PERGAMON

Personality and Individual Differences 35 (2003) 659–669

PERSONALITY AND
INDIVIDUAL DIFFERENCES

www.elsevier.com/locate/paid

Quantifying cognitive complexity: evidence from a reasoning task

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Received 7 February 2002; received in revised form 17 May 2002; accepted 7 August 2002

Abstract

There are some doubts about the nature of *cognitive complexity*. It has been proposed that the loadings on the first un-rotated factor can be taken as a way to quantify the cognitive complexity of a given task. However, the evidence is sparse. The present study tests 1968 participants in a computerized task that comprises linear syllogisms or three-term series problems. The correlation matrix is submitted to a factor analysis. The first un-rotated factor is taken as the vector of cognitive complexity. The vector of task difficulty was obtained after the proportion of participants that failed each syllogism. In addition to task empirical difficulty, three information processing models are taken as predictors of cognitive complexity. Then, regression analyses were carried out to predict cognitive complexity from the information processing (IP) models and task difficulty. Results show that the IP models and task difficulty predict cognitive complexity defined by the loadings on the first un-rotated factor. Therefore, it is concluded that those loadings can be taken as a way to quantify cognitive complexity.

Keywords: Reasoning; Linear syllogisms; Information processing models; Cognitive complexity; Factor loadings

1. Introduction

Jensen (1998a, 1998b) proposed that cognitive complexity could be quantified after the loadings on the first un-rotated factor, because complex tasks show higher loadings than simple tasks on that factor. Tasks are considered more complex if, for instance, they involve a wider range of

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PII: S0191-8869(02)00243-X

elementary cognitive processes (Jensen, 1998a; Stankov, 2000; Stankov & Raykov, 1995). However, the results are far from conclusive.

Some authors established a difference between cognitive complexity and difficulty. Thus, Stankov (2000) considers that task difficulty can be represented by the arithmetic mean calculated after the performance on a test. Test difficulty could be different from its complexity. However, it is easy to find that cognitive complexity of homogeneous tasks and their levels of difficulty covariate (Spilsbury, Stankov, & Roberts, 1990). Some authors found significant correlations between difficulty, as measured by response time and accuracy indexes, and the cognitive complexity derived from the cognitive processes involved in a task. In a number of studies mean reaction time and accuracy have been recorded for a variety of tasks. The correlations found indicated that processing time increases with complexity (Jensen, 1992, 1998a). Thus, variations in cognitive complexity can reflect variations in the amount of mental processes required. The critical aspect is that more complex tasks could imply a higher number of cognitive processes.

In order to test whether the loadings on the first un-rotated factor reflects cognitive complexity, it is interesting to have an independent criterion beyond task difficulty. A way to do this is to select a task for which the cognitive processes involved can be specified. Linear syllogisms seem to be a nice candidate (Johnson-Laird, 1999; Sternberg, 1980).

Linear syllogisms comprise two premises. Each premise describes the relationship between two of three terms (A–B–C). One of the terms (B) overlaps between the premises. This overlap allows the establishment of the relation between the pair of terms not presented in a single premise. One example:

Anna (A) is taller than Vicky (B)
Vicky (B) is taller than Bea (C)
Who is tallest?

Each premise involves a relation between two terms ($A > B : B > C$). The person must combine the information from the two premises in order to make an inference about the relationship between A and C.

This basic structure allows the construction of 32 syllogisms, combining the relation between the terms on the two premises and the localization of the response (see Table 1, column 1). Sixteen syllogisms are characterized by positive comparative forms (taller than) and the remaining are characterized by negative equatives (not taller than).

Experimental manipulations have been done in order to explore the cognitive processes involved when solving the syllogisms. Results were interpreted within the information-processing framework. These models were developed considering different ways by which the information is mentally represented. Task difficulty is specified according to the resources needed to carry out the processes.

DeSoto, London, and Handel (1965) proposed a spatial model in which the information is mentally represented in the form of mental images. The formation and integration of mental images depends on whether the terms within the two premises are in a preferred direction (up–down; left–right). An alternative linguistic model was formalized by Clark (1969). According to the linguistic model, the processes of encoding and recalling the information of the two premises involve a deeper analysis of the words. Both processes are guided by the congruency principle,

Table 1
Vectors of cognitive complexity (first un-rotated factor) and empirical difficulty

Syllogisms	Cognitive complexity	Empirical difficulty (%)
1/ A > B:B > C: >	0.38	6
2/ A > B:B > C: <	0.38	8
3/ B > C:A > B: >	0.43	15
4/ B > C:A > B: <	0.42	13
5/ C < B:B < A: >	0.34	21
6/ C < B:B < A: <	0.29	20
7/ B < A:C < B: >	0.35	30
8/ B < A:C < B: <	–	–
9/ A > B:C < B: >	0.38	12
10/ A > B:C < B: <	0.37	10
11/ C < B:A > B: >	0.39	20
12/ C < B:A > B: <	0.42	10
13/ B < A:B > C: >	0.47	43
14/ B < A:B > C: <	0.47	39
15/ B > C:B < A: >	0.52	27
16/ B > C:B < A: <	0.52	32
17/ A < B:B < C: > neg.	0.36	30
18/ A < B:B < C: < neg.	–	–
19/ B < C:A < B: > neg.	38	35
20/ B < C:A < B: < neg.	0.46	31
21/ C > B:B > A: > neg.	0.43	14
22/ C > B:B > A: < neg.	0.48	26
23/ B > A:C > B: > neg.	0.42	18
24/ B > A:C > B: < neg.	–	–
25/ A < B:C > B: > neg.	0.42	51
26/ A < B:C > B: < neg.	0.43	39
27/ C > B:A < B: > neg.	0.48	46
28/ C > B:A < B: < neg.	0.49	39
29/ B > A:B < C: > neg.	0.43	19
30/ B > A:B < C: < neg.	0.32	17
31/ B < C:B > A: > neg.	0.28	15
32/ B < C:B > A: < neg.	0.45	24

that makes reference to the way the information is going to be searched. Finally, considering both spatial and linguistic models, Sternberg (1980) proposed a mixed model in which both language and images are used. This model is based on the idea that both linguistic and spatial operations are used to solve syllogisms.

Let us examine some similarities and differences among the three IP models. This will serve the purpose of explaining the sources of task difficulty.

The three models agree that there are encoding, negation, marking, and response operations that contribute to the participants' performance. All linear syllogisms contain terms and relations that must be encoded and require a response, but only some problems contain premises with negations and marked adjectives. However, the models disagree as to which of the operations are spatial or linguistic, as well as to what further operations are required. The three models agree that marked adjectives and negations should make the problem harder, but for different reasons. The spatial model states that difficulty increases because processing of negations and marked adjectives requires a more complex encoding of information into a mental spatial array. The linguistic model states that the difficulty increases from a linguistic encoding process. The mixed model states that negations require a more complex linguistic encoding process, while marked adjectives require more complex linguistic encoding and more complex spatial encoding. The models agree that a pivot search is needed under some circumstances, but again for different reasons. The spatial model states that pivot search is required for premises that are not end anchored (the first term, A, is the middle rather than an end of the spatial array). The linguistic model states that pivot search results from compression of the first premise in the deep-structural encoding. If the term dropped from working memory in compression is the pivot (B), the participant has to retrieve that term back from long-term memory. The mixed model states that pivot search is required if the reformulated deep-structural version of a negative second premise does not have the pivot (B) in its most recently available proposition. The spatial and mixed models agree that the terms of the two premises are combined into a unified mental representation. The combination requires a seriation operation. The linguistic model proposes that the two premises are stored separately. The linguistic and mixed models (but not the spatial model) postulate an operation to establish congruence between question and answer. In the spatial model (but not in the linguistic and mixed models) participants are hypothesized to prefer acting top-down between and within premises. Because of that, more difficulty will be encountered in seriation if the term at the preferred end of the array does not occur in the first premise. In the linguistic model, participants search the deep-structural propositions for the term that answers the question. In the mixed model (but not in the spatial or linguistic models), participants have to search for the response if their active location in their combined spatial array is not in the half of the array containing the answer. Finally, the three models agree that the last operation is a response process.

In summary, the information processing models stipulate a specific number of cognitive processes related to the formal structure of the syllogisms (see Appendices A–C). Thus, it is worthwhile to consider the predictions made by each model while studying the cognitive complexity of the reasoning task. If we have a set of tasks ordered according to their levels of difficulty derived from a theory, then they must correlate with their loadings on the first un-rotated factor if, as predicted, those loadings reflect the number of cognitive processes involved.

The main purpose of the present research is to study whether the loadings on the first un-rotated factor can be taken as a representation of the cognitive complexity of a well-known reasoning task. Cognitive complexity must be predicted both by empirical task difficulty and the difficulty postulated by the information-processing models.

2. Method

2.1. Participants

Participants were 1,968 applicants for an Air Traffic Control training course: 983 were females and 985 were males. The mean age was 27.88 and the standard deviation was 4.49. The participants were all university graduates.

2.2. Measures and procedures

The task comprises 32 syllogisms. Items were constructed according to the structure presented in Table 1 (column 1). Half of the syllogisms were positive comparative and half were negative equatives.

The task was self-administered. The first premise appeared on the computer screen and the participant had to "click" with the mouse button on it when mentally coded. Then the second premise appeared and then the question. The instructions were to get the correct response as soon as possible. The computer recorded the processing time for every part of the syllogism (first premise, second premise, question, and response). Errors and hits were also recorded.

2.3. Analyses

There were technical problems to recover information from three syllogisms (there were three messages delivered by the computer program remembering that the answer must be delivered as quick as possible; those messages appeared following the three lost syllogisms and the algorithm failed in those instances). Therefore, the analyses were carried out considering 29 syllogisms. The correlation matrix was submitted to a factor analysis (the correlation matrix comprised tetrachoric correlations because the input data contained a dichotomous value: 1 = correct; 0 = incorrect). A principal axis factoring was computed. Cognitive complexity was quantified by the loadings on the first un-rotated principal factor (Table 1; column 2). Empirical difficulty was defined by the proportion of participants failing the syllogisms (Table 1; column 3).

Several regression analyses were performed to predict cognitive complexity after empirical difficulty and the information processing models enumerated previously (see Appendix).

3. Results

Linear regression analyses were carried out taking the cognitive processes of the information processing models and the task empirical difficulty as the predictor variables. Cognitive complexity was the dependent variable.

Results show that empirical task difficulty predicted cognitive complexity ($R = 0.524$). Moreover, Table 2 shows that the information-processing models predicted cognitive complexity. The spatial model shows the higher R value (0.628), while the linguistic model shows the lower R value (0.469). The mixed model shows an intermediate R value (0.539).

It is noteworthy that the specific cognitive processes postulated by the information processing models do not predict cognitive complexity. There is only one exception: pivot search from the spatial model.

In summary, as predicted, the task empirical difficulty and the difficulty postulated by the information-processing models are related to cognitive complexity. The implication is that the first un-rotated principal factor can be taken as a way to quantify cognitive complexity.

Table 2
R values for the spatial, linguistic and mixed IP models (slopes for specific processes are also shown)

IP processes	Cognitive complexity
Spatial model	$R = 0.628$
Non-preferred direction	−0.131
Negation	0.091
Pivot search	0.589*
Non-preferred direction	0.161
Linguistic model	$R = 0.469$
Mark	−0.138
Negation	0.094
Non-congruence	0.348
Pivot recuperation	−0.247
Mixed model	$R = 0.539$
Mark	−0.225
Negation	−0.067
Pivot search	0.284
Response search	0.288
Congruence	0.328

*$P < 0.01$.

4. Discussion

The findings support the assumption that the loadings on the first un-rotated factor can be taken as a way to quantify cognitive complexity. First, cognitive complexity was predicted by the task empirical difficulty. Second, and perhaps more important, the difficulty postulated by the information processing models predicted cognitive complexity.

These results leave room for the conclusion that the loadings on the first un-rotated factor reflect variations in cognitive complexity. Presumably, these loadings capture an estimation of the number of elementary cognitive processes required to solve the task: the higher the number of cognitive processes needed to solve a cognitive task, the greater its cognitive complexity.

4.1. What is cognitive complexity?

Researchers used basic elements of the cognitive system to understand performance differences. One of those basic elements is working memory (Colom, Flores-Mendoza, & Rebollo, in press; Colom, Palacios, Kyllonen, & Juan-Espinosa, submitted for publication; Jensen, 1998a; Kyllonen & Christal, 1990). Working memory is frequently related to performance differences. Two are the main properties of working memory: (a) its limited capacity, and (b) the short duration of information. Syllogisms vary in the amount of information they entail. Therefore, it is tenable that syllogisms that involve more processing claim more resources. Considering that information is temporarily stored in WM, the time needed to process the information of the two premises is important. If syllogisms involve more information that can be processed by WM, the system can be overloaded and therefore, the information has to be transferred to a more permanent stage. The time taken to process and transfer information can yield a partial loss of information.

However, Spilsbury et al. (1990) have shown that working memory load cannot be considered as an adequate explanation of cognitive complexity. Complexity is defined by these researchers after the Snow, Kyllonen, and Marshalek (1984) study: increasing number of components required for a given task. Nevertheless, Spilsbury et al. (1990) state that complexity and difficulty should be distinguished independently of references to intelligence. The present study does just that. Those researchers propose that "for a task to increase in complexity it may be essential that there is a qualitative change in the task" (p. 1077). The *qualitative* change requires memory span and reasoning. Remember that the present study displays sequentially the premises required to solve the task. Thus, there is a memory span component as well as a reasoning one, which fulfils the requirements proposed by Spilsbury et al. (1990).

The loading of a given syllogism on the first un-rotated factor depends on the correlation between that syllogism and the remaining: the higher the correlation, the greater the loading. The increase in the loadings suggests a broader spectrum of cognitive processes. The syllogism "$B > C : B < A :: <$" shows a loading of 0.52, while the syllogism "$C < B : B < A :: <$" shows a loading of 0.29. The argument suggests that the latter syllogism requires less processing than the former.

In sum, syllogisms vary in their difficulty. There could be several explanations for these variations. One of that explanations is cognitive complexity. Easier syllogisms involve fewer cognitive processes and, therefore, less processing is neccesary to get an answer. Harder syllogisms involve more cognitive processes and, therefore, more processing is required. The number of processes is stipulated by some information processing models. Moreover, complex syllogisms will result in more people failing to get the answer. The present study shows that both empirical task difficulty and the difficulty postulated by three well-known information processing models predicted the cognitive complexity quantified by the first un-rotated factor. Although the prediction is far from perfect, it can be stated that the loadings on the first un-rotated factor represent the amount of resources (processes and/or capacity) claimed for the task. When no other information is available, the loadings on the first un-rotated factor can be taken as a way to quantify cognitive complexity.

However, we must acknowledge that further research is needed. The present study analyzed a clearly specified task. Furthermore, the task format is relatively homogeneous. Could the same result be obtained for other tasks? The analogies studied by Sternberg (1977) are one obvious candidate. Although we predict similar findings, it remains to be seen.

Acknowledgements

This research was supported by the project AENA-UAM/785001.

Appendix A. Prediction of difficulty from spatial model

Syllogisms	Non-preferred direction	Negation	Pivot search	Non-preferred direction
1/ A > B:B > C: >	0	0	1	0
2/ A > B:B > C: <	1	0	1	0
3/ B > C:A > B: >	0	0	1	1
4/ B > C:A > B: <	1	0	1	1
5/ C < B:B < A: >	2	0	1	1
6/ C < B:B < A: <	3	0	1	1
7/ B < A:C < B: >	2	0	1	0
8/ B < A:C < B: <	3	0	1	0
9/ A > B:C < B: >	1	0	0	0
10/ A > B:C < B: <	2	0	0	0
11/ C < B:A > B: >	1	0	0	1
12/ C < B:A > B: <	2	0	0	1
13/ B < A:B > C: >	1	0	2	0
14/ B < A:B > C: <	2	0	2	0
15/ B > C:B < A: >	1	0	2	1
16/ B > C:B < A: <	2	0	2	1
17/ A < B:B < C: > neg.	2	2	1	0
18/ A < B:B < C: < neg.	3	2	1	0
19/ B < C:A < B: > neg.	2	2	1	1
20/ B < C:A < B: < neg.	3	2	1	1
21/ C > B:B > A: > neg.	0	2	1	1
22/ C > B:B > A: < neg.	1	2	1	1
23/ B > A:C > B: > neg.	0	2	1	0
24/ B > A:C > B: < neg.	1	2	2	0
25/ A < B:C > B: > neg.	1	2	2	0
26/ A < B:C > B: < neg.	2	2	2	0
27/ C > B:A < B: > neg.	1	2	2	1
28/ C > B:A < B: < neg.	2	2	2	1
29/ B > A:B < C: > neg.	1	2	0	0
30/ B > A:B < C: < neg.	2	2	0	0
31/ B < C:B > A: > neg.	1	2	0	1
32/ B < C:B > A: < neg.	2	2	0	1

Appendix B. Prediction of difficulty from linguistic model

Syllogisms	Mark	Negation	Non-congruence	Pivot recuperation
1/ A > B:B > C: >	0	0	0	1
2/ A > B:B > C: <	1	0	1	1
3/ B > C:A > B: >	0	0	0	0
4/ B > C:A > B: <	1	0	1	0
5/ C < B:B < A: >	2	0	1	1
6/ C < B:B < A: <	3	0	0	1
7/ B < A:C < B: >	2	0	1	0
8/ B < A:C < B: <	3	0	0	0
9/ A > B:C < B: >	1	0	0	0
10/ A > B:C < B: <	2	0	0	0
11/ C < B:A > B: >	1	0	0	0
12/ C < B:A > B: <	2	0	0	0
13/ B < A:B > C: >	1	0	1	0
14/ B < A:B > C: <	2	0	1	0
15/ B > C:B < A: >	1	0	1	0
16/ B > C:B < A: <	2	0	1	0
17/ A < B:B < C: > neg.	2	2	1	0
18/ A < B:B < C: < neg.	3	2	0	0
19/ B < C:A < B: > neg.	2	2	1	1
20/ B < C:A < B: < neg.	3	2	0	1
21/ C > B:B > A: > neg.	0	2	0	0
22/ C > B:B > A: < neg.	1	2	1	0
23/ B > A:C > B: > neg.	0	2	0	1
24/ B > A:C > B: < neg.	1	2	1	1
25/ A < B:C > B: > neg.	1	2	1	0
26/ A < B:C > B: < neg.	2	2	1	0
27/ C > B:A < B: > neg.	1	2	1	0
28/ C > B:A < B: < neg.	2	2	1	0
29/ B > A:B < C: > neg.	1	2	0	0
30/ B > A:B < C: < neg.	2	2	0	0
31/ B < C:B > A: > neg.	1	2	0	0
32/ B < C:B > A: < neg.	2	2	0	0

Appendix C. Prediction of difficulty from mixed model

Syllogisms	Mark	Negation	Pivot search	Response search	Congruence
1/ A > B:B > C: >	0	0	0	1	0
2/ A > B:B > C: <	1	0	0	0	1
3/ B > C:A > B: >	0	0	0	0	0
4/ B > C:A > B: <	1	0	0	1	1
5/ C < B:B < A: >	2	0	0	0	1
6/ C < B:B < A: <	3	0	0	1	0
7/ B < A:C < B: >	2	0	0	1	1
8/ B < A:C < B: <	3	0	0	0	0
9/ A > B:C < B: >	1	0	0	1	0
10/ A > B:C < B: <	2	0	0	0	0
11/ C < B:A > B: >	1	0	0	0	0
12/ C < B:A > B: <	2	0	0	1	0
13/ B < A:B > C: >	1	0	0	1	1
14/ B < A:B > C: <	2	0	0	0	1
15/ B > C:B < A: >	1	0	0	0	1
16/ B > C:B < A: <	2	0	0	1	1
17/ A < B:B < C: > neg.	2	2	0	1	1
18/ A < B:B < C: < neg.	3	2	0	0	0
19/ B < C:A < B: > neg.	2	2	1	0	1
20/ B < C:A < B: < neg.	3	2	1	1	0
21/ C > B:B > A: > neg.	0	2	0	0	0
22/ C > B:B > A: < neg.	1	2	0	1	1
23/ B > A:C > B: > neg.	0	2	1	1	0
24/ B > A:C > B: < neg.	1	2	1	0	1
25/ A < B:C > B: > neg.	1	2	1	1	1
26/ A < B:C > B: < neg.	2	2	1	0	1
27/ C > B:A < B: > neg.	1	2	1	0	1
28/ C > B:A < B: < neg.	2	2	1	1	1
29/ B > A:B < C: > neg.	1	2	0	1	0
30/ B > A:B < C: < neg.	2	2	0	0	0
31/ B < C:B > A: > neg.	1	2	0	0	0
32/ B < C:B > A: < neg.	2	2	0	1	0

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