Developmental Psychology

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Online First Publication, September 26, 2011. doi: 10.1037/a0025510

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0012-1649/11/$12.00 DOI: 10.1037/a0025510

Cognitive Predictors of Achievement Growth in Mathematics: A 5-Year Longitudinal Study

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The study's goal was to identify the beginning of 1st grade quantitative competencies that predict mathematics achievement start point and growth through 5th grade. Measures of number, counting, and arithmetic competencies were administered in early 1st grade and used to predict mathematics achievement through 5th ($n = 177$), while controlling for intelligence, working memory, and processing speed. Multilevel models revealed intelligence and processing speed, and the central executive component of working memory predicted achievement or achievement growth in mathematics and, as a contrast domain, word reading. The phonological loop was uniquely predictive of word reading and the visuospatial sketch pad of mathematics. Early fluency in processing and manipulating numerical set size and Arabic numerals, accurate use of sophisticated counting procedures for solving addition problems, and accuracy in making placements on a mathematical number line were uniquely predictive of mathematics achievement. Use of memory-based processes to solve addition problems predicted mathematics and reading achievement but in different ways. The results identify the early quantitative competencies that uniquely contribute to mathematics learning.

Keywords: longitudinal study, mathematics, mathematical cognition, reading, achievement growth

Each additional year of education improves employability and results in higher wages once employed (Ashenfelter & Krueger, 1994), with a particular premium for strong mathematical skills: Independent of reading competence, intelligence, and ethnic status, competence in arithmetic and basic algebra influence employability, wages, and on-the-job productivity (Rivera-Batiz, 1992). Entry into technical occupations requires an even deeper understanding of mathematics (Paglin & Rufolo, 1990). Clearly, the development of mathematical competence has individual benefits as well as benefits to the wider society (National Mathematics Advisory Panel, 2008), and yet we do not fully understand the mechanisms that influence children's mathematical learning or the sources of individual differences in this learning (Geary, 1994). We do know, however, that children who begin school behind their peers in their understanding of number, counting, and simple arithmetic are at high risk of staying behind throughout their schooling (Duncan et al., 2007), and in adulthood, they will have difficulties with many activities that are dependent on mathematical knowledge (Every Child a Chance Trust, 2009).

The development of effective strategies for improving the educational trajectory of these individuals is contingent on identifying areas of early quantitative knowledge that influence later mathematics achievement. Relevant longitudinal studies have tracked the relation between early mathematics achievement and later achievement (Duncan et al., 2007); early quantitative knowledge and later achievement (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008); and early cognitive abilities, such as working memory, and later achievement or later performance on specific quantitative tasks (Bull, Espy, & Wiebe, 2008; Krajewski & Schneider, 2009). None of the studies, however, have longitudinally tracked the contributions of early quantitative knowledge to growth in mathematics achievement while simultaneously controlling for the domain general cognitive abilities that are known to broadly influence academic learning, such as working memory and general intelligence (e.g., Gottfredson, 1997). Without such controls, it is difficult to identify the unique contributions of early quantitative competencies to subsequent mathematics learning.

The current study details the contributions of competence in number, counting, and arithmetic at the beginning of first grade to growth in basic mathematics achievement through fifth grade, while controlling for intelligence, working memory, and processing speed. As a further control, the early predictors of mathematics achievement were compared and contrasted with those that predict word reading achievement to determine if there are quantitative competencies that are unique to mathematics learning. The first section provides a brief overview of the domain general cognitive abilities that predict outcomes across academic domains, and the

Editor's Note. Katherine Magnuson served as the action editor for this article.—JSE

I acknowledge support from the Eunice Kennedy Shriver National Institute of Child Health and Human Development Grants R01 HD38283 and R37 HD045914. I thank Mary Hoard, Lara Nugent, Drew Bailey, Linda Coutts, Jennifer Byrd-Craven, Rachel Christensen, Caitlin Cole, Mike Coutts, Patricia Hoard, Rehab Mojid, Cy Nadler, Robyn Tenney, Jonathan Thacker, Leah Thomas, Jasmine Tilghman, Chatty Numtee, Erin Twellman, Katie Waller, and Jessica Wallace for help on various aspects of the project.

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second overviews the mathematical cognition domains covered and measures used in this study.

Domain General Cognitive Abilities

The domain general abilities that influence learning across many if not all academic areas include general intelligence, working memory, and processing speed (Carroll, 1993; Gottfredson, 1997). Measures of these competences are correlated but each assesses unique abilities.

Intelligence

Independent of the contributions of working memory and processing speed to performance on intelligence tests, general intelligence includes the ability to think logically and systematically (Embretson, 1995) and is the best individual predictor of achievement across academic domains, including mathematics (e.g., Deary, Strand, Smith, & Fernandes, 2007; Jensen, 1998; Stevenson, Parker, Wilkinson, Hegion, & Fish, 1976; Taub, Floyd, Keith, & McGrew, 2008; Walberg, 1984). As just one illustration, in a 5-year prospective study of more than 70,000 students, Deary et al. (2007) found that intelligence assessed at age 11 years explained nearly 60% of the variation on national mathematics tests at age 16 years. Despite the high heritability of intelligence and the shared genes contributing to the correlation between intelligence and mathematics achievement (Kovas, Harlaar, Petrill, & Plomin, 2005), findings such as these do not indicate educational interventions will not affect academic outcomes.

The relative contributions of common environmental factors, such as schooling, and heritable ones on educational outcomes vary across the distribution of intellectual ability; heritable contributions are strongest at the higher end of intellectual ability and common environmental ones at the lower end (W. Johnson, Deary, & Iacono, 2009). Aside from these issues, a substantial portion of individual differences in children's mathematics achievement cannot be explained by general intelligence.

Working Memory and Processing Speed

Working memory represents the ability to hold a mental representation in mind while simultaneously engaging in other mental processes. The core component is the central executive, which is expressed as attention-driven control of information represented in two systems (Baddeley, 1986; Baddeley & Hitch, 1974; Cowan, 1995). These are a language-based phonological loop (Baddeley, Gathercole, & Papagno, 1998) and a visuospatial sketch pad (Logie, 1995). Measures of general intelligence and working memory, especially the central executive, are moderately to highly correlated (e.g., Ackerman, Beier, & Boyle, 2005; Conway, Cowan, Bunting, Therriault, & Minkoff, 2002) but capture independent components of ability. Performance on both types of measures requires attentional and inhibitory control, but these mechanisms appear to be more important for tests of the central executive.

The relation between performance on measures of working memory and on mathematics achievement tests and specific mathematical cognition tasks (below) is well established (DeStefano & LeFevre, 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; McLean & Hitch, 1999; Swanson & Sachse-Lee, 2001). Whether assessed concurrently or 1 or more years earlier, the higher the capacity of the central executive the better the performance on measures of mathematics achievement and cognition (Bull et al., 2008; Mazzocco & Kover, 2007; Passolunghi, Vercelloni, & Schadee, 2007). The importance of the phonological loop and visuospatial sketch pad varies with the complexity and content of the mathematics being assessed. The phonological loop appears to be important for processes that involve the articulation of numbers, as in counting (Krajewski & Schneider, 2009), and may be related to arithmetic fact retrieval (Fuchs et al., 2006; Geary, 1993), whereas the visuospatial sketch pad appears to be involved in a broader number of mathematical domains (De Smedt et al., 2009; Geary, Saults, Liu, & Hoard, 2000; Swanson, Jerman, & Zheng, 2008).

The relation between working memory and processing speed is currently debated—specifically, whether individual differences in working memory are driven by more fundamental differences in speed of cognitive processing and decision making (Ackerman et al., 2005) or whether the attentional focus associated with the central executive speeds information processing (Engle, Tuholski, Laughlin, & Conway, 1999). Whatever the direction of the relation, processing speed itself has several subcomponents that appear to be independent of working memory (Carroll, 1993) and is sometimes found to be a better predictor of mathematics outcomes than is working memory (Bull & Johnston, 1997). A systematic assessment of the potential mechanisms contributing to individual differences in children's achievement and achievement growth requires measurement of both working memory and processing speed, as well as intelligence.

Mathematical Cognition

Studies of infants, preschoolers, and young children have identified a core suite of basic quantitative competencies that include an implicit and sometimes explicit understanding of numerical magnitude (Starkey, Spelke, & Gelman, 1990), the rules for counting (Gelman & Gallistel, 1978; Briars & Siegler, 1984), and how the addition and subtraction of one to several objects from a collection of objects increases or decreases quantity, respectively (Levine, Jordan, & Huttenlocher, 1992; Starkey, 1992; Wynn, 1992). This core suite of competencies appears to provide the foundation for the early learning of formal mathematics in school (Geary, 1994; Spelke, 2000). The tasks used in the current study assess a mix of these competencies and the early formal mathematical knowledge that is correlated with later mathematics achievement (Booth & Siegler, 2006; Geary, Bow-Thomas, & Yao, 1992; Ginsburg & Baroody, 2003; Jordan et al., 2009; Locuniak & Jordan, 2008; Passolunghi et al., 2007), although their independent contributions to this achievement above and beyond domain general abilities have not been fully established (Fuchs, Geary, Compton, Fuchs, & Hamlett, 2010).

Children's core numerical competencies include the ability to subitize—that is, quickly apprehend, without counting—the quantity of collections of up to four objects or a short sequence of actions (Starkey & Cooper, 1980; Wynn, Bloom, & Chiang, 2002), and to represent the relative approximate magnitude of larger collections of objects (Halberda & Feigenson, 2008; Xu & Spelke, 2000). The Number Sets Test is the first of two number assessments used in this study and was designed to assess children's fluency in combining sets of objects (e.g., ●●●) and Arabic numerals (e.g., 2) to match a target number (e.g., 5; Geary et al., 2007; Geary, Bailey, & Hoard, 2009). In theory, performance is dependent on several of children's early quantitative competencies, including their ability to subitize and map Arabic numerals onto representations of small quantities and to perform simple addition with small sets and Arabic numerals (Levine et al., 1992; Rousselle & Noël, 2007).

The second numerical assessment is Siegler and Opfer's (2003) number line task. Learning the linear, mathematical number line is educationally important in and of itself, and the pattern of children's placements on the line may reflect how they represent approximate large numerical magnitudes. Placements that conform to the natural logarithm of the numbers may reflect dependence on the core system that represents approximate magnitudes (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 1992), whereas linear placements indicate that the child is learning the mathematical number line. Whatever the underlying representational system, accuracy in making linear placements is predictive of later mathematics achievement (Booth & Siegler, 2006).

Gelman and Gallistel (1978) proposed that children's early counting is constrained by five implicit, potentially inherent principles, such as one-one correspondence (only one number word can be assigned to each counted object), whereas Briars and Siegler (1984) proposed that children's early counting knowledge is induced as they observe others' counting behavior. The counting task used in this study assesses Gelman and Gallistel's core one-one and order-irrelevance (items can be counted in any order) principles (Gelman & Meck, 1983); the latter also assesses Briars and Siegler's adjacency rule. Geary et al. (1992) found that first graders who were sensitive to violations of these principles used more sophisticated counting procedures to solve addition problems (below), and LeFevre et al. (2006) found that first graders with high mathematics achievement scores were more sensitive to these violations than were their lower achieving peers. Paradoxically, this heightened sensitivity sometimes resulted in the rejection of unusual counts that were correct, resulting in lower overall scores.

By the time children begin first grade, most of them have merged their implicit arithmetic knowledge with their counting abilities such that they can solve formal problems. First graders often use counting to solve such problems (Siegler & Shrager, 1984), sometimes using their fingers (finger counting strategy) and sometimes not using them (verbal counting strategy). The min and sum procedures are two common ways children count (Groen & Parkman, 1972). The min procedure involves stating the larger-valued addend and then counting a number of times equal to the value of the smaller addend. The sum procedure involves counting both addends starting from 1; the less common max procedure involves stating the smaller addend and counting the larger one. Counting results in the development of long-term memory representations of basic facts, which then support the use of memory-based processes (Siegler & Shrager, 1984); direct retrieval of arithmetic facts and decomposition (e.g., $6 + 7$ is solved by retrieving the answer to $6 + 6$, then adding 1). The focus of this study was the frequency with which memory-based processes were used for problem solving and the sophistication and accuracy of the counting procedures that were used when a memory-based process was not.

Current Study

The current study provides a unique picture of the foundational quantitative competencies needed by children at the beginning of first grade to be successful in learning mathematics through the elementary school years. The mathematics achievement test used in the study, Numerical Operations (Wechsler, 2001), primarily assesses computational arithmetic, including fraction and decimal problems in the grade ranges assessed, but is highly correlated with performance on mathematical reasoning tests that include more complex problems, including word problems, measurement, simple geometry, and statistics items (rs = .74 to .78; Wechsler, 1992). The results are thus likely to broadly apply to mathematics learning, making the early quantitative competencies that predict mathematics achievement and achievement growth above and beyond domain general abilities prime targets for early intervention.

Method

Participants

All kindergarten children from 12 elementary schools that serve children from a wide range of socioeconomic backgrounds were invited to participate. Parental consent and child assent were received for 37% (n = 311) of these children, and 287 of them completed the 1st year of testing. The mathematics curriculum when the children began the study was Investigations in Number, Data, and Space (Foresman, 1999), and they continued with this curriculum throughout the grades analyzed here.

Complete mathematics and reading achievement scores, first and fifth grade working memory assessments, and first grade mathematical cognition data were available for 177 children. These children composed the current sample. At the end of first grade, their mean IQ (M = 102, SD = 14) and standard scores for mathematics (M = 94, SD = 13) and reading (M = 106, SD = 14) were average with respect to national norms but higher than the respective scores of the 110 children who did not complete all of the assessments (ps < .05): IQ (M = 94, SD = 14), mathematics (M = 90, SD = 12), and reading (M = 104, SD = 17). There are, of course, ways to estimate missing values, assuming the data were lost randomly (e.g., Luke, 2004). Given the group differences in intelligence and achievement scores, this assumption has not been met. Despite this limitation, the retained sample is in the average range with respect to national norms and a substantial range of scores is maintained for these tests: intelligence (74 to 149), mathematics (minimum range of 3rd to 99th national percentile ranking per grade), and reading (minimum range of 3rd to 99th national percentile ranking per grade).

The mean ages were 74 (SD = 4) and 82 (SD = 4) months, respectively, at the times of the kindergarten achievement and first grade mathematical cognition assessments. Fifty-four percent of the sample were girls, and 74% were White; most of the remaining children were Black (14%, including mixed race), or Asian (5%); 7% of the sample identified their ethnicity as Hispanic.

Standardized Measures

Intelligence. The Vocabulary and Matrix Reasoning subtests of the Wechsler Abbreviated Scale of Intelligence (WASI) were used to estimate IQ as per manual instructions (Wechsler, 1999).

Achievement. Mathematics and reading achievement were assessed using the Numerical Operations and Word Reading subtests from the Wechsler Individual Achievement Test–II–Abbreviated (Wechsler, 2001), respectively. The easier Numerical Operations items assess number discrimination, rote counting, number production, and basic addition and subtraction. More difficult items include multidigit addition and subtraction, multiplication and division, and rational number problems solved with pencil and paper. The easier Word Reading items require matching and identifying letters, rhyming, beginning and ending sounds, and phoneme blending. The more difficult items assess accuracy of reading increasingly difficult words.

Mathematical Tasks

Counting knowledge. The child was first introduced to a puppet that was just learning how to count and therefore needed assistance to know if his counting was okay or not okay. During each of the 13 trials, a row of seven, nine, or 11 poker chips of alternating color (e.g., red, blue, red) were aligned behind a screen. The screen was then removed and the puppet counted the chips. The child was queried on the correctness of the counting (i.e., the ability to detect violations of counting rules; Briars & Siegler, 1984; Gelman & Meck, 1983), and the experimenter recorded whether the child stated the puppet's count was "OK" or "Not OK and wrong."

The four types of trials were correct, right-left, pseudo-error, and error. For correct trials, the chips were counted sequentially and correctly, from the child's left to the child's right. Right-left involved counting the chips sequentially and correctly but starting from the child's right. For pseudo-error trials, the chips were counted correctly from left to right, but first one color was counted, and then, returning to the left-hand side of the row, the count continued with the other color. For error trials, the chips were counted sequentially from left to right, but the first chip was counted twice. Each trial type occurred once for each array size (i.e., seven, nine, 11), with one additional pseudo-error count (for seven chips) as the last trial. Previous studies indicate that children's performance on pseudo-error (assesses order-irrelevance principle) and error (assesses one-one correspondence) trials are related to individual differences in mathematics achievement (Geary et al., 1992; LeFevre et al., 2006). Thus, the two variables from this task were the percentage of correct identifications of pseudo-error and error counts.

Addition strategy choices. Fourteen simple addition problems and six more complex problems were horizontally presented, one at a time, at the center of a computer monitor. The simple problems consisted of the integers 2 through 9, with the constraint that the same two integers (e.g., 2 + 2) were never used in the same problem; half of the problems summed to 10 or less, and the smaller valued addend appeared in the first position for half of the problems. The complex problems were six double-digit/single-digit problems (e.g., 16 + 7, 3 + 18).

The child was asked to solve each problem (without pencil and paper) as quickly as possible without making too many mistakes. It was emphasized that the child could use whatever strategy was easiest to get the answer and was instructed to speak the answer into a microphone that was interfaced with the computer which in turn recorded reaction time (RT) from onset of problem presentation to microphone activation. After solving each problem, the child was asked to describe how they got the answer. Based on the child's description and the experimenter's observations, the trial was classified based on problem solving strategy; the four most common were counting fingers, verbal counting, retrieval, and decomposition. Counting trials were further classified as min, sum, max, or other. The combination of experimenter observation and child reports immediately after each problem is solved has proven to be a useful measure of children's strategy choices (Geary, 1990; Siegler, 1987). The validity of this information is supported by findings showing that finger counting trials have the longest RTs, followed, respectively, by verbal counting, decomposition, and direct retrieval (e.g., Siegler, 1987).

Four summary variables, two for simple problems and two for complex ones, were created to represent children's competence in solving addition problems. The first variable represented the extent to which memory-based processes were used in problem solving and was the total number of problems solved correctly using direct retrieval or decomposition. The second, procedural competence variable was coded such that high scores represented frequent and accurate use of the min procedure, whether or not they used their fingers, and low scores frequent counting errors: [(2 × frequency of min counts) + (frequency of sum counts) – (total frequency of counting errors)]. For simple addition, 19% of the problems were correctly solved using a memory-based process and most of these (65%) involved direct retrieval, and therefore, the variable is termed simple addition retrieval. For complex addition, 7% of the problems were correctly solved using a memory-based process, and most of these (69%) involved decomposition (e.g., 17 + 6 = 17 + 3 = 20 + 3 = 23), hereafter referred to as complex addition decomposition.

Number sets. Two types of stimuli were used: objects (e.g., stars) in a 0.5-in. (1.27-cm) square and an Arabic numeral (18-point font) in a 0.5-in. square. Stimuli were joined in domino-like rectangles with different combinations of objects and numerals. These dominos were presented in lines of 5 across a page. The last two lines of the page showed three 3-square dominos. Target sums (5 or 9) were shown in large font at the top the page. On each page, 18 items matched the target, 12 were larger than the target, six were smaller than the target, and six contained "0" or an empty square.

The tester began by explaining two items matching a target sum of 4, then used the target sum of 3 for practice. The measure was then administered. The child was told to move across each line of the page, from left to right, without skipping any; to "circle any groups that can be put together to make the top number, 5 (9)"; and to "work as fast as you can without making many mistakes." The child had 60 s per page for the target 5 and 90 s per page for the target 9. Time limits were chosen to avoid ceiling effects and to assess fluent recognition and manipulation of quantities. Performance was consistent across target number and item content (e.g., whether the rectangle included Arabic numerals or shapes) and thus combined to create an overall frequency of hits ($\alpha = .88$), correct rejections ($\alpha = .85$), misses ($\alpha = .70$), and false alarms ($\alpha = .90$; Geary et al., 2007). Using signal-detection methods, Geary et al. (2009) found that a sensitivity measure, d' (z scores for hits – z scores for false alarms; MacMillan, 2002) was predictive of mathematics but not reading achievement above and beyond the influence of domain general abilities. This measure was used in the current analyses.

Number line estimation. A series of twenty-four 25-cm number lines containing a blank line with two endpoints (0 and 100) was presented, one at a time, to the child with a target number (e.g., 45) in a large font printed above the line. The child's task was to mark the line where the target number should lie (for a detailed description, see Siegler & Booth, 2004). Siegler and Opfer (2003) used group-level median placements fitted to linear and log models to make inferences about the modal numerical representation children were using to make the placements, and for individual difference analyses, they used an accuracy measure. Accuracy is defined as the absolute difference between the child's placement and the correct position of the number. For the number 45, placements of 35 and 55 each produce difference scores of 10. The overall score is the mean of these differences across trials.

Other potential individual differences measures include the frequency with which children make placements consistent with a linear representation of the line or placements that conform to the natural log of the numbers (Geary et al., 2007). To determine the best measure of children's understanding of the linear number line, first grade Numerical Operations scores were correlated (using data from all available children, $n = 287$) with absolute number line error, with the percentage of trials consistent with use of a linear representation and degree of error for these trials and the percentage of trials consistent with use of a log representation and degree of error for these trials. The best single predictor was absolute number line error, $r(285) = -.46$; lower degree of error is associated with higher Numerical Operations scores. Absolute error scores were then simultaneously regressed on the percentage of linear and log trials and error rates. The degree of absolute error increased with increases in the percentage of log trials, $\beta = .40$, $t(278) = 7.45$; degree of error on log trials, $\beta = .64$, $t(278) = 40.05$; and degree of error on linear trials, $\beta = .17$, $t(278) = 11.09$, $R^2 = .96$. The analyses indicate the absolute difference variable provides a good summary measure of the extent to which children have learned the linear, mathematical number line and thus was used in the current analyses.

Working Memory and Processing Speed

The Working Memory Test Battery for Children (WMTB–C; Pickering & Gathercole, 2001) consists of nine subtests that assess the central executive, phonological loop, and visuospatial sketchpad. All of the subtests have six items at each span level. Across subtests, the span levels range from one to six to one to nine. Passing four items at one level moves the child to the next. At each span level, the number of items (e.g., words) to be remembered is increased by one. Failing three items at one span level terminates the subtest. Working memory spans for the central executive, phonological loop, and visuospatial sketch pad are the mean span scores for the corresponding subtests. The means of the first and fifth grade span scores were used in these analyses, as these represent an estimate of individual differences in working memory across the grades assessed here; first and fifth grade scores were significantly correlated for the central executive ($r = .58$), phonological loop ($r = 63$), and visuospatial sketch pad ($r = .48$; $ps < .0001$).

Central executive. The central executive is assessed using three dual-task subtests. Listening Recall requires the child to determine if a sentence is true or false and then recall the last word in a series of sentences. Counting Recall requires the child to count a set of four, five, six, or seven dots on a card and then to recall the number of counted dots at the end of a series of cards. Backward Digit Recall is a standard format backward digit span.

Phonological loop. Digit Recall, Word List Recall, and Nonword List Recall are standard span tasks with differing content stimuli; the child's task is to repeat words spoken by the experimenter in the same order as presented. In the Word List Matching task, a series of words, beginning with two words and adding one word at each successive level, is presented to the child. The same words, but possibly in a different order, are then presented again, and the child's task is to determine if the second list is in the same or different order than the first list.

Visuospatial sketch pad. Block Recall is another span task, but the stimuli consist of a board with nine raised blocks in what appears to the child as a "random" arrangement. The blocks have numbers on one side that can only be seen from the experimenter's perspective. The experimenter taps a block (or series of blocks), and the child's task is to duplicate the tapping in the same order as presented by the experimenter. In the Mazes Memory task, the child is presented a maze with more than one solution, and a picture of an identical maze with a path drawn for one solution. The picture is removed and the child's task is to duplicate the path in the response booklet. At each level, the mazes get larger by one wall.

Processing speed. Two rapid automatized naming (RAN) tasks assessed processing speed (Denckla & Rudel, 1976; Mazzocco & Myers, 2003). Although the RAN does not assess all of the multiple components of processing speed (Carroll, 1993), it does assess the educationally relevant facility of serially encoding arrays of visual stimuli as with words and multidigit Arabic numerals (Wolf, Bowers, & Biddle, 2000). The child is presented with five letters or numbers to first determine if the child can read the stimuli correctly. After these practice items, the child is presented with a 5×10 matrix of incidences of these same letters or numbers and is asked to name them as quickly as possible without making mistakes. RT is measured via a stopwatch, and errors and reversals for the letters b and d and p and q are recorded. Errors and reversals were too infrequent for meaningful analysis, and thus, only RTs were used. RTs for letter and number naming were highly correlated in each grade ($rs = .74$ to $.81$, $p < .0001$). The mean across-grade RTs for combined letter and number naming RTs were also highly correlated ($r = .88$, $p < .0001$).

Despite the very high correlation between speed of number and letter naming, it is possible that speed of retrieving domain-specific content is more important for performance on achievement tests than speed of retrieval more generally. To assess this possibility, a series of preliminary analyses compared and contrasted the predictive utility of combined letter and number naming RAN RT, RAN RT for number naming, and RAN RT for letter naming. The results indicated that number naming RT resulted in better fitting models for predicting mathematics achievement and letter naming for word reading achievement. Thus, mean across-grade number naming RT was used for mathematics and letter naming RT for reading.

Procedure

Assessments. Achievement tests were administered every spring beginning in kindergarten and the WASI (Wechsler, 1999) in the spring of first grade. The mathematical cognition tasks were administered in the fall of first grade; the RAN was administered in the fall from first to fourth grade. The majority of children were tested in a quiet location at their school site and occasionally on the university campus or in a mobile testing van. Testing in the van occurred for children who had moved out of the school district and for administration of the WMTB–C (e.g., on the weekend or after school). The mean ages at the times of the first and fifth grade WMTB–C assessments were 84 (SD = 6) and 128 (SD = 5) months, respectively. The mathematical cognition and achievement assessments required between 20 and 40 min and the WMTB–C about 60 min. Table 1 shows the timing of the assessments; the WMTB-C was administered either the fall or spring semester in first and fifth grades.

Analyses. Kindergarten to fifth grade raw scores from the Numerical Operations and Word Reading tests were analyzed using multilevel modeling, specifically, PROC MIXED (SAS Institute, 2004). Linear and quadratic (grade^2) slopes for grade and intercept values were random effects, and the predictors were the above described measures, as summarized in Table 2; correlations among these predictors are shown in the Appendix. All of the predictor variables were standardized (M = 0, SD = 1), and the number line error scores and RAN RTs were reversed so that higher values indicate better performance. The intercept values estimate the mean raw scores in kindergarten (coded 0), and the grade variables represent rate of change from kindergarten to fifth grade; first, second, third, fourth, and fifth grades were coded 1, 2, 3, 4, and 5, respectively.

The first step was to specify a model using only the domain-general predictors and their interactions with the linear and quadratic grade (slope) variables. The corresponding negative log likelihood, Bayesian information criterion (BIC), and t tests for the maximum likelihood estimates for individual predictors were used in model selection (for accessible review, Luke, 2004; Raftery, 1995). Differences in the negative log likelihood values for nested models can be evaluated using a chi-square, with the change in the number of predictors as the degrees of freedom. BIC values can be derived from the negative log likelihood, specifically, with a correction factor that evaluates model fit in terms of the overall number of parameters. The BIC favor parsimonious models.

The second step was to drop all quadratic slope effects with nonsignificant t tests and evaluate change in overall model fit using the chi-square and change (Δ) in BIC. A nonsignificant chi-square indicates the trimmed model fit the data as well as the model with more parameters, and a lower BIC indicates better overall fit, given the number of parameters. The ΔBIC is not evaluated using p values, but differences >10 are considered very strong evidence for the model with the smaller BIC, and differences >3 are considered positive evidence (Raftery, 1995). The odds that the lower valued BIC provides better estimates for the data can be estimated by $e^{.5(\Delta BIC)}$, such that a ΔBIC of 10 yields 150:1 odds that the lower valued BIC provides better estimates. The third and fourth steps involved dropping nonsignificant linear slope effects and nonsignificant predictors, respectively. The final, trimmed, domain-general model was then used as the start point for evaluating the mathematical cognition predictors. Specifically, the eight predictors and their linear and quadratic slope effects were added to the trimmed model, and the stepwise process was repeated.

Table 1
Timing of Assessments

Grade	Assessment
Kindergarten	
Spring	Achievement
First grade	
Fall	Mathematical Cognition, RAN, Working Memory
Spring	IQ, Achievement, Working Memory
Second grade	
Fall	RAN
Spring	Achievement
Third grade	
Fall	RAN
Spring	Achievement
Fourth grade	
Fall	RAN
Spring	Achievement
Fifth grade	
Fall	Working Memory
Spring	Achievement, Working Memory

Note. Achievement = Numerical Operations and Word Reading achievement tests from Wechsler Individual Achievement Test–II–Abbreviated (Wechsler, 2001); Math Cognition = the counting knowledge, simple and complex addition strategy choice tasks, the number sets measure, and the number line task; RAN = rapid automatized naming for numbers and letters; Working Memory = Working Memory Test Battery for Children (Pickering & Gathercole, 2001); IQ = composite based on Vocabulary and Matrix Reasoning subtests of the Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999).

Results

The results are presented in three sections. The first provides descriptive information on change in mathematics and reading achievement and in mean working memory span scores across grades. The second and third respective sections describe the mixed models for mathematics and reading achievement.

Academic Achievement and Working Memory

Standard scores (M = 100, SD = 15; Wechsler, 2001) for the Numerical Operations and Word Reading tests are shown in Table 3, and the corresponding raw scores are shown in the rows below these. With the possible exception of reading scores in kindergarten, the means and standard deviations are consistent with national norms for these tests (Wechsler, 2001); that is, the mean mathematics and reading achievement of the sample and variation in achievement are at about the national average. The average grade-by-grade improvement in raw Numerical Operations scores indicates that the children correctly solved 3.6 more problems from one grade to the next. The corresponding mean for Word Reading is 11.

Mean working memory span scores are also consistent with test norms (Pickering & Gathercole, 2001). Span increased from first to fifth grade for the phonological loop (M = 3.3, SD = 0.6 for

Table 2
Cognitive Predictors of Achievement

Variable	Task	Coding
	Domain general	
Intelligence	WASI	Standard scores from national norms
Central executive	WMTB–C	The mean of span scores across first and fifth grade
Phonological loop	WMTB–C	The mean of span scores across first and fifth grade
Visuospatial sketch pad	WMTB–C	The mean of span scores across first and fifth grade
Processing speed	RAN	The mean of number naming (mathematics) or letter naming (reading) RTs across first to fourth grade, inclusive
	Mathematical cognition	
Counting error	Counting knowledge	Number of counting errors detected as errors
Counting pseudo	Counting knowledge	Number of pseudo counting errors detected as correct counts
Simple addition retrieval	Addition strategy choice	Frequency of problems correctly solved with direct retrieval
Simple addition procedural competence	Addition strategy choice	Sophistication and accuracy of using counting procedures
Complex addition decomposition	Addition strategy choice	Frequency of problems correctly solved with decomposition
Complex addition procedural competence	Addition strategy choice	Sophistication and accuracy of using counting procedures
Number line	Number line	Mean of absolute difference between correct placement and child's actual placement
d′	Number sets	*z* score for hits − *z* score for false alarms

Note. WASI = Wechsler Abbreviated Scale of Intelligence (Wechsler, 1999); WMTB–C = Working Memory Test Battery for Children (Pickering & Gathercole, 2001); RAN = Rapid Automatized Naming (Denckla & Rudel, 1976; Mazzocco & Myers, 2003); RT = reaction time.

first grade; $M = 4.0$, $SD = 0.6$ for fifth grade; $d = 1.17$), visuospatial sketch pad ($M = 2.8$, $SD = 0.6$; $M = 4.2$, $SD = 0.8$; $d = 2.0$), and central executive ($M = 2.1$, $SD = 0.5$; $M = 3.0$, $SD = 0.6$; $d = 1.64$; $ps < .0001$).

Mathematics

Fit indexes for the multilevel models are shown in Table 4. Considering first the domain general predictors, none of the *t* tests for the quadratic slope effects were significant in the full model, and, thus, all of them were dropped, yielding a model with only linear grade effects (Model 2). The result was a nonsignificant drop in the chi-square value ($p > .10$) and a strong improvement in model fit based on the ΔBIC. The same pattern emerged when nonsignificant linear grade effects were dropped (Model 3) and when the nonsignificant effect for the phonological loop was dropped (Model 4). The difference in negative log likelihood values comparing Model 4 to Model 1 was not significant, $\chi^2(9) = 4.2$, $p > .10$, and the ΔBIC was substantial (–42.4), favoring Model 4. Moreover, all of the parameter estimates, except that for the central executive, were significant, as shown in the second and third columns of Table 5; the nonsignificant central executive variable was retained because of the significant central executive on slope effect.

The 7.8 intercept value for the final domain general model is the estimated Numerical Operations raw score in the spring of kindergarten when all other effects in the model are set at 0 and is consistent with the raw mean kindergarten score of 7.9 (see Table 3). The linear and quadratic slope effects estimate grade-by-grade change in these scores (see Table 5). The predictors on intercept effects estimate a constant grade-to-grade benefit of being above average on this variable or disadvantage if the coefficient is negative. As an example, the estimate for the visuospatial sketch pad indicates that at each grade level being 1 *SD* above average results in a 0.49 increase in raw Numerical Operations scores above and beyond the grade-level increases of children with average visuospatial scores. A significant predictor on slope effect

Table 3
Standard and Raw Scores for Mathematics and Reading Achievement

Score type	Kindergarten *M*	Kindergarten *SD*	First *M*	First *SD*	Second *M*	Second *SD*	Third *M*	Third *SD*	Fourth *M*	Fourth *SD*	Fifth *M*	Fifth *SD*
Numerical Operations												
Standard	103	13	94	13	96	16	95	14	96	16	99	17
Raw	7.9	1.9	10.4	2.0	14.1	3.1	17.4	4.0	21.8	4.5	26.1	5.4
Word Reading												
Standard	113	14	108	16	106	14	104	12	104	12	104	11
Raw	51	13	72	16	87	14	95	11	101	10	106	9

Table 4
Fit Indexes for Multilevel Models

Model	Negative log likelihood	χ^2	Parameters	p	BIC	ΔBIC
Domain general predictors: Numerical Operations						
1: Full	4,931.3	—	24	—	5,055.5	—
2: Drop non-significant quadratic slope effects	4,934.2	2.9	19	>.10	5,032.5	−23.0
3: Drop non-significant linear slope effects	4,935.5	1.3	16	>.10	5,018.3	−14.2
4: Drop PL	4,935.5	0	15	>.10	5,013.1	−5.2
Trimmed domain general and mathematical cognition predictors: Numerical Operations						
1: Model 4 above and mathematical cognition predictors	4,811.6	—	39	—	5,013.5	—
2: Drop non-significant quadratic slope effects	4,816.8	5.2	31	>.10	4,977.3	−36.2
3: Drop non-significant linear slope effects	4,824.6	7.8	25	>.10	4,954.0	−23.3
4: Drop simple addition procedural and counting error variables	4,825.9	1.3	23	>.10	4,944.9	−9.1
5: Drop IQ on slope effect	4,826.9	1.0	22	>.10	4,940.8	−4.1
6: Add number line on slope effect	4,822.9	4.0	23	<.05	4,941.9	1.1
Domain general predictors: Word Reading						
1: Full	6,980.2	—	24	—	7,109.6	—
2: Drop non-significant quadratic slope effects	6,980.8	<1	20	>.10	7,089.4	−20.2
3: Drop non-significant linear slope effects	6,981.8	1.0	18	>.10	7,080.2	−9.2
4: Drop VSSP	6,982.3	<1	17	>.10	7,075.5	−4.7
Trimmed domain general and mathematical cognition predictors: Word Reading						
1: Model 4 above and mathematical cognition predictors	6,951.2	—	41	—	7,168.6	—
2: Drop non-significant quadratic slope effects	6,956.9	5.7	33	>.10	7,132.9	−35.7
3: Drop non-significant linear slope effects	6,962.4	5.5	25	>.10	7,097.0	−35.9
4: Drop all but one mathematical cognition variable	6,972.7	10.3	18	>.10	7,071.0	−26.0

Note. BIC = Bayesian Information Criterion; PL = phonological loop; VSSP = visuospatial sketch pad. Em dashes indicate that no value can be estimated for these parameters for the full model.

indicates the magnitude of the benefit varies across grades. The benefit of being 1 *SD* above average on the central executive does not result in an advantage in first grade, but does result in a 1.75-point advantage in fifth grade, holding other factors constant. To put this in perspective, the latter value is about half the mean across-grade change in raw scores (i.e., 3.6), or about half a grade-level difference.

The second set of values in Table 4 results from the inclusion of the mathematical cognition predictors with the final domain general predictors. Again, none of the quadratic slope effects were significant, and thus, all of them were dropped, resulting in a nonsignificant change in model fit, $\chi^2(8) = 5.2$, $p > .10$, and a lower BIC value (Model 2). In the resulting model, two of the eight mathematical cognition predictor on linear slope effects were significant (ps $< .05$; simple addition retrieval and complex addition decomposition). The six remaining effects were dropped and the model re-estimated (Model 3). The resulting chi-square was not significant, and the BIC value improved. Dropping the procedural competence variable for simple addition and the counting error variable from the counting knowledge task produced Model 4, which, in turn, resulted in a nonsignificant t value ($p > .25$) for the intelligence on slope effect. The latter was dropped, yielding a nonsignificant change in model fit, $\chi^2(1) = 1.0$, $p > .10$, and a lower BIC value (Model 5). To ensure that the inclusion of the intelligence on slope effect in Models 2 to 4 did not mask any mathematical cognition predictor on slope effects, the parameters dropped in Model 3 (i.e., the nonsignificant mathematical cognition on linear slope effects) were added one at a time and model fit was reevaluated. The resulting Model 6 yielded a significant number line on slope effect, $\chi^2(1) = 4.0$, $p < .05$, and little change in the BIC value.

To determine if mean working memory scores (i.e., mean for first and fifth grade) were better predictors than first and fifth grade scores as independent predictors or first grade scores alone, Model 6 was rerun twice. The first was with separate estimates for the first and fifth grade visuospatial sketch pad and central executive predictors and the central executive on slope effects replacing their across-grade means. The second model was estimated with first grade scores replacing mean scores. The first model resulted in a nonsignificant change in the negative log likelihood value, $\chi^2(3) = 6.2$, $p > .10$, and both models produced higher (worse fit) BIC values (ΔBICs = 14.5 and 6.0, respectively). Mean working memory scores were thus retained in the final model.

The final full model in columns four and five in Table 5 shows that, holding the domain general factors constant, children who had above average scores on the d' variable and the variables representing use of decomposition and efficient counting procedures for solving complex addition problems at the beginning of first grade had a consistent grade-to-grade advantage on the Numerical Operations test. Frequent detection of pseudo counting errors, in contrast, was associated with slightly lower Numerical Operations scores. The number line on slope and addition retrieval on slope (for simple problems) effects suggest that the benefits to early advantages in these competences increase with successive grades. As an example, being 1 *SD* above average on the number line task at the beginning of first grade results in a nonsignificant

Table 5
Maximum Likelihood Estimates (and Standard Errors) for Numerical Operation and Word Reading

Effect	Numerical Operations				Word Reading			
	Final domain general model	t	Final full model	t	Final domain general model	t	Final full model	t
Intercept	7.81 (0.14)	57.83	7.81 (0.13)	59.20	52.10 (0.72)	72.08	52.10 (0.72)	72.53
Grade (linear slope)	2.61 (0.15)	17.80	2.61 (0.15)	16.92	20.54 (0.46)	45.08	20.54 (0.46)	45.08
Grade^2 (quadratic slope)	0.21 (0.03)	6.84	0.21 (0.03)	6.57	−1.99 (0.07)	−27.01	−1.99 (0.07)	−27.01
Intelligence on intercept	0.38 (0.13)	2.87	0.24 (0.11)	2.23	4.01 (0.77)	5.23	3.63 (0.78)	4.68
Processing speed on intercept	0.35 (0.13)	2.78	0.26 (0.10)	2.48	4.36 (0.81)	5.40	4.19 (0.80)	5.22
Phonological loop on intercept	—	—	—	—	1.62 (0.54)	3.01	1.82 (0.53)	3.45
Visuospatial sketch pad on intercept	0.49 (0.13)	3.71	0.26 (0.11)	2.31	—	—	—	—
Central executive on intercept	−0.05 (0.16)	<1	−0.29 (0.14)	−2.03	3.43 (0.87)	3.93	3.19 (0.87)	3.68
Intelligence on slope	0.16 (0.06)	2.43	—	—	−0.36 (0.16)	−2.30	−0.36 (0.16)	−2.30
Central executive on slope	0.40 (0.06)	6.18	0.34 (0.06)	5.53	−0.42 (0.17)	−2.45	−0.42 (0.17)	−2.45
Processing speed on slope	—	—	—	—	1.36 (0.46)	2.93	1.36 (0.46)	2.93
Processing speed on slope^2	—	—	—	—	−0.34 (0.07)	−4.67	−0.34 (0.07)	−4.67
Number line on intercept	—	—	0.14 (0.14)	<1	—	—	—	—
d' on intercept	—	—	0.47 (0.14)	3.41	—	—	—	—
Addition retrieval on intercept	—	—	−0.16 (0.16)	−1.04	—	—	1.40 (0.44)	3.16
Addition decomposition on intercept	—	—	0.67 (0.15)	4.50	—	—	—	—
Complex addition procedural on intercept	—	—	0.36 (0.12)	3.15	—	—	—	—
Pseudo counting on intercept	—	—	−0.19 (0.09)	−2.00	—	—	—	—
Number line on slope	—	—	0.13 (0.07)	2.04	—	—	—	—
Addition retrieval on slope	—	—	0.38 (0.08)	4.78	—	—	—	—
Addition decomposition on slope	—	—	−0.22 (0.07)	−2.99	—	—	—	—

Note. For tabled estimates, $t < 1.96$, $p < .05$; $t < 2.58$, $p < .01$; $t < 3.29$, $p < .001$. For Numerical Operations, domain general variances for intercept (0.00), grade (0.95, $SE = .41$, $z = 2.34$, $p < .01$), and quadratic grade (0.06, $SE = .018$, $z = 3.48$, $p < .001$); full model variances for intercept (0.00), grade (1.49, $SE = .43$, $z = 3.44$, $p < .001$), and quadratic grade (0.08, $SE = .02$, $z = 4.14$, $p < .001$). For Word Reading, domain general variances for intercept (72.9, $SE = 9.9$, $z = 7.35$, $p < .001$), grade (19.4, $SE = 4.0$, $z = 4.78$, $p < .001$), and quadratic grade (0.32, $SE = .11$, $z = 2.91$, $p < .01$); full model variances for intercept (71.7, $SE = 9.8$, $z = 7.33$, $p < .001$), grade (19.4, $SE = 4.0$, $z = 4.78$, $p < .001$), and quadratic grade (0.32, $SE = .11$, $z = 2.91$, $p < .001$). Em dashes indicate that no value can be estimated for these parameters for the full model.

0.14 raw-score advantage on the Numerical Operations test at the end of first grade, but this improves to a 0.79-point advantage by the end of fifth grade. The significant effect for decomposition combined with the negative coefficient for the decomposition on slope effect indicate the relative benefits of being able to use decomposition to solve complex addition problems in first grade fade across subsequent grades.

Reading

Use of these procedures revealed the same set of domain general predictors for Word Reading, as found for Numerical Operations, with one exception: The visuospatial sketch pad was replaced by the phonological loop in the set of domain general predictors. Again, replacing phonological loop and central executive means with separate variables for first and fifth grade or first grade alone resulted in a worsening of model fits (ΔBICs > 12). The corresponding fit indexes are shown in the third section of Table 4, and the parameter estimates for the final domain general model are shown in the sixth and seventh columns in Table 5. Higher intelligence and higher scores on the phonological loop and central executive components of working memory, as well as faster speed of articulating letters, all independently contributed to spring of kindergarten Word Reading scores. The importance of intelligence and the central executive declined linearly across grades, whereas the importance of processing speed increased in early grades and then declined. For the mathematical cognition predictors of Word Reading, none of the quadratic (Model 2) or linear grade (Model 3) effects were significant (see the bottom section of Table 4). Only one mathematical cognition variable was retainable in these analyses (Model 4), as shown in the final two columns of Table 5, specifically, simple addition retrieval. More frequent use of fact retrieval to correctly solve addition problems was predictive of early Word Reading scores.

Discussion

The results demonstrate that individual differences in mathematics achievement and achievement growth are driven, in part, by a combination of domain general abilities that affect learning in many academic domains as well as by early quantitative competencies that may be unique to learning mathematics. The implications are addressed in terms of our understanding of the relation between domain-general abilities and academic achievement in general and then for mathematics achievement in particular.

Domain General Abilities

The results for both mathematics and reading are consistent with many other studies that have shown the utility of intelligence tests for predicting academic achievement (Deary et al., 2007; Jensen, 1998; Walberg, 1984). The importance of intelligence for performance on the Number Operations test increased across grades but decreased for Word Reading. This pattern is likely due to the quickly increasing difficulty of the items on the former test and the relatively simple items, combined with greater reading fluency across grades, for the latter test.

The current findings add to the domain general literature by demonstrating that working memory and processing speed contribute to these individual differences above and beyond the contributions of intelligence. Although performance on working memory and intelligence tests are often highly correlated (Conway et al., 2002; Kyllonen & Christal, 1990), potentially because of the attentional and inhibitory demands of these tests (Engle et al., 1999), the independent effects found here suggest they are tapping some nonoverlapping abilities. The design of the current study does not allow for a determination of what these abilities might be (see Geary, 2005) but does support their existence, following Carroll (1993) and Embretson (1995) and contra proposals that intelligence and working memory are one in the same (e.g., Kyllonen & Christal, 1990).

Of the components of working memory, the central executive, was an important predictor of both mathematics and reading achievement, confirming previous studies (Bull et al., 2008; Cormier & Dea, 1999; Geary, Hoard, Byrd-Craven, & Desoto, 2004; Swanson et al., 2008). The unique contribution here is found with the across grade changes in the importance of the central executive for performance on these achievement tests, as was found for intelligence. The easier items on the Numerical Operations test do not appear to require extensive engagement of the central executive, but with successive grades and more difficult test items, the central executive emerges as an important contributor to individual differences in mathematics achievement. The opposite pattern emerged for Word Reading. Here, the central executive is important in early grades, when most children are learning the basics of word decoding (Bradley & Bryant, 1983; Wagner & Torgesen, 1987), but its importance declines for later items that likely tap automatic retrieval of word names during the act of reading. Of course, the central executive (and intelligence) may still contribute to individual differences in other aspects of reading, especially comprehension (Stevenson et al., 1976; Swanson & Ashbaker, 2000). Overall, the central executive appears to contribute to individual differences on more complex and unfamiliar academic tasks, with its importance lessening as task performance becomes more dependent on automatic, long-term-memory-based processes.

The finding that the visuospatial representational system predicted mathematics but not reading achievement, holding other domain general abilities constant, strengthens results from other studies and extends them to learning across the elementary school years (Bull et al., 2008; De Smedt et al., 2009; Swanson et al., 2008). Although the phonological loop may be engaged on more circumscribed mathematical tasks (Geary et al., 2007; Krajewski & Schneider, 2009), the ability to generate visuospatial representations contributes to mathematics learning more broadly than do phonological processes; discussion of potential mechanisms can be found elsewhere (Geary, 1996; E. S. Johnson, 1984; Lewis, 1989). The phonological loop and not the visuospatial sketch pad, in contrast, predicted word reading achievement, a result that is not surprising given the well-established foundational importance of phonological abilities for reading (Bradley & Bryant, 1983; Swanson, Trainin, Necoechea, & Hammill, 2003). The more important finding is the contrast between the contributions of the visuospatial sketch pad and the phonological loop to individual differences in mathematics and word reading achievement, respectively, and the corresponding support for Carroll's (1993) hypothesis that these are broad cognitive abilities that, unlike intelligence, support learning in many but not all domains.

The highly significant correlations between letter and number naming RTs are consistent with common mechanisms that influence speed of serially processing visual symbols (Wolf et al., 2000), and the final models for mathematics and reading indicate an effect of processing speed on achievement above and beyond the influence of intelligence and working memory; processing speed and intelligence tend to be correlated (Jensen, 1998), but the correlations are larger for speeded tasks that are more complex (i.e., they require a choice) than the RAN tasks used in this study (Deary, Der, & Ford, 2001; Der & Deary, 2003). The practical results here are that speed of retrieving letter names predicted Word Reading scores, as is commonly found (Denckla & Rudel, 1976; Swanson et al., 2003), and speed of retrieving number names predicted Numerical Operation scores, which has not been as systematically studied (Lachance & Mazzocco, 2006). The combination of findings is nonetheless consistent with Dark and Benbow's (1991) finding that mathematically and verbally gifted adolescents had respective advantages in speed of recognizing and naming digits and words. There may be low-level perceptual mechanisms that support retrieval speed generally (Wolf et al., 2000), but there appear to be other mechanisms that result in differences in speed of accessing numbers and letters that, in turn, has differential effects on mathematics and reading achievement, respectively. The design of the current study, however, does not provide insights as to what these mechanisms might be.

Mathematics Achievement

The core findings of this study are found with the mathematical cognition measures that predicted mathematics achievement and achievement growth above and beyond the contributions of the domain general abilities. The current findings also contribute by demonstrating that only one mathematical cognition predictor of mathematics achievement emerged as a predictor of word reading achievement. This one predictor, simple addition retrieval, may reflect some overlap in the mechanisms that support addition fact and word retrieval (Geary, 1993), but this is not certain. The more important point is that the remaining mathematical cognition predictors were unique to mathematics.

Among previous longitudinal studies, Jordan et al.'s (2009) is the most similar to the current one. They administered a battery of number, counting, and simple arithmetic tasks to children four times across their kindergarten year and twice in the fall of first grade and tracked the relation between these early competencies and mathematics achievement through the end of third grade but did not control for domain general abilities. Their results indicated that quantitative competence at the beginning of kindergarten explained 66% of the variation in mathematics achievement at the end of third grade, and growth in quantitative competence across kindergarten and early first grade explained an additional 10% of the variance (Jordan et al., 2009). Of the quantitative items, early competence in simple arithmetic appeared to be especially important. In a study of risk of mathematical learning disability (MLD), Mazzocco and Thompson (2005) found that poor addition skills in kindergarten predicted risk of MLD at the end of third grade. The current results confirm these findings and demonstrate that early arithmetic skills are important for later mathematics achievement, above and beyond the influence of domain-general abilities and several other quantitative competencies. Skilled use of counting procedures to solve addition problems and the ability to decompose numbers to solve these problems appear to be particularly important, with the benefits of knowing basic facts in first grade increasing with each successive grade.

The importance of more basic numerical competencies for later achievement has also emerged in several studies (Landerl, Bevan, & Butterworth, 2004; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005). Mazzocco and Thompson (2005), for instance, found that children at risk for later MLD were behind their peers on number reading and number comparison (e.g., "Which is larger, 8 or 5?") tasks in kindergarten, as did Landerl et al. (2004). Locuniak and Jordon (2008) found the same, controlling for working memory, intelligence, and addition skills. The current study suggests that fluency in apprehending the quantity of small sets of items and Arabic numerals and in combining these, as measured by the Number Sets Test (Geary et al., 2009), may be a critical aspect of early competence with number. The ability to map Arabic numerals onto corresponding quantities may be a related critical skill (Rousselle & Noël, 2007). In any case, it is not simply number recognition and naming, although this is important, but also apprehension of the corresponding quantities and skill at composing and decomposing these as related to task demands. The latter is reflected in the likely demands of the Number Sets Test and is reflected in the use of decomposition to solve arithmetic problems.

The results for the number line task support Siegler and colleagues' findings that number line performance correlates with later mathematics achievement (e.g., Booth & Siegler, 2006; Siegler & Booth, 2004). The current study extends these findings by demonstrating the correlation is not due to confounds with domain general or other quantitative abilities and adds nuance to them. In particular, children's knowledge of the mathematical number line in first grade did not predict Numerical Operations scores in the early grades, but this knowledge did predict later achievement. As with early knowledge of basic facts, an early advantage in the ability to accurately map numbers onto the mathematical number line appears to be part of the foundation for later mathematics learning (National Mathematics Advisory Panel, 2008).

The only seemingly contradictory finding was for pseudo counting, whereby children who correctly identified these unusual counts as correct had lower Numerical Operations scores. This result is, in fact, consistent with those of LeFevre et al. (2006), who showed that low ability first graders had higher scores on these types of counting tasks than did their average and high-ability peers, a trend that reversed in second grade. They argued that lower ability children tend to say all counts are correct, unless the error is quite obvious, whereas children who are more sensitive to nuances in counting often identify unusual counts as incorrect, including pseudo error counts. With experience in different ways of counting, these perceptive children learn that pseudo and other irregular counts can be correct, if other rules (e.g., no item is double counted) are not violated. More practically, the overall trend in the literature, including the current findings, suggests that the relation between children's conceptual and procedural competence in counting and their mathematics achievement is more nuanced than is the case for other early quantitative competencies (Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009), and thus, counting-task results for any single time of measurement need to be interpreted with caution.

Summary and Limitations

An important limitation of the current study is the noninclusion of instructional, classroom (e.g., in-class attention) and other student centered (e.g., organization of class work) variables (e.g., Crosnoe, et al., 2010; Dettmers, Trautwein, Lüdtke, Kunter, & Baumert, 2010; Fuchs et al., 2006). Presumably some combination of these types of factors would explain some of the variation in Numerical Operations scores that were not accounted for by the cognitive variables used in the current study. Another limitation is the exclusion of children with missing data. Although the retained sample is typical with respect to national norms, the children who did not complete all of the testing had lower average intelligence and achievement scores than the retained sample. Their attrition likely resulted in lower variability in outcomes and, thus, less power to detect potentially important effects. Although scores on the Numerical Operations test are highly correlated with scores on mathematical reasoning tests (Wechsler, 1992), it is not likely that the current study identified all of the quantitative competencies that support more abstract mathematical reasoning. The results do, nonetheless, suggest that the basic quantitative competencies identified in this study are an important foundation for learning more complex mathematics.

Despite these limitations, the current study contributes to our understanding of and ability to assess the foundational quantitative skills that support long-term mathematics learning. This is critical, as Duncan et al. (2007) demonstrated that individual differences on mathematics achievement tests at the beginning of formal schooling are maintained throughout the remainder of schooling, above and beyond the influence of intelligence. The current results, and those from other recent longitudinal projects (e.g., Jordan et al., 2009), indicate that the critical early quantitative competencies that children must possess to learn mathematics include an understanding of the relation between number words, Arabic numerals, and the underlying quantities they represent, as well as skill at fluently manipulating these representations, knowledge of the mathematical number line, and basic skills in arithmetic (i.e., skilled use of counting procedures, decomposition, and fact retrieval in problem solving). These skills are easily assessed in young children and many have been shown to be highly responsive to instructional interventions (Locuniak & Jordan, 2008; Siegler & Ramani, 2008).

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Appendix

Table A1
Correlations Among Predictors of Academic Achievement

Predictor	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Intelligence	1.00												
2. Central executive	.45	1.00											
3. Phonological loop	.41	.59	1.00										
4. Visuospatial sketch pad	.31	.53	.34	1.00									
5. Processing speed	.21	.47	.36	.35	1.00								
6. Counting error	.06	.13	.01	.12	.08	1.00							
7. Pseudo counting	.19	.22	.14	.12	.12	.00	1.00						
8. Simple addition retrieval	.34	.27	.13	.26	.20	.05	−.02	1.00					
9. Simple addition procedural competence	.28	.37	.34	.21	.25	.12	.18	−.15	1.00				
10. Complex addition decomposition	.14	.11	.00	.22	.07	.08	−.08	.67	−.21	1.00			
11. Complex addition procedural competence	.27	.43	.43	.23	.39	.13	.14	.13	.68	−.16	1.00		
12. Number line	.41	.43	.31	.38	.26	.21	.14	.45	.31	.31	.40	1.00	
13. d'	.48	.56	.36	.48	.32	.18	.26	.43	.45	.24	.48	.62	1.00

Note. $|r| > .15$, $p < .05$; $|r| > .20$, $p < .01$.

Received June 11, 2010
Revision received May 12, 2011
Accepted June 21, 2011 ■