An Investigation of the Construct of Number Sense

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Abstract
Through a comprehensive review of the literature, this poster presents the results of an investigation of previous efforts to define and identify the essential components of number sense. A review of 40 relevant research studies found 34 components of number sense, including skills such as the ability to compare quantities, estimate, count, and complete simple computations. Results indicate the need for a tighter operational definition of the construct as well as future research into the predictive characteristics of each component.

Introduction
Background and Rationale
Number sense has been highlighted as a vital prerequisite to success in mathematics (National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008). A well-developed number sense allows students to understand number facts and algorithms more quickly, recognize errors, and ultimately perform mathematical computations with greater ease. Given the importance of developing a sound sense of number, it seems obvious that number sense should be a well-defined construct that can be easily observed, assessed, and explicitly taught through a developmentally systematic scope and sequence. This, however, is not the case; number sense has historically been poorly defined in the literature (Berch, 2005; Gersten, Jordan, & Flojo, 2005; Mcintosh, Reys, & Reys, 1992). As Gersten et al. (2005) have noted, number sense is a complex, intricate set of skills that "no two researchers have defined in precisely the same fashion" (p. 296). In fact, Berch's (2005) brief review of the literature found approximately thirty alleged components of number sense, ranging from the ability to estimate, to the understanding of number meanings, to the skill of having a non-algorithmic "feel" for numbers. Current definitions of number sense are often vague and overly extensive, containing abstract principles and far too many components. Despite the recent interest in investigating the foundational skills needed to succeed in mathematics, the lack of consensus regarding the definition and components of number sense has made it increasingly difficult to observe, assess, and teach.

Purpose
The purpose of this poster presentation is to examine how the mathematical construct of number sense has been operationally defined throughout the extant literature. Through a comprehensive review of the literature, this poster presents the results of an investigation of previous efforts to define number sense by analyzing the several proposed components of the construct.

Method

- Studies were gathered by searching the following databases: ERIC, PsycINFO, Google Scholar
- Keywords used to identify relevant studies included:
    - Number sense, sense of number, numerosity, defining number sense, number sense components, assessing number sense, measuring number sense
- Inclusionary criteria:
    - Peer-reviewed research studies
    - Published from 1980 to present
    - Studies conducted with or regarding students in pre-school through high school

Review articles, theoretical papers, dissertations, and research studies involving infants or adults were excluded from this investigation.

Studies that met the inclusionary criteria were carefully reviewed for their proposed components of number sense. Components from each study were extracted, organized, and compiled into a table of frequently cited components of number sense. This table was then converted into a figure depicting the number of studies in which each component appears (see Figure 1).

Results

- A total of 40 studies were included in this analysis
- Among the 40 studies, 34 different components of number sense were identified
- The number of components each study listed ranged from 1 to 13
- Figure 1 shows the number of studies in which each component of number sense appears
- Components that appeared in two or fewer studies are listed below

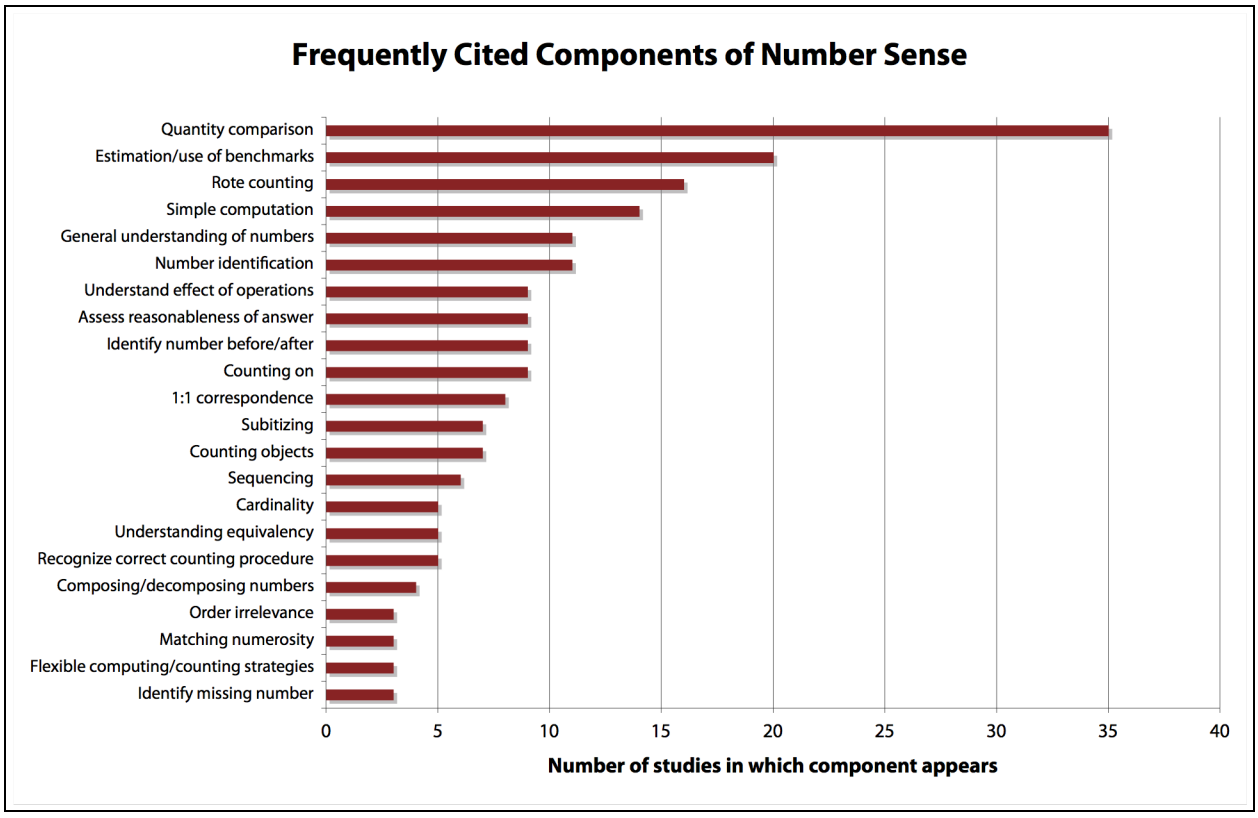

Figure 1. Components of number sense appearing in three or more studies

Components of number sense found in two or fewer studies:

- Number writing
- Measurement concepts
- Understanding patterns
- Resultative counting
- Classification
- Ordinality
- Inversion principle
- Part-whole relationships
- Communicativity
- Moving between number representations
- Counting out objects to match spoken number
- Understanding symbols used to denote numbers

Discussion

A comprehensive review of the literature on number sense clearly demonstrates the general lack of consensus regarding which skills are essential components of the construct. Among the 40 research studies analyzed, 34 different components of number sense were identified, with quantity comparison being the only component that appeared in the majority of the studies. A number of other components, such as the ability to estimate, count, and complete simple computations, were cited as components of number sense in at least ten of the reviewed studies.

Future Directions

This investigation highlights the need for a tighter operational definition of the construct of number sense as well as additional research into each proposed component of number sense. Future research should seek to answer the following questions:

- Of the skills listed, which are truly essential components of number sense and which perhaps belong to another related construct?
- Mastery of which components is predictive of later success in mathematics?
- What other skills or factors contribute to/moderate a child's development of number sense?
- How do the essential components of number sense change as students progress through mathematics/grade levels?
- What other foundational skills are necessary for success in mathematics?

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