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The cylindrical structure of the Wechsler Intelligence Scale for Children — IV: A retest of the Guttman model of intelligence

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Abstract

A previous study on the underlying structure of the Wechsler intelligence test (WISC-R; [Wechsler, D. (1974). Manual WISC-R: Wechsler intelligence scale for children—Revised. New York: Psychological Corporation]), using smallest space analysis (SSA) [Guttman, L., and Levy, S. (1991). Two structural laws for intelligence tests. *Intelligence*, 15, 79–103] had indicated a three-dimensional cylindrical solution. The first described level of abstract thinking (rule inferring, rule applying, rule following or new learning tasks), the second related to mode of representation (verbal, numeral, visual), while the third dimension related to output mode (oral, manual, or pencil and paper). In view of the appearance of the recent version of this test (WISC-IV; [Wechsler, D. (2003). Manual for the Wechsler Intelligence Scale for Children—Fourth edition. San Antonio, TX: The Psychological Corporation]), the purpose of the present study is to test Guttman's model of intelligence on the current version of the scale. Thus, the intercorrelation matrix of the WISC-IV subtests of the entire normative sample of 2200 children was submitted to SSA. This solution replicates Guttman and Levy three-dimensional cylindrical structure almost completely, and it offers further differentiation of the visual mode into geometric and pictorial modes and implies that the Block Design subscale relates to the "rule inferring" category. The Guttman model organization of the present solution provides an elegant description of the structure of intelligence and suggests the construction of new subscales for measuring different aspects of intelligence.
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The pursuit of the underlying structure of intelligence has intrigued psychologists since the dawn of the previous century. The most common method of examining the underlying nature of a set of variables in general, and the subtests of an intelligence test in particular, is factor analysis. In fact, factor analysis was originally invented by Spearman (1904) in order to conceptualize the nature of intelligence. Its underlying assumption is that observed variables are the functions of latent variables called factors and its goal is to identify these factors. Indeed, most widely accepted models of intelligence today are based on factor analytic research (e.g. Cattell–Horn–Carroll Theory; McGrew, 1997). Factor analytic studies of cognitive abilities such as Carroll's meta-analysis yield a hierarchical structure, with *g* at the apex, an array of broad cognitive abilities underneath, and narrow abilities under those (Carroll, 1993).

The Wechsler scales, the most commonly used tests of intelligence, have traditionally been divided conceptually into verbal and non-verbal (performance) sections.

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However, with the publication of the WISC-R, factor analyses revealed the presence of a third factor separate from the verbal and performance factors (Kaufman, 1979), calling into question the two-part model of intelligence. During the development of the WISC-III, therefore, an additional subtest was developed to attempt to clarify the factor structure of the test, leading to the emergence of four factors (Wechsler, 1991), though the venerable Verbal and Performance IQs remained. The most recent, fourth edition (WISC-IV; Wechsler, 2003) has embraced and expanded the four factors, leading to the abandonment of the Verbal and Performance dichotomy in favor of four scores: the *Verbal Comprehension Index* (Similarities, Vocabulary, Comprehension, plus supplemental subtests Information, Word Reasoning), *Perceptual Reasoning Index* (Block Design, Picture Concepts, Matrix Reasoning, plus supplemental Picture Completion), *Working Memory Index* (Digit Span, Letter–Number Sequencing, plus supplemental Arithmetic), and *Processing Speed Index* (Coding, Symbol Search, plus supplemental Cancellation).

While factor analysis is the most widely used multivariate analysis method, some non-metric alternatives have been developed to analyze relationships in psychological phenomena. Some of these alternatives relate to multidimensional scaling (MDS, Kruskal, 1964) and Smallest Space Analysis (SSA, a variant of MDS developed by Guttman, 1968). These methodologies represent variables as points in Euclidian space with interpoint distances corresponding to proximities measured among the variables (e.g. intercorrelations). The underlying rationale of this approach is that the isomorphism between the proximity measures among the variables and their interpoint distances in the Euclidian space enables direct observation of intercorrelation matrix. Another assumption of this approach is that geometric representation of order relations among variables, rather than mathematical expression of items' loadings on factors, may highlight in the data structures that are not so apparent in factor analytic solutions. The loss function for solutions based on this approach is Stress (Kruskal, 1964) or coefficient of alienation (Borg & Shye, 1995 p. 129; Guttman, 1968). These measures range from 1 to 0 (0 represents a perfect match).

For an intuitive explanation of MDS let us assume that we want to obtain a spatial representation of the intercorrelation matrix among *N* variables. In order to do that let us assume that we have *N* small metal balls — each representing a different variable. Furthermore, let us assume that all metal balls incorporate inside them magnetic forces that represent the intercorrelation among the variables. In this manner a very high correlation between two variables will be represented in a strong magnetic attraction between these balls while a negative correlation between two other variables will be represented by a repulsion between those balls. If we now put these balls on a square board, the overall configuration of the balls will depict the overall configuration of the intercorrelations among the *N* variables. Nevertheless, as a perfect match between the configuration of the intercorrelations and the configuration of the balls may not always be possible, the loss function will indicate to what degree the balls' configuration represents the intercorrelation matrix. Furthermore, while in the physical world only up to three-dimensional configuration among the balls is possible, mathematically any N-variable matrix may be perfectly represented in $N-2$ dimensions.

In view of the potential of MDS "in revealing insights that classical factor analytic techniques seem to have hidden (Sternberg, 1984; p. xii)," the scarcity of its application to the study of intelligence is surprising. Nevertheless, the few of these studies that exist yield interesting insights into the underlying structure of intelligence in general and I.Q. in particular. Indeed, the few studies that have focused on the study of intelligence by employing MDS approach indicated a radex model (L. Guttman, 1954). This model describes the simultaneous appearance of two orderings of the variables, one from the center to the periphery and the other a circular order around the center. This arrangement forms a disc in two dimensions or a sphere in three dimensions with sectors or conic areas relating to different characteristics of the variables. In ability testing this structure suggests the appearance of more cognitively complex tasks closer to the center of the figure and less complex tasks farther away from the center, with the appearance of verbal, numerical, and figural-spatial test content in separate sectors. The radex model of intelligence was tested through MDS (Marshalek, Lohman, & Snow, 1983; Snow, Kyllonen, & Marshalek, 1984) or SSA (Adler & Guttman, 1982; Beauducel, Brocke, & Liepmann, 2001; Koop, 1985; Guttman, 1965a,b; Peled, 1984; Schlesinger & Guttman 1969; Tziner & Rimmer, 1984, 1991; Ziedner & Feitelson, 1991). In all the above studies, encompassing various ability test batteries and various sample characteristics, the findings were more or less similar, indicating a two-dimensional solution where the tests were ordered from the most complex, abstract, inferential tasks near the origin, to the more simple tasks requiring associative learning at the periphery. Furthermore, in most studies, subtests that include items in the verbal mode (like vocabulary), figural-spatial mode (like matrices), and numerical mode (like mathematical

Table 1
The WISC-IV subscales and their categorization

The subtest	Code	Level of abstraction	Presentation mode	Response mode
Block design	BD	High	Geom/pict	Manual
Similarities	SI	High	Verbal	Oral
Digit span	DS	Low	Numeric	Oral
Picture concepts	PC	High	Geom/pict	Manual
Coding	CD	Low	Geom/pict	Pencil
Vocabulary	VC	Medium	Verbal	Oral
Letter–number sequencing	LN	Low	Numeric	Oral
Matrix reasoning	MR	High	Geom/pict	Manual
Comprehension	CO	Medium	Verbal	Oral
Symbol search	SS	Low	Geom/pict	Pencil
Picture completion	PCM	Medium	Geom/pict	Manual
Cancellation	CA	Low	Geom/pict	Pencil
Information	IN	Medium	Verbal	Oral
Arithmetic	AR	Medium	Numeral	Oral
Word reasoning	WR	Medium	Verbal	Oral

exercises), clustered separately in different sectors around the center.

Interestingly enough, although the Wechsler IQ test is the most popular measure of intelligence, the radex model was tested on the WISC only once. In that study responses to the WISC-R (Wechsler, 1974) of two large of American and Israeli samples, Guttman and Levy (1991) obtained a three-dimensional solution using SSA. Based on these findings they offered a modified three-dimensional version of the radex theory of intelligence.

The first dimension in the Guttman and Levy (1991) model relates to the level of abstract thinking required of the examinee, with three levels: 1. Rule inferring tasks — tasks that require abstract thinking. 2. Rule applying tasks — tasks that require knowledge of data presented in an item, such as defining a vocabulary word or solving an oral arithmetic problem. 3. Tasks that require new associative learning or rule following, such as clerical or short-term memory tasks.

The second dimension in this model relates to the mode of representation or content of the item. This dimension has three categories: 1. Verbal. The item involves words, such as answering general knowledge questions. 2. Numerical. The item involves numbers, such as recalling digits or solving math problems. 3. Visual. The item involves geometrical or pictorial representations, such as finding the missing part of a picture or matching a design with colored blocks. The third dimension relates to the output mode required of the examinee. This dimension has three categories: 1. Oral. The examinee responds orally. 2. Manual. The examinee manually manipulates the stimulus materials. 3. Paper and pencil. The examinee writes or marks a response on paper.

In view of the recent publication of the new version of the WISC (WISC-IV) and the introduction of new subtests into the battery, the purpose of the present study is to examine the applicability of the radex model of intelligence to the new version of the scale. More specifically, the purpose of this study is to examine whether the theoretical categorization of the new subscales would fit the actual MDS solution. Thus, Table 1 includes the 15 subscales of the WISC-IV and their categorization according to Guttman's three-dimensional model.

1. Method

The intercorrelation matrix of the WISC-IV subtests for the entire normative sample of the scale was obtained from the Technical and Interpretive Manual (Wechsler, 2003). The total sample consisted of 2200 children, 100 male and 100 female in each of the 11 age groups between six and sixteen, and was very representative of the U. S. population based on census data from March, 2000. The intercorrelation matrix of the WISC-IV total sample was analyzed through Weighted SSA1 (WSSA1, Amar & Toledano, 2001), a technique appropriate for use with a symmetrical matrix of observed relationships. The present matrix was analyzed employing the procedures taken from Amar and Toledano (2001, formulas (1) and (2)).[1] The matrix, in this case of Pearson's r, is analyzed to minimize the Euclidean distance between the points in

[1] "Given a symmetric matrix of dissimilarity coefficients $\{D_{ij}\}$, D_{ij} being the coefficient between elements V_i and V_j, we want to represent the elements (V_k;k=1,..n) as points in an m-dimensional Euclidean space such that the following monotonicity condition is fulfilled "as well as possible":

$$D_{ij} < D_{kl} \Leftrightarrow d_{ij} < d_{kl}$$

for each quadruplet (i,j,k,l), d_{ij} being the computed Euclidean distance between points representing V_i and V_j, in the m-dimensional space:

$$d_{ij} = \sqrt{\sum_{a=1}^{m} (x_{ia} - x_{ja})^2}$$

The monotonicity condition is fulfilled as well as possible for dimensionality m thought to be the smallest. The solution is noted:

$$x = \{x_{ia}\}_{i=1,2,\ldots n;a=1,2,\ldots m}$$

Note: If the input matrix is that of similarity coefficients (as correlations for example), $\{R_{ij}\}$, the monotonicity condition becomes: $R_{ij}>R_{kl} \Leftrightarrow d_{ij}<d_{kl}$. But then, we can always find a simple monotone function which transform similarities into dissimilarities: $D_{ij}=M(R_{ij})$."

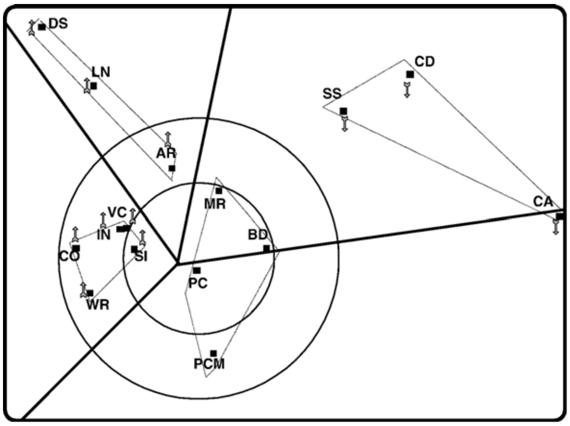

Fig. 1. Three-dimensional Guttmon model of the WISC-IV.

m-dimensional space, then a loss function is calculated (the coefficient of alienation) to determine the dimension that yields the closest fit to the data.

2. Results

The SSA solution for the total normative sample of the WISC-IV yielded a three-dimensional model with a coefficient of alienation equal to .07 (a two-dimensional model yielded a c.o.a. of .11). Fig. 1 presents a two-dimensional projection of the SSA solution.

The figure demonstrates a two-dimensional projection with a circular base around the axis (radex). The central circle includes subscales of high inference ability – rule-inferring tasks – i.e. Similarities (SI), Matrix Reasoning (MR), Block Design (BD), and Picture Concepts (PC). The second circle includes subscales that require less inferential ability – rule-applying tasks – i.e., Arithmetic (AR), Information (IN), Vocabulary (VC), Comprehension (CO), Word Reasoning (WR), and Picture Completion (PCM). The third circle includes subscales that require learning or low inferential ability — Letter–Number Sequencing (LN), Symbol Search (SS), Coding (CD), and Cancellation (CA). Thus, the first dimension from the center to the periphery indicates level of inference.

The second dimension is apparent in the various sectors, dividing the circles into subscales of a verbal nature (SI, VC, IN, CO, WR), numerical (AR, LN, DS), geometrical (MR, SS, BD, CD), and pictorial (PC, PCM, CA). The third dimension is represented in Fig. 1 by small arrows. Arrows pointing up represent points above the plane and include subscales that require an oral mode of response — SI, AR, LN, DS, VC, IN, CO, and WR. Arrows pointing downward represent points below the horizontal plane and include subscales that require pencil and paper mode of response — CD, CA and SS. The remaining points without arrows represent items in the vicinity of the plane and subscales that require pointing or manual mode of response — MR, PC, PCM, and BD. Finally, for comparison, the four factors of the WISC-IV (Wechsler, 2003) are identified in Fig. 1 by four rectangular forms.

3. Discussion

The current SSA solution of the WISC-IV supports Guttman's model of the structure of intelligence. The three-dimensional solution differs only slightly from Guttman's original model, derived from the WISC-R (Guttman & Levy, 1991). In the first dimension – level of abstraction – almost all subscales appear in the same regions as in the original study. Only BD appeared in a different region, in the rule-inferring region rather than in the rule-applying region. Indeed, BD involves the use of abstract reasoning ability (Hale & Fiorello, 2004; Mayman, Schafer, & Rapaport, 1964; McCloskey & Maerlender, 2005). The second dimension – mode of representation – has an additional segment, which differentiates the geometric from the pictorial. This may be a reflection of the greater number and variety of subtests available on the more recent revision. However, even the 15 subtests of the WISC-IV provided relatively few points in space to define the model, necessitating further validation with additional instruments. The third dimension – mode of response – includes the CD subscale, as in the original study, as well as two new subscales, SS and CA.

Interpretation of intelligence test results is a complex process, and the radex model does not eliminate this complexity. Rather, it adds additional factors that should be taken into account when interpreting cognitive functioning. Where factor analysis emphasizes *g* and broad categories of cognitive ability, SSA highlights groupings based on content and mode of response while simultaneously representing the *g* loading of the subtests. In doing so, it makes clear that the content, or mode of representation, of a subtest is less important for the more abstract, *g*-loaded tasks, while becoming more important for the more automatic, low-level tasks. The *g* loading in this analysis appears to represent what John Horn (e.g., Horn & Noll, 1997) calls Gf, or Fluid Reasoning, rather than global intelligence. However, neither of these models explicates the neuropsychological processes or information processing underlying cognitive performance, necessitating the integration of another level of analysis in individual assessment.

The organization of the radex model provides an elegant description of the structure of intelligence, while still maintaining both broad cognitive abilities and the concept of level of abstraction, a reflection of fluid reasoning across modalities. Furthermore, the model's depiction of the whole of the structure allows theorists to identify gaps and generate new tasks to assess previously unmeasured aspects of intelligence. For example, in this analysis of the WISC-IV, the most highly abstract tasks are figural and verbal, with no coverage of numeric reasoning. The somewhat less abstract rule application level is heavily verbal, and has no figural content, and the least abstract, automatic tasks are numerical and figural without any subscale in the verbal mode. This suggests the possibility of the addition of new tasks to cover those areas untapped by the current test, such as a numerical reasoning task or a short-term memory task using words as well as the use of manual manipulation of verbal tasks.

In addition to the obvious implications for test development, the radex model may also be useful diagnostically. Creating a cross-battery assessment that fully covers all three dimensions may allow identification of strong or weak modes for use in planning remediation or accommodation for children with learning disabilities or other school difficulties.

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