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Keeping Synchrony While Tempo Changes: Accelerando and Ritardando

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We studied synchronization with a metronome that smoothly changes tempo, from slow to fast (accelerando) or from fast to slow (ritardando). During the transition phase, systematic alternations of underadjustment and overadjustment of period and phase were observed. We analyzed the synchronization error ("asynchrony") sequences in terms of two models that both assume linear period and phase correction mechanisms but differ in terms of how the timekeeper period is adjusted to the tempo change. In the interval-based model, period corrections are based on comparisons between timekeeper and metronome intervals, whereas in the asynchrony-based model, period corrections are based on the deviations of taps from metronome events. The qualitative data pattern is more compatible with an asynchrony-based model than with an interval-based model. Additional mechanisms that switch on and off period adjustment seem to be needed, however, for a quantitative fit of this model to our data.

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SYNCHRONIZATION of movements with a sequence of external events has been studied for a long time. The simplest task, synchronization with a metronome that produces auditory sounds equally spaced in time, is well

We dedicate this paper to the memory of our friend and colleague Andras Semjen.
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understood. Synchronization performance under such conditions is described well by the Wing-Kristofferson two-level model (Wing & Kristofferson, 1973) augmented by a linear phase-error correction mechanism (Pressing, 1998; Vorberg & Schulze, 2002). A central notion is the assumption of an internal timekeeper that controls the interval between taps and triggers the motor system correspondingly. Error correction is necessary because the timekeeper and the motor system are subject to temporal jitter; without correction, the produced taps and the metronome sequence will run out of phase. A linear error-correction mechanism that uses the asynchronies between taps and metronome clicks for corrective phase shifts without changing the timekeeper period gives an excellent account of synchronization performance when the metronome's period is constant and when it is subject to small perturbations (Semjen, Vorberg, & Schulze, 1998; Semjen, Schulze, & Vorberg, 2000; Repp, 2000, 2001).

Synchronization becomes a more demanding task when the metronome's tempo changes systematically. In this case, phase correction alone is not sufficient for keeping synchrony. Rather, the internal timekeeper must be adjusted to the changing period of the metronome. Several models have been proposed for period adjustment. Mates (1994a, 1994b) has suggested that timekeeper adjustments are based on the discrepancy between the current timekeeper interval and the previous metronome interval. This error signal is used to adjust the timekeeper by adding or subtracting a fixed proportion of the interval difference. A simple alternative that we pursue in this article is that, like phase, the timekeeper period is adjusted on the basis of the asynchronies, that is, the temporal difference between perceived events, rather than the difference between time intervals.

Almost all previous studies with changes in the metronome period used either step (sudden change to a new tempo) or rectangular changes (sudden change, followed by abrupt return to the initial tempo after a certain number of intervals; e.g., Michon & Van der Valk, 1964). Large, Fink, and Kelso (2002) studied the time course of the return to the base period after a rectangular tempo change ("phase relaxation curve"). They found smooth exponential returns of the relative phase error to zero after the tempo perturbation. This result is expected, assuming a phase correction model (Vorberg & Schulze, 2002). A third variation of the metronome was used by Thaut, Tian, and Azimi-Sadjadi (1998), who modulated the period of the metronome by a cosine function.

In this study, we investigate the synchronization to a metronome with acceleration and ritardando. In music, both types of tempo changes occur as an expressive tool. It is a challenge for musicians in ensemble playing to do this tempo change in synchrony. How do subjects adapt to smooth tempo changes? A theoretical possibility that seems promising in view of the success of the linear-phase correction model just sketched is to extend it by an adaptive timekeeper. The original model (Vorberg & Wing, 1996) assumes that the internal timekeeper generates time intervals T_i that control the spacing between successive motor commands; at the onset of each interval, a motor response is triggered that leads to an observed response after a motor delay M_i. The timekeeper intervals T_i are independent random variables with constant mean and variance; they are adjusted each, however, by the preceding asynchrony: If a tap lags the metronome click, the next interval is reduced by some proportion α of the asynchrony; if it leads, the next interval is lengthened correspondingly. These phase corrections are local, that is, the timekeeper mean and variance remain unchanged. It can be shown that this model implies the following characteristic difference equation (Vorberg & Schulze, 2002):

$$A_i = (1 - \alpha)A_{i-1} + T_{i-1} - C_{i-1} + M_i - M_{i-1}, \tag{1}$$

where A_i is the asynchrony and C_i the metronome interval at cycle *i*.

To incorporate period correction in the model, we assume that additional adjustments are made to the timekeeper mean. A straightforward way is to do so analogously to linear phase correction, that is, by linearly updating the timekeeper mean by a fixed proportion of the preceding asynchrony. Let τ_i be the timekeeper mean in cycle *i*. Then our assumption of linear (first-order) period correction can be stated as

$$\tau_i = \tau_{i-1} - \beta A_{i-1}, \tag{2}$$

where β is the period correction gain factor.

A well-known alternative to our model is the more ambitious phase and period correction model proposed by Mates (1994a, 1994b), which in a linearized version without threshold can be shown to imply

$$\tau_i = (1 - \beta)\tau_{i-1} + \beta C_{i-1} \tag{3}$$

$$= \tau_{i-1} + (C_{i-1} - \tau_{i-1}), \tag{4}$$

provided its internal feedback delays are assumed to be equal in mean. In the Mates model, the timekeeper interval in a cycle thus follows a distribution with mean given by a weighted average of the timekeeper mean and the metronome interval in the previous cycle; if $\beta = 1$, the timekeeper mean perfectly mimics the metronome, except for a one-interval delay. A comparison of Equations 2 and 4 shows that the first model uses the asynchrony as the basis for period correction, whereas the second model uses the difference between the metronome interval and the timer interval.

In the following, we will refer to these alternative models as asynchrony-based and interval-based, respectively.

Experiment

The purpose of the experiment was to compare the linear error-correction models for synchronization under experimental conditions that cannot be handled by phase-error correction alone. Therefore, we extended the synchronized tapping paradigm to a situation in which the metronome undergoes smooth tempo changes. The situation we used was motivated by the tempo changes common in classical music, where *accelerando* (i.e., becoming faster) and *ritardando* (i.e., gradually slackening in speed) are musical means for creating or reducing tension, often when the end of a musical piece is approached. Performing gradual tempo changes gracefully as well as accurately is quite demanding, particularly in ensemble playing. How important is the knowledge of when an accelerando or ritardando begins, and of how fast the tempo change is going to be? As a first step toward studying such questions, we constructed metronome sequences that consisted of three phases: in the initial phase, tempo was constant; then during the transition phase, it gradually increased or decreased; until it smoothly approached the final tempo in the third phase. While subjects were fully informed about the duration of the tempo transition phases, information about transition onset was manipulated experimentally. The direction of the change (accelerando vs. ritardando) was always known, but there was uncertainty about the rate of change.

METHODS

Participants

Five female volunteers were tested. Their ages were in the range from 21 to 30 years. Four of them had moderate training in playing a music instrument (4–6 years); the fifth was musically untrained.

Stimuli

The tempo of the metronome sequence was constant for an initial phase of 16 periods, then smoothly changed to another tempo within 16 periods, and remained constant at that tempo for another 16 periods. Start and goal tempi, expressed in terms of their interonset intervals (IOIs) C_{start} and C_{goal}, could be either 300, 370, 440, or 510 ms. All unequal pairings were used, resulting in 12 different metronome sequences. During the tempo transition, the IOIs C_i followed a sigmoidal function defined by

$$C_i = p_i C_{start} + (1 - p_i) C_{goal}$$

with

$$p_i = \begin{cases} 1 & \text{if } i < 17 \\ \frac{1}{2}[1 + \cos(\frac{i-16}{17}\pi)] & \text{if } 17 \leq i \leq 32 \\ 0 & \text{if } i > 32 \end{cases}$$

(see Figure 1). This transition function was chosen to produce musically acceptable tempo changes, which resemble accelerando and ritardando in performed music. To make the onset of the tempo change unpredictable, each sequence was preceded by an additional R

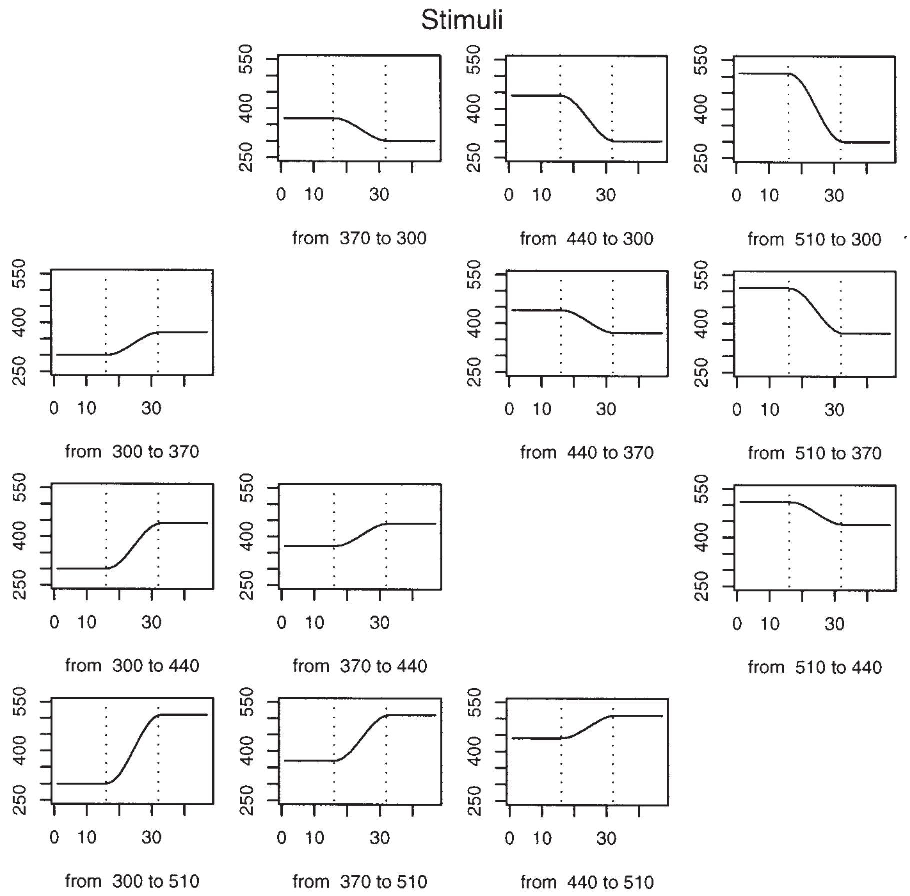

Fig. 1. Stimuli: The ordinate of each figure is the period in milliseconds; the abscissa is the index of the period. The same arrangement of the figures (columns are ordered by starting period, rows are ordered by ending period) is used in the following figures. The stimuli in the (lower, upper) diagonal are (ritardando, accelerando).

periods, where *R* was randomly chosen between 1 and 6. These initial taps were excluded from the analysis.

Design and Conditions

Two independent variables were crossed factorially: tempo (4 × 3 start-goal tempo combinations) and information condition (signal, no signal). On signal trials, subjects were informed about the beginning and end of the tempo transition phase, which remained unknown on no-signal trials. Signal and no-signal trials were blocked; all other conditions were randomized within blocks of 12 trials each. A session contained four signal and four no-signal blocks, occurring in random order.

Apparatus

The metronome sounds (synthesized grand piano tones of 30 ms duration) were produced by a Roland D70 synthesizer that was controlled via MIDI by an ATARI Mega/STE.

They were presented by headphones, at a volume level adjusted by the subject. Tone pitch was constant within a trial, but randomly chosen from [$F\sharp_3$, G_3, $G\sharp_3$, A_3, $A\sharp_3$]. Subjects responded by tapping on the A_4 key, which elicited the corresponding piano sound.

Procedure

The task was to keep close synchrony with the metronome. Subjects were informed that the metronome changed tempo on every trial. Before each trial, visual information was given about the direction of the impending tempo change (accelerando vs. ritardando), whereas the amount of tempo change remained uncertain. On trials in which the tempo change was cued, the pitch of the metronome tone increased or decreased by a semitone at the beginning or end of the transition on accelerando trials, respectively, and decreased and increased on ritardando trials. In trials, the metronome pitch remained constant throughout.

Trials were started by pressing a button, which initiated the metronome sequence that kept going at a constant rate until the subject entered. At the end of a trial, knowledge of results was provided on the screen by displaying the intertap intervals (ITIs) superimposed on the metronome IOIs. Sequences with responses that were more than half a period off from the corresponding metronome event were discarded and recollected. This was mainly due to skipped taps and occurred in less than 5% of the trials. In six 1-hour sessions, a total of 24 replications per condition were collected. The first session was considered practice, leaving 20 sequences per condition for analysis.

Basic Empirical Findings

The trial data consist of the metronome IOI sequence, $\{C_i\}$, and the realized ITI sequence, $\{I_i\}$, from which the corresponding asynchrony sequence, $\{A_i\}$, was derived. Graphical data inspection did not reveal any systematic effects on synchronization performance for signal trials as compared to trials in which the onsets of tempo changes were signaled. The statistical analysis was performed with a random-effect linear model with synchronization performance (variance of asynchrony) as dependent variable and subject as random factor and information as fixed factor. The result was that there was no effect of information (p = .88). All further analysis and model tests were therefore performed on the data lumped across signal and no-signal trials. Note that, given the IOI sequence, the asynchronies contain the same information as the corresponding ITIs because $I_i = C_i + A_{i+1} - A_i$, by definition (Vorberg & Schulze, 2002). Therefore, we focus on the asynchrony trajectories almost exclusively in the following. By convention, negative asynchronies indicate responses anticipating metronome events.

Figure 2 shows the evolution of the average synchronization error during accelerando (upper right triangle) and ritardando (lower left triangle), for all start-goal tempo combinations. Since our focus is on quantitative evaluation of the models, we just the sketch the findings that hold for most conditions and subjects, and we single out constraints on possible synchronization models. To get an indication of individual differences,

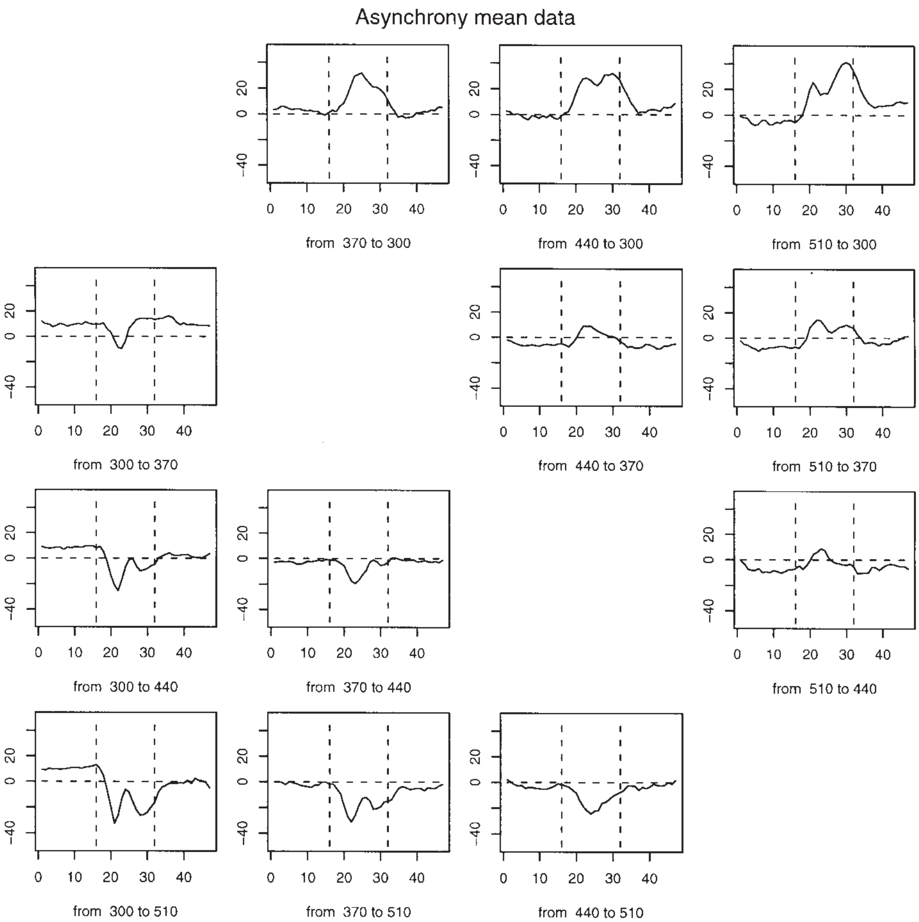

Fig. 2. Mean trajectories of asynchrony. The ordinate is the asynchrony. The abscissa is the index of the sequence. Data are averaged over replications and subjects.

Figure 3 shows individual trajectories. Note the differences in the stationary first phase: Subjects differ not only in the amount of the asynchrony but also in the sign. Positive asynchrony is more common.

TRAJECTORY SHAPE

In the tempo transition phase, the asynchrony trajectories deviate from zero in a characteristic M-shaped or W-shaped fashion when the metronome speeds up or slows down, respectively. This pattern is most clearly seen for the largest tempo differences, namely, in accelerando from 510 ms to 300 ms (top right) and in ritardando from 300 ms to 510 ms (bottom left), but is noticeable for the less extreme changes as well. Obviously, initial adjustment of the timekeeper period is late, leading to

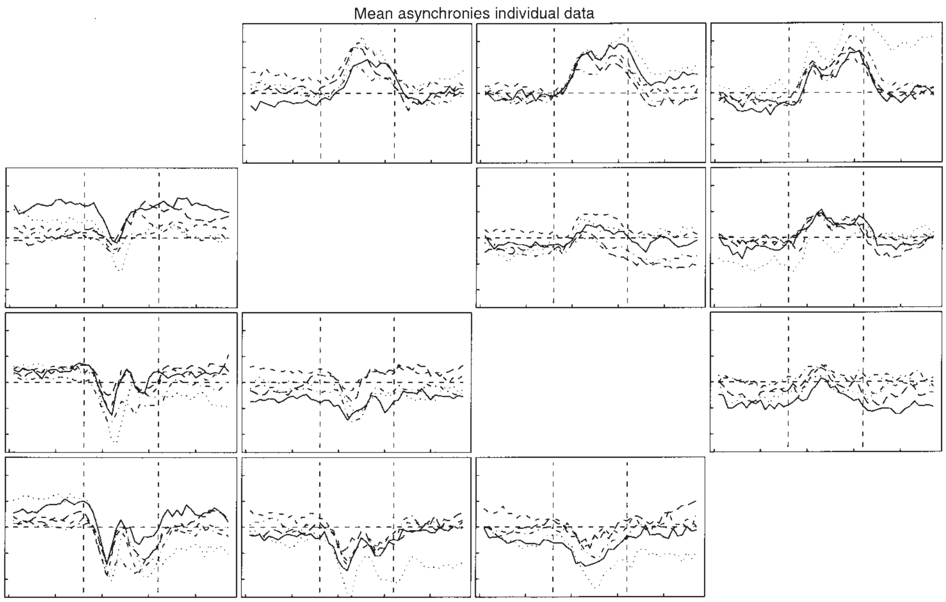

Fig. 3. Individual trajectories of asynchrony. The ordinate is the asynchrony in the range of –50 to 50 ms. The abscissa is the index of the sequence. Data are averaged over replications.

phase lagging (leading) in accelerando (ritardando). After six to eight taps, however, subjects catch up and even overcompensate, then fall behind and overcompensate a second time, as evidenced by the ITI-IOI trajectories before the new tempo is established (Figure 4). Note that these patterns hold for all subjects, despite the considerable individual differences.

NEAR-MISS SYMMETRY

For accelerating tempo, the trajectory shape is almost the inverse of that for ritardando, but not quite. Note the first trough in the W is lower than the second one, whereas in the M, the second peak exceeds the first one.

NONZERO ASYNCHRONY ASYMPTOTES

The mean asynchrony at the stationary part before and after the tempo transition seems to differ from zero systematically. Whereas the final level does not seem to follow an obvious pattern, the initial level does: The slower the tempo, the more negative the asynchronies, that is, the larger the amount of anticipation of the metronome.

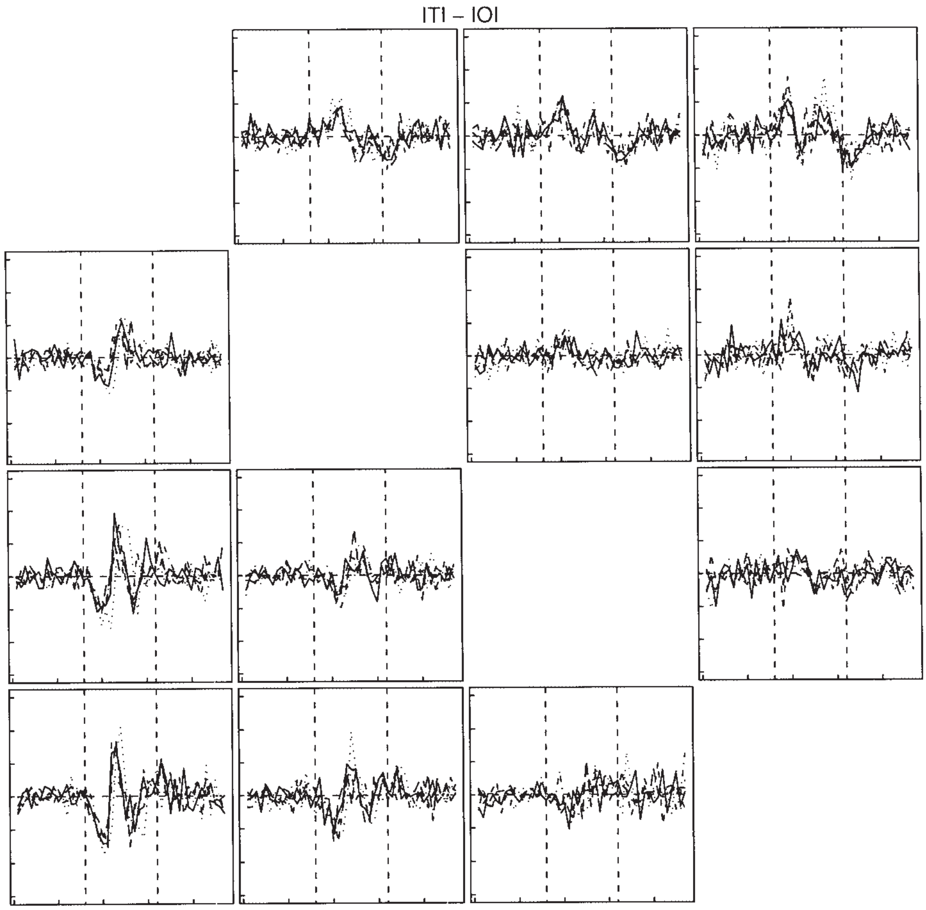

Fig. 4. Individual trajectories of the difference between the intertap interval (ITI) and the metronome interval (IOI). The ordinate is the difference in the range of –30 to 30 ms. The abscissa is the index of the sequence. Data are averaged over replications.

Parameter Estimation and Model Evaluation

Because both the Mates (1994a, 1994b) model and the alternative proposed here involve phase and period correction mechanisms that are linear, fitting model predictions and estimating parameters can be based on the mean asynchrony sequences without loss of information (assuming constant parameters within subjects); averaging was across replications per condition. Parameters were estimated by numerical minimization, using the algorithm *nlm* provided in the statistical programming language **R** (Ihaka & Gentleman, 1996). The function uses a Newton-type algorithm to find the minimum of a function. The sum of squared deviations between theoretical and observed asynchronies was used as minimization criterion. Parameters were estimated separately per subject and tempo condition.

Elsewhere (Vorberg & Schulze, 2002), we have argued from statistical principles that when the metronome period is known to be stable, period adjustment may be suboptimal while simply adjusting phase error may be a better strategy under these conditions, even if the internal timekeeper period is based on a rough tempo estimate. This argument is supported by the findings of Repp (2001), who found strong evidence for phase adjustment after experimental perturbations, but very little evidence for period adjustment. To check on the validity of our argument, we fitted the models in two different ways, applying them either to the complete asynchrony sequences ("complete analysis") or to the tempo transition phase only ("transient analysis").

NONZERO BASELINES

A problem in fitting synchronization models is that the mean asynchronies are often different from zero even when the metronome period is stationary, which may be outside a model's scope. Some authors have attributed nonzero asynchronies to differential afferent delays for metronome events and taps (e.g., Aschersleben & Prinz, 1995). However, this is a partial explanation at best, because the mean asynchrony often depends on the period of the metronome in a systematic way, as we saw above: for all subjects, asynchronies before the tempo transition were mostly positive at the fastest tempo and turned toward more negative values the slower the tempo (see Figure 2). Another explanation (Vorberg & Wing, 1996) is that the timekeeper period systematically differs from the metronome period. To cope with this problem, which might be unrelated to the issue of phase and period correction in synchronization, we extended the models such that they accommodate without explaining nonzero asynchronies, and we developed a pragmatic approach for fitting them. We illustrate the rationale by sketching the approach for the asynchrony-based period correction model.

Assume that the amounts by which phase and timekeeper period are adjusted after a tap are proportional to the deviation of the current asynchrony from an unknown internal baseline (denoted by γ_i) rather than to A_i directly. This amounts to replacing Equations 1 and 2 by

$$(A_i - \gamma_i) = (1 - \alpha)(\mathrm{A}_{i\text{-}1} - \gamma_{i\text{-}1}) + T_{i\text{-}1} - C_{i\text{-}1} + M_i - M_{i\text{-}1}$$

and

$$\tau_i = \tau_{i\text{-}1} - (A_{i\text{-}1} - \gamma_{i\text{-}1}).$$

If γ_i is constant, the revised phase and period correction scheme implies $E(T_i) = C_i$ and $E(A_i) = \gamma$ asymptotically, which shows that the asynchrony mean mirrors the internal baseline. For nonconstant γ_i, closed-form prediction is not possible in general, but for subsequent trials with constant internal baseline, the expected asynchrony tends toward this value. Since the observed mean asynchronies were rather stable during the constant tempo synchronization phases, we assume that γ_i equals s or g in the initial or final metronome phases, respectively, and changes linearly in between. We estimated $\hat{s}$ and $\hat{g}$ from the stationary phase data before and after the tempo transition, and fitted the revised models, using

$$\hat{\gamma}_i = \frac{n-i}{n-1}\hat{s} + \frac{i-1}{n-1}\hat{g},\ 1 \leq i \leq n,$$

as internal baseline estimates in the tempo transition phase. For the analysis of the complete sequences, the estimates were based on the initial and the three asynchronies within a sequence, for the transient analysis, they were based on the last (first) three asynchronies preceding (following) the tempo transition phase.

RESULTS OF TRANSIENT ANALYSIS

Model 1

Figure 5 compares the observed synchronization error trajectories in the transient phase with best-fitting predictions of Model 1, assuming asynchrony-based timekeeper period adjustment. For each tempo-condition, best-fitting predictions were obtained for individual subjects, which were then averaged to yield the mean asynchrony predictions that correspond to the observed mean trajectories (averaged across replications and subjects). Although quantitatively the fit is far from being perfect, it is remarkable that the model does capture some of the qualitative properties of the data, the W-shaped asynchrony trajections in the ritardando conditions in particular. In accelerando, the M-shape is captured less well (this failure is not due to averaging predictions; it is seen in the individual data also).

Model 2

Figure 6 shows the corresponding predictions under the assumption that period correction is interval-based, compared with the same data. Obviously, the fit of this model is much inferior to that of Model 1, and there is no hint that the model can reproduce the observed M-shaped and W-shaped trajectories. Instead, predicted error trajectories were single-peaked throughout.

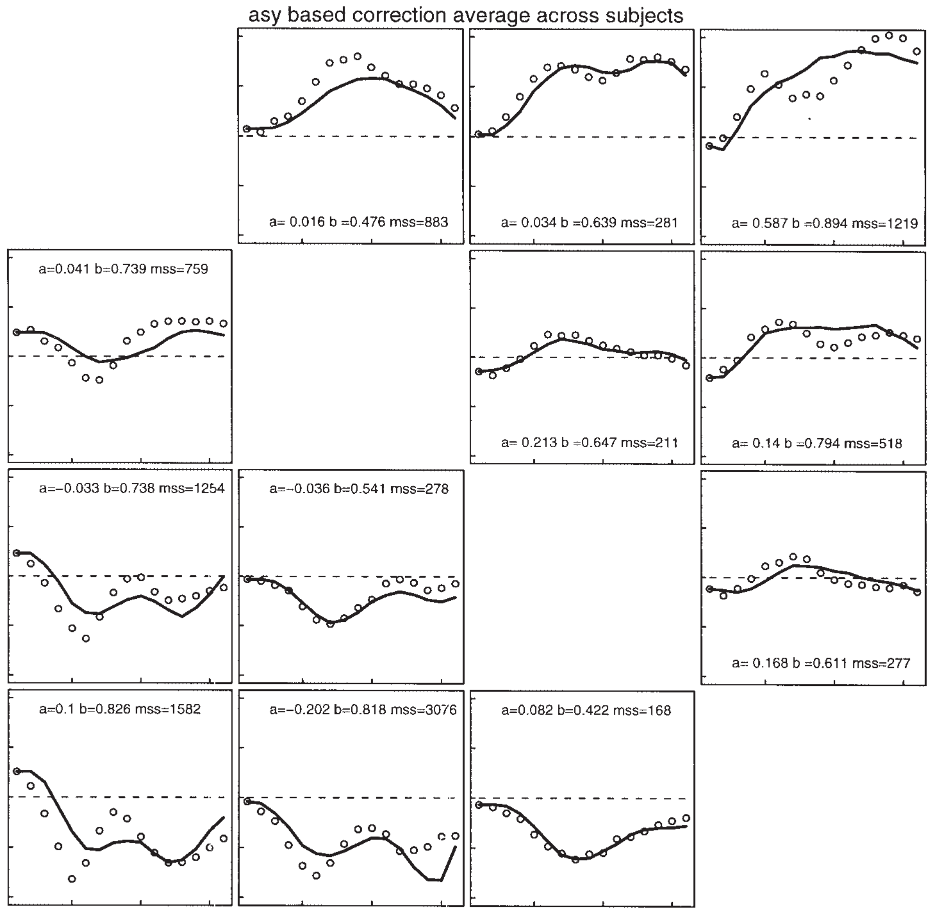

Fig. 5. Observed and predicted asynchronies of the asynchrony based model. Transient phase of data. a and b are the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$; mss is the mean sum of squared deviation (averaged over subjects).

Comparison of the models (which have the same number of free parameters) in terms of the quantitative goodness of fit also leads to the conclusion that Model 2 is inferior. In 10 of 12 cases, the average sum of squares was smaller for Model 1 than for Model 2. Not surprisingly, the difference in goodness of fit was largest for the ritardando conditions, where asynchrony-based period adjustment accommodated the double troughs in the trajectories.

RESULTS OF COMPLETE ANALYSIS

One might argue that the model comparison does not treat Model 2 fairly, as it does not give it a chance to fit the full asynchrony trajectories;

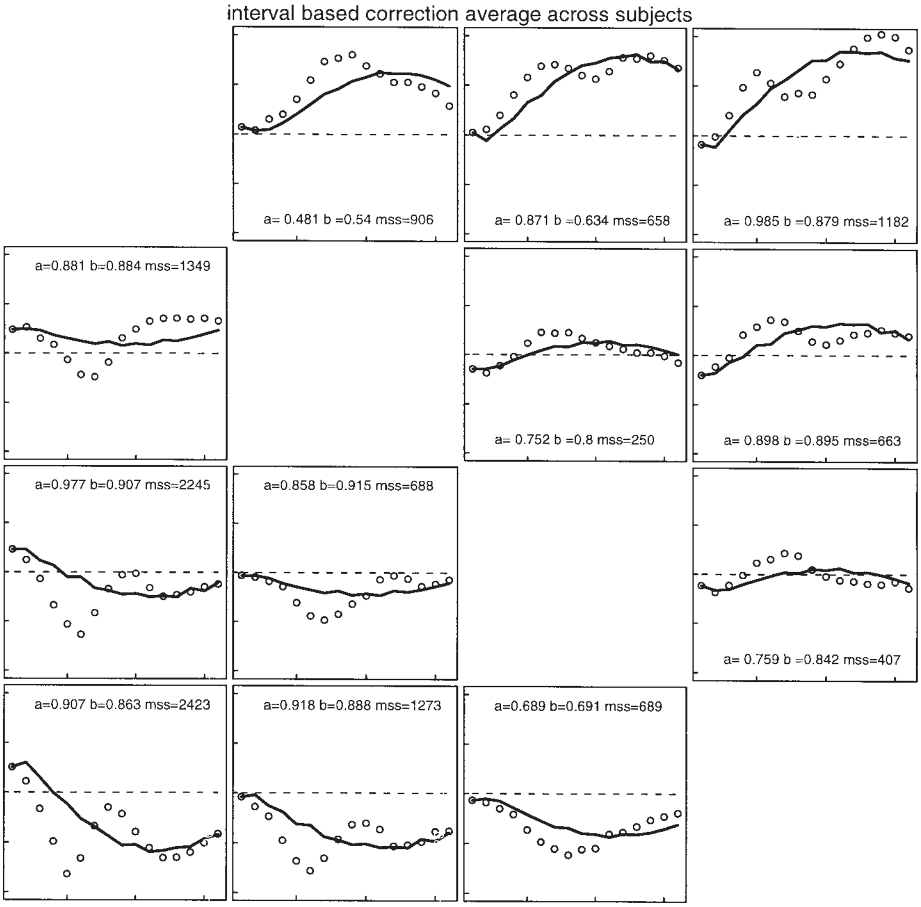

Fig. 6. Observed and predicted asynchronies of the interval-based model. Transient phase of data. a and b are the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$; mss is the mean sum of squared deviation (averaged over subjects).

originally, the model was designed to hold for synchronization under both stationary and nonstationary tempo conditions (Mates, 1994a, 1994b). Therefore, we fitted either model to the complete data, including the initial and final stationary phases of each sequence.

Figures 7 and 8 show the resulting model fits. When the asynchrony-based model is forced to fit both the stationary and nonstationary sections with constant parameters, it can no longer track the qualitative asynchrony patterns characteristic for the tempo-transition (Figure 7). The same is true for the interval-based model (Figure 8). Surprisingly, however, Model 2 does not fare better in the full than in the transient analysis. In fact, it is even inferior to Model 1, as evidenced by visual impression and by comparing the goodness-of-fit indices, on which Model 1 wins 8 of 12 comparisons.

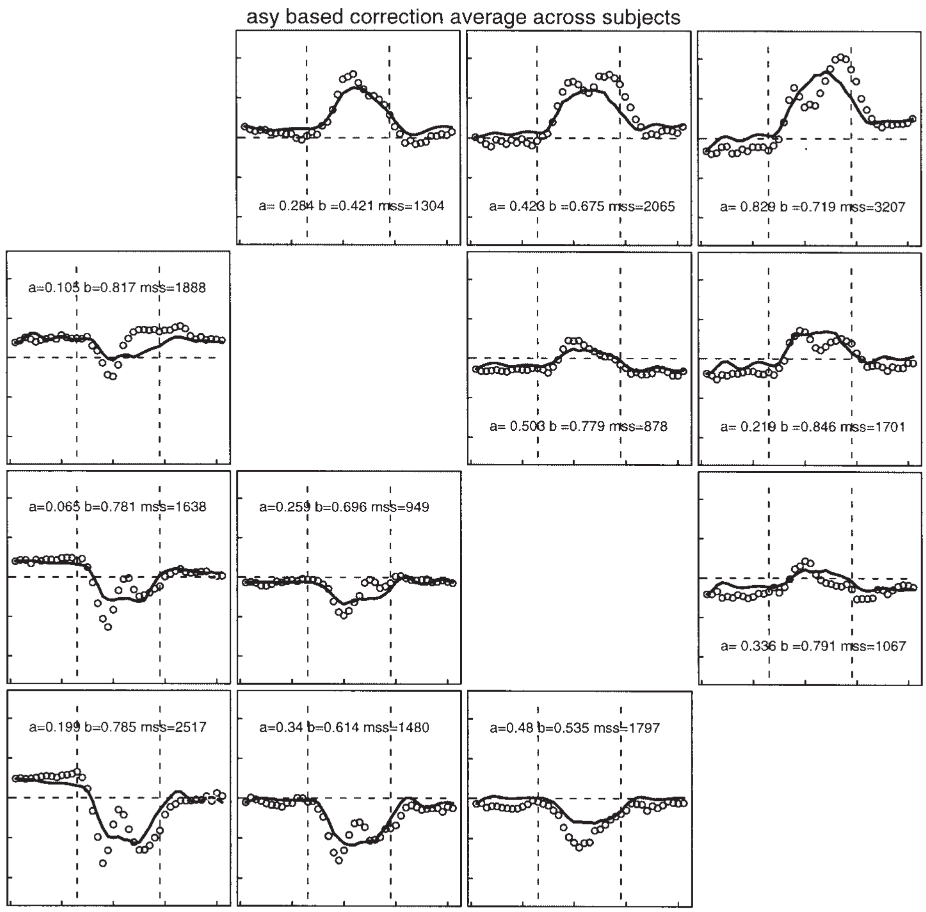

Fig. 7. Observed and predicted asynchronies of the asynchrony-based model: complete analysis. a and b are the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$; mss is the mean sum of squared deviation (averaged over subjects).

Discussion

In the experiment reported, we observed a novel phenomenon: When synchronizing with a metronome that undergoes accelerando or ritardando, the synchronization errors follow a systematic pattern in the transient phase, which held for all subjects. If the initial and final tempi differ widely, subjects first undershoot the smoothly changing tempo, then catch up and overadjust twice before settling on the goal tempo. The question is whether linear period-adjustment models are compatible with such data patterns. Surprisingly, a new version of the highly successful two-level phase-error correction model (Pressing, 1998; Vorberg & Wing, 1996; Vorberg & Schulze, 2002) fared rather well. Based on the assumptions

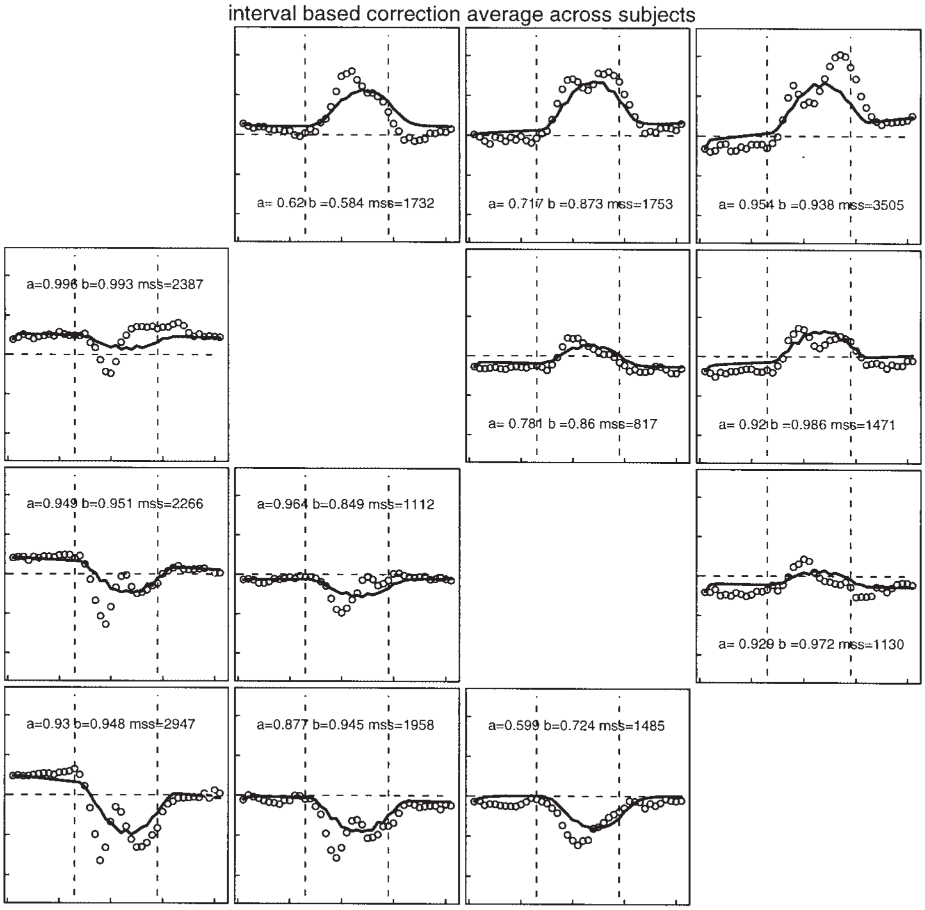

Fig. 8. Observed and predicted asynchronies of the interval-based model: complete analysis. a and b are the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$; mss is the mean sum of squared deviation (averaged over subjects).

that (1) local phase adjustments and global period adjustments are active simultaneously during noticeable tempo changes, that (2) both mechanisms are fed by the same error information, and that (3) adjustments are first-order linear, the model accounts for the qualitative data pattern quite well, although quantitative goodness-of-fit leaves much to be desired. However, even the qualitative fit was satisfactory only when restricted to the transient phase of the data, which implies that there must exist additional control mechanisms that determine when the period adjustment mechanism is started and stopped (e.g., by setting the period correction gain).

The competitor model, a linear version of the phase and period adjustment model originally proposed by Mates (1994a, 1994b), was less successful. We found no evidence for the more complex assumption that period adjustment acts on the difference between metronome and self-generated *intervals*, rather than on the asynchronies, that is, the time difference between perceived external and motor *events*. From the observed lack of fit and from simulation studies, it seems to us that the Mates model is unable to account for the qualitative effects we observed during the tempo transitions, although we have been unable to show this analytically. Note that this model is also inferior to the asynchrony-based model on quantitative grounds, whether fit to the complete sequences including the stationary phases or to the transient phase only. It remains to be seen whether the superiority of the asynchrony-based over the interval-based adjustment assumption persists when models are extended to second-order correction, and when control mechanisms are added for switching them on and off. We are currently pursuing such questions.

A radically different approach to modeling synchronization assumes internal oscillators entrained by the external stimuli and interacting with each other (Eck, 2000; Large & Jones, 1999; Large & Palmer, 2002). This approach results in complicated mathematical structures, which are difficult to evaluate empirically because fitting the model to data and parameter estimation are quite a challenge. This is one reason why we have preferred to start from conceptually simpler models, adding complex assumptions if needed only.

How is the present experimental paradigm related to synchronization in ensemble playing? We see at least two differences: First, in ensemble playing, there is visual contact with other players. Second, in rehearsal, the same accelerando or ritardando can be practiced many times, and there are additional musical cues to which a particular tempo trajectory is tied, whereas in our experiments, the amount and direction of the tempo change varied from trial to trial, with very few cues that reduced uncertainty, neither visual nor musical. It is to be expected that performance improves with certainty about the rate of change. Subjects may even be able to develop timekeeping routines for particular accelerando and ritardando trajectories, routines that predict tempo changes rather adjust for deviations thereof. Studies that vary the shape of the trajectory and compare performance under blocked and varied tempo change conditions are likely to give insight into the development of such timekeeping programs dedicated to particular tempo changes.

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