and .30 for ease of interpretation (Evans et al., 2002; Floyd et al., 2003; McGrew, 1993; McGrew & Hessler, 1995; McGrew & Knopik, 1993). The parallel lines included in the figures represent thresholds for interpretation established in previous research. Standardized regression coefficients at or above .10 to .30 represent a *moderate* relation between the broad CHC ability and mathematics achievement. Standardized regression coefficients above .30 represent a *strong* relation between the broad CHC ability and mathematics achievement (Evans et al., 2002).

Math calculation skills and CHC clusters

The median R^2value across age groups for the math calculation skills multiple regression models is R^2 = .59, with a range of R^2 = .54 to R^2 = .64 (see the supporting information for a list of all standardized regression coefficients and R^2 values). The CHC abilities that have consistent relations with math calculation skills are Gf, Gc, Gs, and Gwm, although the strength of the relations between math calculation skills and each of these broad CHC abilities varies from moderate to strong. Gf demonstrated the strongest relation with math calculation skills across the school years with standardized regression coefficients between .40 and .50 for all ages included in the analyses. The relation between Gf and math calculation skills was also highly consistent from the age of 6 to the age of 19 (Figure 2). Gc, however, demonstrated variability in its relation with math calculation skills —a weaker relation was observed during the early school years, followed by a gradual increase to a moderate to strong relation to math calculation skills in late adolescence. Gs also demonstrated a strong and consistent relation with math calculation skills throughout the school years (Figure 3).

The relations between the other three broad CHC abilities (Ga, Gv, and Glr) and mathematics achievement were of no practical significance with standardized regression coefficients consistently below .10 across all school ages. It should be noted that Ga, Gv, and Glr do display significant relations with math calculation skills when considered in isolation (see correlation matrices in Appendix F of the WJ IV technical manual). The results suggesting no significant relations in the current study are based on models controlling statistically for the concurrent contributions of all other CHC abilities.

Math problem solving and CHC clusters

The median R^2 value across age groups for the math problem solving multiple regression models is R^2 = .68, with a range of R^2 = .57 to R^2 = .73 (see the supporting information for a list of all standardized regression coefficients and R^2 values). Results of the relations between math problem solving and broad CHC clusters are similar to those seen between math calculation skills and the broad CHC clusters. Specifically, the CHC abilities with consistent relations with math problem solving are Gf, Gc, and Gwm, with the strength of the relations again varying from moderate to strong depending on the CHC ability being examined (Figure 4). Gs, however, was an exception to the comparison between math problem solving and math calculation skills, as it demonstrated no relation of practical significance to math problem solving, when controlling statistically for the contributions of other CHC abilities in the model. Gf, again, demonstrated the strongest relation across the school years. In this case, the standardized regression coefficients were greater than those seen for math calculation skills, with values ranging between .50 and .60 for all ages included in the analyses.

DISCUSSION

The purpose of this study involved two objectives. First, we sought to examine the relations between broad WJ IV COG clusters and the WJ IV mathematics achievement clusters beyond the contributions of *g* to academic achievement in mathematics. The results suggest that broad CHC abilities demonstrate significant relations above and beyond those accounted for by general intelligence (i.e., *g*). Specifically, Gf, Gc, and Gs appear to have significant relations with math calculation skills throughout the school years. In addition, Gf and Gc also demonstrate consistent relations with math problem solving throughout the school years. Second, we examined the developmental trajectories of the relations between the broad WJ IV COG clusters and the WJ IV mathematics achievement clusters across the school years. When considering the strength of the relations of Gf and Gc with both math calculation skills and math problem solving over time, the developmental trajectories appear to be relatively stable throughout the school years. The developmental trajectory of the relation between Gs and math calculation skills also follows a consistent pattern with regard to the strength of the relation over time. These findings, as well as variations in the relations and developmental trajectories observed for the other broad CHC abilities, are discussed in more detail in the following sections.

Relations between Gf and mathematics

Floyd et al. (2003) suggested three hypotheses as they considered the consistent, moderate to strong relations between fluid reasoning (Gf) and mathematics achievement, which continue to be supported by additional empirical evidence. First, they argue that quantitative reasoning abilities are a narrow Gf ability (Carroll, 1993; Schneider & McGrew, 2012). Second, Gf plays a significant role in the development of specific mathematical skills, such as rational number calculations (Seethaler, Fuchs, Star, & Bryant, 2011), whole-number line estimation (Namkung & Fuchs, 2016), and algebra (Singley & Bunge, 2014). Third, there is considerable evidence demonstrating the relationship between Gf and mathematics achievement (McGrew & Wendling, 2010). In this study, Gf was the strongest predictor of both math calculation skills and math problem solving across all age levels from the age of 6 to the age of 19. It appears that based on the latest versions of WJ COG and WJ ACH, Gf has remained as the most prominent indicator of the variation in mathematics achievement during the school years.

The strong relations between Gf and math problem solving observed in the current investigation, however, must be interpreted with caution, as the math problem solving cluster includes the Number Matrices test, a test that loaded at similar levels on the Gf and Gf-RQ (quantitative reasoning) factors in the WJ IV internal confirmatory factor analysis validity studies (McGrew et al., 2014). This suggests possible predictor-criterion contamination. The authors of the WJ IV noted a potential for predictor-criterion contamination when they excluded the Number Series test as a potential predictor in the development of the Scholastic Aptitude Cluster designed to predict the math problem solving cluster. Predictor-criterion contamination may be the reason that the R^2 values for math problem solving and Gf tend to be greater than the R^2 values for math calculation skills and Gf, with an average R^2 value of .69 for math problem solving and Gf, and an average R^2 value of .62 for math calculation skills and Gf from ages 6 to 19. Although a comprehensive examination of the criterion-predictor contamination effect is beyond the scope of this paper, the observed relation between Gf and math calculation skills, where criterion-predictor contamination is not an issue, suggests that Gf generally has a strong relation to math performance.

Relations between Gc and mathematics

Previous research has suggested a moderate to strong relation between Gc and mathematics achievement, particularly for later school years (Floyd et al., 2003; Taub et al., 2008; Williams, McCallum, & Reed, 1996). Floyd et al. (2003) suggested that the increasing strength of relations between Gc and mathematical reasoning abilities later on during the school years is due to the hierarchical relationship between learning basic mathematics skills, such as simple addition and multiplication, and learning more complex mathematics skills thereafter. The findings of this study regarding the relation between Gc and mathematics achievement are also aligned with the findings of Floyd et al. (2003), and therefore, this study provides further evidence supporting this developmental trend.

Relations between Gs and mathematics

Findings of the current investigation suggest that Gs has a strong and consistent relation with math calculation skills throughout the school years. However, Gs seems to have no practically significant relation to math problem solving, after controlling for relations between math problem solving and other CHC abilities. These findings are in agreement with those from Floyd et al. (2003). They indicated that despite Gs having a strong relation with math calculation skills throughout the school years, it has a moderate relation with math reasoning only until age 14. The reason for the decreased strength in the relation between Gs and mathematical reasoning may be that speed of processing is more influential during the early stages of academic skill acquisition (Fry & Hale, 2000; Necka, 1999; Weiler et al., 2000). However, as individuals develop more complex cognitive abilities and develop their academic skills, they may focus on more complex mathematical problems where fast processing of information may no longer be one of the primary skills needed to successfully complete a given task.

It should be noted that the strong relation between Gs and math calculation skills must be interpreted with caution because the Math Facts Fluency test, which provides information for one half of the math calculation skills cluster, demonstrated a consistent moderate loading ($r \approx .50$) on the Gs factor reported in the WJ IV internal validity studies (McGrew et al., 2014). This could be interpreted to suggest Gs predictor-criterion contamination. Conversely, this may reflect the real importance of cognitive processing speed in math fluency. Additional research is needed to investigate this issue.

Relations between Gwm and mathematics

Gwm demonstrated weak but steady relations to both math problem solving and math calculation skills. This is consistent with the findings from previous studies with the WJ III (Floyd et al., 2003), although the relations between Gwm and components of mathematics achievement (e.g., math calculation skills and math problem solving) are slightly weaker in the current study. Floyd and colleagues focused their analysis on two components of memory: working memory and the Gwm cluster. Here, we only examined the relations between Gwm and mathematics. Regardless, the finding that Gwm demonstrates significant relations to the two mathematics clusters is consistent with related research. The Gwm cluster in the WJ IV COG involves two tasks: verbal attention and numbers reversed. Verbal attention requires the maintaining of complex relations between unordered words and numbers to be retained (relational complexity) in working memory and significant attentional control (Heitz, Unsworth, & Engle, 2004). Numbers reversed is considered to be a measure of working memory capacity and attention (Hale, Hoeppner, & Fiorello, 2002). Bull and Scerif (2001) indicated that lack of inhibition and poor working memory were the primary contributing factors to poor performance in mathematics. Children between the ages of 5 and 11 with poor working memory appear to have difficulty maintaining attention (i.e., poor attention span), are highly distractible, and engaging in effective problem solving (Alloway, Gathercole, Kirkwood, & Elliott, 2009). Despite the contributions of individual tests being beyond the scope of this paper, working memory and the Gwm cluster appear to play a significant role predicting the acquisition of mathematics skills throughout the school years.

Non-significant relations with mathematics achievement

The broad CHC clusters, Ga, Gv, and Glr, did not demonstrate significant relations with math problem solving or math calculation skills throughout the school-age years. It appears that the bulk of the variance in mathematics achievement is explained by the other broad CHC abilities (i.e., Gf, Gc, Gs, and Gwm). It should be noted, however, that this does not imply that these three abilities do not contribute to

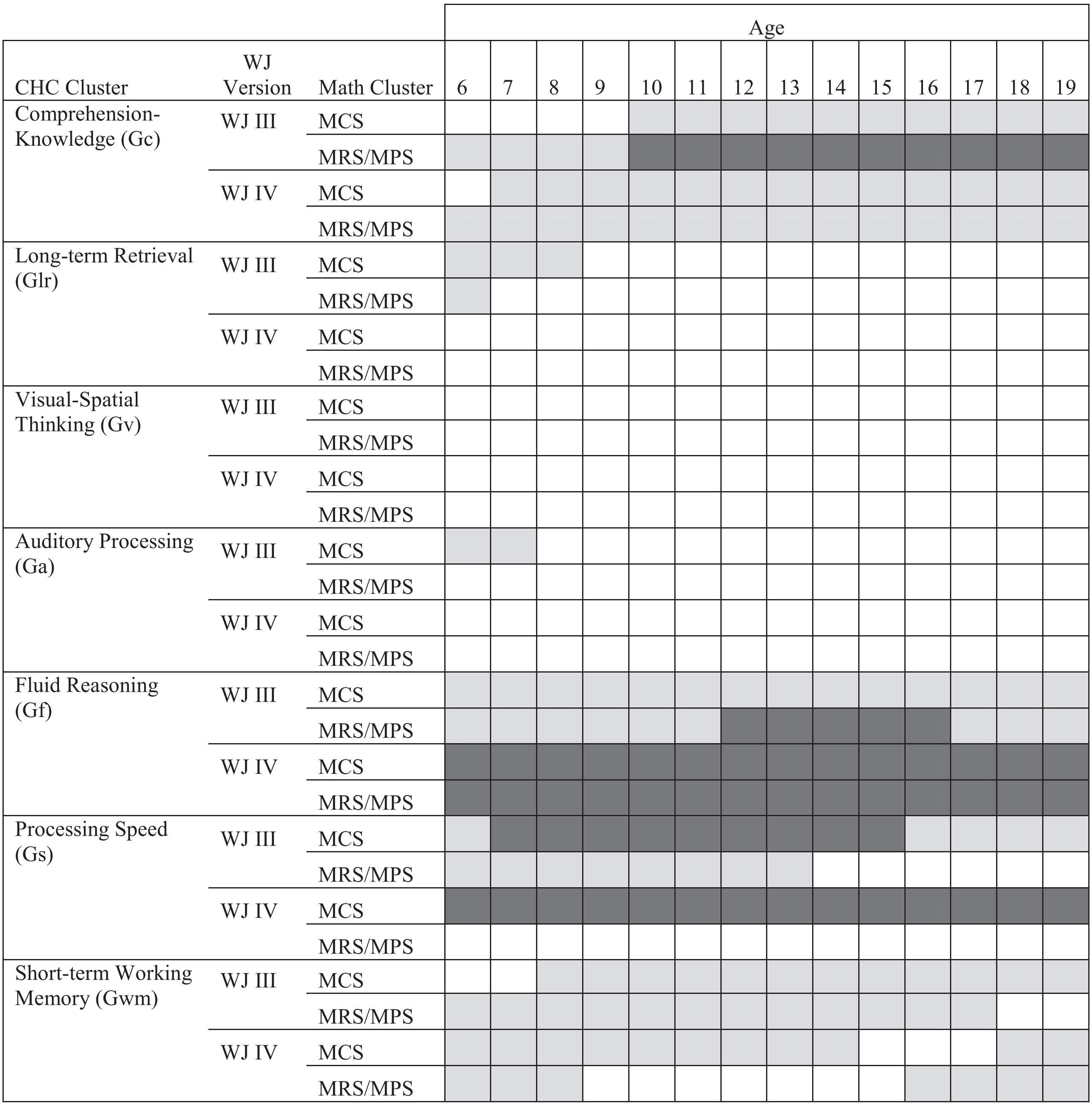

Figure 5. A comparison of the strength of relations between WJ broad CHC abilities and areas of mathematics achievement across the school age years. The white cells indicate no significant association; the shaded cells indicate a moderate association; the black cells indicate a strong association. MCS = math calculation skills; MRS = math reasoning skills (WJ III); MPS = math problem solving (WJ IV). The data for the WJ III are sourced from Floyd et al. (2003)

learning in general or the acquisition of mathematics skills specifically. The results herein simply show their contribution, *relative to* other cognitive ability clusters.

Limitations

The results presented herein should be considered within the context of a couple limitations. First, inferences regarding the relations between CHC abilities and academic achievement in mathematics were drawn from the measurement of these abilities by using the WJ IV battery. Thus, these results may not generalize to other measures of CHC abilities. Future research with other cognitive batteries that also measure broad CHC abilities may provide convergent evidence of these results. Second, the design of this study does not allow for causal links to be made between broad CHC abilities and components of mathematics achievement. However, the identified relations between cognitive abilities and mathematics achievement may serve as a foundation for future research that may want to test these relationships experimentally. This may provide a deeper understanding of the moderate or strong relations between cognitive broad CHC abilities and mathematics achievement in the current study.

CONCLUSION

There are clearly a number of cognitive abilities that contribute to academic achievement in mathematics. Moreover, there is variability over the course of the school years in how cognitive abilities contribute to academic achievement in mathematics. The results of this study suggest that previously established relations between CHC cognitive abilities and mathematics achievement are relatively consistent when comparing the results from previous studies with the WJ III to our results with the WJ IV (Figure 5). For example, although the strength of the relations change at certain age levels, many of the significant relations remained significant across measures (i.e., the WJ III and WJ IV). One of the noteworthy differences between the cognitive batteries is perhaps the relation between Gs and math problem solving, as this CHC ability had a significant relation with math problem solving from age 6 to 13 for the WJ III, but did not show a significant relation with math problem solving at any age level for the WJ IV. This difference could be explained by the increased strength of the relation between Gf and math problem solving in the early school years (i.e., moderate for the WJ III compared with strong for the WJ IV). Regardless, the general trend in the findings across batteries suggests a consistency in the construct representation of CHC abilities in both batteries.

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