Psychology in the Schools, Vol. 40(2), 2003
Published online in Wiley InterScience (www.interscience.wiley.com).

DOI: 10.1002/pits.10083

RELATIONS BETWEEN MEASURES OF CATTELL-HORN-CARROLL (CHC) COGNITIVE ABILITIES AND MATHEMATICS ACHIEVEMENT ACROSS THE SCHOOL-AGE YEARS

RANDY G. FLOYD

The University of Memphis

JEFFREY J. EVANS

Evans Consulting, St. Cloud, Minnesota

KEVIN S. McGREW

University of Minnesota

Cognitive clusters from the Woodcock-Johnson III (WJ III) Tests of Cognitive Abilities that measure select Cattell-Horn-Carroll broad and narrow cognitive abilities were shown to be significantly related to mathematics achievement in a large, nationally representative sample of children and adolescents. Multiple regression analyses were used to predict performance on the Math Calculation Skills and Math Reasoning clusters from the WJ III Tests of Achievement for 14 age groups ranging in age from 6 to 19 years. Comprehension-Knowledge (Gc) demonstrated moderate relations with Math Calculation Skills after the early school-age years and moderate to strong relations with Math Reasoning. Fluid Reasoning (Gf), Short-term Memory (Gsm), and Working Memory generally demonstrated moderate relations with the mathematics clusters. Processing Speed (Gs) demonstrated moderate relations with Math Reasoning during the elementary school years and moderate to strong relations with Math Calculation Skills. During the earliest ages of the analysis, Long-term Retrieval (Glr) demonstrated moderate relations with the mathematics clusters, and Auditory Processing (Ga) demonstrated moderate relations with Math Calculation Skills. Visual-Spatial Thinking (Gv) generally demonstrated nonsignificant relations with the mathematics clusters.

The acquisition and application of mathematics skills, such as counting and simple addition and subtraction, hold great societal importance due to the demands of formal schooling, daily living activities, and employment (Mullis et al., 2001; Rivera-Batiz, 1992; Rourke & Conway, 1997). However, in comparison to reading-related competencies, relatively little is known about the development and maintenance of mathematics skills. Several recent investigations have focused attention on instructional, home, and community variables and their influence on mathematics achievement (Fleischner, 1994). For example, recent research conducted as part of the Third International Math and Science Study (TIMMS) indicated the importance of educational resources found in the home (viz., books in the home, availability of study aids, and parents' educational levels) in predicting mathematics success and failure (Mullis et al., 2001). Teacher training in mathematics, the focus and content of curricular materials, and time spent during mathematics instruction also appear to be strong predictors of mathematics skill development (Carnine, 1991; Lyon, Vaasen, & Toomey, 1989; Mullis et al., 2001; Russell & Ginsburg, 1984).

Less is known about the underlying cognitive processes that contribute to mathematics achievement and mathematical disabilities (Geary, 1993, 1994, in press; Rourke & Conway, 1997). Most research investigating the influence of cognitive processes on mathematics skill development has included only a narrow set of domain-specific conceptual and procedural competencies related to circumscribed numerical and arithmetical domains (Bryant & Rivera, 1997; Fleischner, 1994; Hoard, Geary, & Hamson, 1999). For example, failure to develop number sense (i.e., the implicit awareness of quantitative concepts and relationships; Gersten & Chard, 1999), and numerical deficits, such as "developmentally immature arithmetic procedures and a high frequency of procedural

Correspondence to: Randy G. Floyd, The University of Memphis, Department of Psychology, Memphis, TN 38152. E-mail: rgfloyd@memphis.edu.

errors" (Geary, 1993, p. 346) have been implicated as the primary causes of disabilities in arithmetical achievement. Furthermore, when domain-general cognitive processes that influence mathematics achievement have been studied, they appear to have been limited in breadth. Research has focused primarily on three cognitive processes: (a) immediate phonological or working memory, (b) storage and retrieval of information in long-term memory, and (c) visual processing abilities (Ashcraft, 1995; Dark & Benbow, 1990, 1991; Furst & Hitch, 2000; Geary & Burlington-Dubree, 1989; Geary & Hoard, 2001; Hulme & Roodenrys, 1995; McLean & Hitch, 1999; Noel, Desert, Aubrun, & Seron, 2001; Robinson, Abbott, Berninger, & Busse, 1996; Rourke, 1993; Swanson, 1993). However, other cognitive processes, such as inductive and deductive reasoning and language processing, as well as processing speed appear to have been omitted from the extant research.

The omission of potentially important variables in predictive or explanatory research is considered a form of specification error, a type of modeling error that can lead to biased estimates of the effects of predictive variables (Pedhazur & Schmelkin, 1991). As an analogy, a college basketball coach could construct a regression model that included season-long player statistics of starters in order to predict post-season performance. If the coach omitted the game statistics for the point guards and included only the statistics for the remaining four starters, over- or under-estimates of the importance of one or more of the other four starting positions would probably result. These biased findings might lead the coach to make erroneous decisions about the strengths of certain player positions when developing game strategies. Because of the failure to include measures of potentially important constructs in the extant mathematics research, specification error may cloud the current understanding of the cognitive predictors of mathematics achievement. One of the most effective means to avoid specification error is to use formal, well-validated models about the phenomena of interest to guide the selection of measures that should be included in analyses (Licht, 1995).

Cattell-Horn-Carroll Theory of Cognitive Abilities

The Cattell-Horn-Carroll (CHC) theory of cognitive abilities is supported by a large network of validity evidence, which includes more than half a century of factor analytic, developmental, heritability, external outcome validity, and neurocognitive research evidence (Flanagan & Ortiz, 2001; McGrew & Flanagan, 1998). The theory stems from a recent synthesis of the Cattell-Horn Gf-Gc theory (Horn, 1991; Horn & Noll, 1997) and the Carroll three-stratum theory of cognitive abilities (Carroll, 1993, 1997, in press) by McGrew (1997). CHC theory is a hierarchical framework of human cognitive abilities that consists of three strata: general intelligence or g (stratum III), broad cognitive abilities (stratum II), and narrow cognitive abilities (stratum I). The broad cognitive abilities include Fluid Reasoning (Gf), Comprehension-Knowledge (Gc), Short-term Memory (Gsm), Visual Processing (Gv), Auditory Processing (Ga), Long-term Retrieval (Glr), Processing Speed (Gs), and Decision/Reaction Time or Speed (Gt), Reading and Writing (Grw), and Quantitative Knowledge (Gq; McGrew & Flanagan, 1998). These broad cognitive abilities subsume approximately 70 narrow cognitive abilities. The structure of CHC theory and its supporting validity evidence provide rich theoretical grounds on which to understand the human cognitive abilities and their relations with a variety of life outcomes. Thus, research organized according to CHC theory that examines the cognitive predictors that contribute to the development and maintenance of mathematics skills provides one remedy for the specification error seen in the extant mathematics research.

Several studies have examined the relations between measures of CHC broad and narrow cognitive abilities and mathematics achievement.[1] McGrew and Hessler (1995) examined the

---

[1] See McGrew and Flanagan (1998), Flanagan, McGrew, and Ortiz (2000), and Flanagan, Ortiz, Alfonso, and Mascolo (2002) for CHC-organized summaries of research examining the relations between cognitive abilities and mathematics achievement.

relations between the seven Woodcock-Johnson Psycho-Educational Battery/Revised (WJ-R; Woodcock & Johnson, 1989) Gf-Gc cognitive clusters and the WJ-R Basic Mathematics Skills and Mathematics Reasoning clusters. Comprehension-Knowledge (Gc) demonstrated the strongest relations with the mathematics clusters from age 5 through late adulthood. Fluid Reasoning (Gf) generally displayed moderate relations with both mathematics clusters. Processing Speed (Gs) demonstrated strong to moderate relations with Basic Mathematics Skills throughout the lifespan and moderate relations with Mathematics Reasoning through young adulthood. Long-term Retrieval (Glr) was moderately related to Basic Mathematics Skills only during late adolescence and early adulthood. Short-term Memory (Gsm) demonstrated moderate relations with Mathematics Reasoning primarily during the early elementary school years; however, it generally demonstrated negligible relations with Basic Mathematics Skills. With the exception of moderate relations between Auditory Processing (Ga) and Mathematics Reasoning during middle adulthood, Ga and Visual-Spatial Thinking[2] (Gv) demonstrated nonsignificant relations with the mathematics clusters.

Similar findings have surfaced from independent research using the WJ-R Gf-Gc clusters and from additional analyses of the WJ-R standardization sample that included g in the analysis. Using multiple regression analyses of the WJ-R Gf-Gc clusters and a group-administered achievement test with a sample of school-age children, Williams, McCallum, and Reed (1996) reported that Gf and Gc were the best predictors of mathematics achievement. Hale, Fiorello, Kavanagh, Hoeppner, and Gaither (2001) reported that when CHC-organized commonality analyses were applied to the Wechsler Intelligence Scale for Children, Third Edition (Wechsler, 1991) Full Scale IQ and factor indexes, the broad cognitive ability factors Gc, Gsm, and Gq were significantly associated with mathematics achievement above and beyond the predictive effects of the Full Scale IQ. After using structural equation modeling to control for the effects of g, McGrew, Flanagan, Keith, and Vanderwood (1997) found that Gf, Gc, and Gs were strong predictors of Mathematics Reasoning. Keith (1999) also found that measures of mathematics achievement were strongly influenced by g and a number of other CHC broad cognitive abilities, such as Gc, Gf, and Gs.

Purpose of Current Study

The studies reviewed above provide evidence of the external validity of CHC broad and narrow cognitive abilities in predicting mathematics achievement, even when the effect of g is present in the analyses. The purpose of this study is extend the research examining these relations using new measures that were constructed to ensure better construct validity via increased construct representation. As a first step in examining the relations between CHC cognitive abilities and mathematics achievement, this study used multiple regression analyses to investigate the validity of the cognitive clusters from the Woodcock-Johnson III (WJ III) Tests of Cognitive Abilities (COG) in predicting mathematics calculation and mathematics reasoning skills.

Method

Participants

Participants were drawn from the nationally representative standardization sample of the WJ III (Woodcock, McGrew, & Mather, 2001). Standardization participants between 6 and 19 years of age were included in the current analyses if they completed the tests that form the WJ III Tests of Achievement (ACH; Woodcock & Mather, 2001) Math Calculation Skills and Math Reasoning

[2] The name for the Gv cluster on the WJ-R was Visual Processing. To increase readability, the WJ III cluster names are used when referring to the analogous WJ-R clusters.

Table 1
Sample Sizes and Descriptive Statistics Based on W*-scores for Each Age Group Included in the Regression Models*

Age Group In Years	Math Calculation Skills			Math Reasoning		
	n	*M*	*SD*	*n*	*M*	*SD*
6	252	464.76	11.08	207	447.69	15.64
7	324	476.45	10.72	218	468.14	17.31
8	376	487.61	10.45	269	487.90	15.75
9	461	496.11	10.96	266	496.59	16.72
10	468	503.40	10.86	328	504.67	16.80
11	384	509.57	11.52	256	513.44	16.87
12	338	514.22	11.57	218	520.27	19.21
13	304	519.47	12.43	208	523.00	21.90
14	283	523.10	11.52	194	528.71	19.70
15	305	524.72	13.17	209	531.31	20.53
16	302	529.62	13.46	201	537.65	23.07
17	241	529.17	13.36	165	539.64	19.69
18	273	531.94	13.36	194	540.75	20.95
19	187	534.87	13.10	131	546.25	19.53
Total	4,498			3,064		

clusters (see Table 1). Thus, two subsamples were formed (n = 4,498 for Math Calculation Skills and n = 3,064 for Math Reasoning).

Select demographic variables were examined to determine the representativeness of the two subsamples used in these analyses. Each participant was assigned a unique weight that represented each participant's required contribution to the final norms as per the United States census statistics used during norming (see McGrew & Woodcock, 2001). For both the Math Calculation and Math Reasoning subsamples, the mean participant weights differed by only .02 points from the mean value (1.0). A mean weight of 1.0 would indicate that the subsamples have the same average subject characteristics as the complete WJ III national standardization sample. A comparison of weighted and unweighted descriptive statistics yielded nearly identical cluster means and standard deviations. Furthermore, the weighted and unweighted percentages for the demographic variable values for race and gender in each subsample varied by no more than 2.4 percentage points. The only notable discrepancy was the finding that both subsamples had approximately 5 percent more participants classified as Hispanic than in the total WJ III standardization sample. Altogether, the representativeness analysis suggests that the two subsamples included in this study were largely similar to the United States population of 6- to 19-year-olds.[3]

Measures

Mathematics clusters. The WJ III ACH Math Calculation Skills and Math Reasoning clusters operationalized mathematics achievement in this study (see Table 2). Correlations between

[3] Due to space limitations, the specific results of these analyses are not reported. The analyses focusing on the representativeness of the samples and the specific statistics from the regression analyses can be obtained by contacting the first author or by visiting the website address http://www.iapsych.com/resrpts.htm.

Table 2
Descriptions of WJ III Mathematics Clusters and Cognitive Clusters

WJ III Cluster	Description of Cluster	Tests Forming Cluster
Math Calculation Skills	Ability to complete mathematics calculations and to perform basic operations quickly	Calculation Math Fluency
Math Reasoning	Ability to complete mathematics operations based upon real-world scenarios and to understand math concepts and quantitative relationships	Applied Problem Quantitative Concepts
Comprehension-Knowledge	Ability to use language and acquired knowledge effectively	Verbal Comprehension General Information
Long-term Retrieval	Ability to store and readily retrieve information in long-term memory	Visual-Auditory Learning Retrieval Fluency
Visual-Spatial Thinking	Ability to recognize spatial relationships and to analyze and manipulate visual stimuli	Spatial Relations Picture Recognition
Auditory Processing	Ability to perceive, attend to, and analyze patterns of sound and speech that may be presented in distorted conditions	Sound Blending Auditory Attention
Fluid Reasoning	Ability to form and recognize logical relationships among patterns and to make deductive and inductive inferences	Concept Formation Analysis-Synthesis
Processing Speed	Ability to perform simple cognitive tasks quickly, especially when under pressure to maintain focused attention and concentration	Visual Matching Decision Speed
Short-term Memory	Ability to understand and store information in immediate awareness and then use it within a few seconds	Numbers Reversed Memory for Words
Working Memory	Ability to temporarily store and mentally manipulate information held in immediate memory	Numbers Reversed Auditory Working Memory

these mathematics clusters and those from the Kaufman Test of Educational Achievement (Kaufman & Kaufman, 1985) and the Wechsler Individual Achievement Test (Wechsler, 1992) provide validity evidence for these clusters. These correlations ranged from .29 to .67 in a sample of first- to eighth-grade children (McGrew & Woodcock, 2001).

Cognitive clusters. The seven CHC factor clusters from the COG operationalized the CHC broad (or stratum II) cognitive abilities. Two tests contribute to each of the seven CHC factor clusters (see Table 2). One COG clinical cluster, Working Memory, was also included to operationalize the narrow cognitive ability of the same name, which is subsumed by Short-term Memory (Gsm). Reliability and validity evidence for the CHC factor clusters and clinical clusters is presented in McGrew and Woodcock (2001), and validity evidence for these clusters is reviewed in Floyd, Shaver, and McGrew (in press).

Analysis

The WJ III W-score was the metric of analysis. W-scores are transformations of raw scores into equal interval units that are derived through application of Rasch scaling (Woodcock, 1978; Woodcock & Dahl, 1971). The W-score scale of each test is centered on a value of 500, which is the approximate average performance of a 10-year-old child. The W-score for each cluster represents the arithmetic average of the W scores of the tests forming the cluster.

In this analysis, the two subsamples were divided into 14 different age-based groups representing one year of age starting at age 6 and continuing through age 19 (see Table 1). Simultaneous multiple regression analysis was completed at each age level. In contrast to sequential or hierarchical multiple regression, which allows researchers to use prior knowledge to select the order in which predictor variables (i.e., independent variables) are entered into the equation, simultaneous multiple regression allows for all predictor variables to be entered at once into the equation to determine their contributions to the prediction of the criterion variable (i.e., dependent variable). During simultaneous multiple regression, the predictor variables are evaluated according to their relative contribution to prediction of the criterion variable, and they are entered into the equation according to the strength of their contributions and the uniqueness of their predictive value after other variables have been include as predictors. For the primary regression analyses in this study, the predictor variables were the seven CHC factor clusters. In one model, the criterion variable was the Math Calculation Skills cluster, and in the other model, the criterion variable was the Math Reasoning cluster. In addition, regression analyses were completed to investigate the relations between the Working Memory cluster and the achievement clusters. In these regression analyses, the Working Memory cluster was substituted for the Short-term Memory (Gsm) cluster.

For each cognitive cluster and each mathematics model, 14 standardized regression coefficients yielded by the regression analysis were plotted on a graph with age representing the x-axis. Standardized regression coefficients indicate the proportion of standard deviation units that the criterion variable changes as a function of one standard deviation change in a predictor variable. For example, a standardized regression coefficient of .25 representing the relations between Gc and Math Calculation Skills means that, for every standard deviation change observed in Gc scores, there is an average of .25 standard deviation change in Math Calculation Skills scores. Given that sampling error is present to an unknown degree at each age level in the analysis, population estimates of the age-related changes in the relations between the cognitive and achievement clusters were identified through the application of the distance weighted least squares smoothing function to the plot of the standardized regression coefficients (Wilkinson, 1990; see McGrew & Wrightson, 1997).

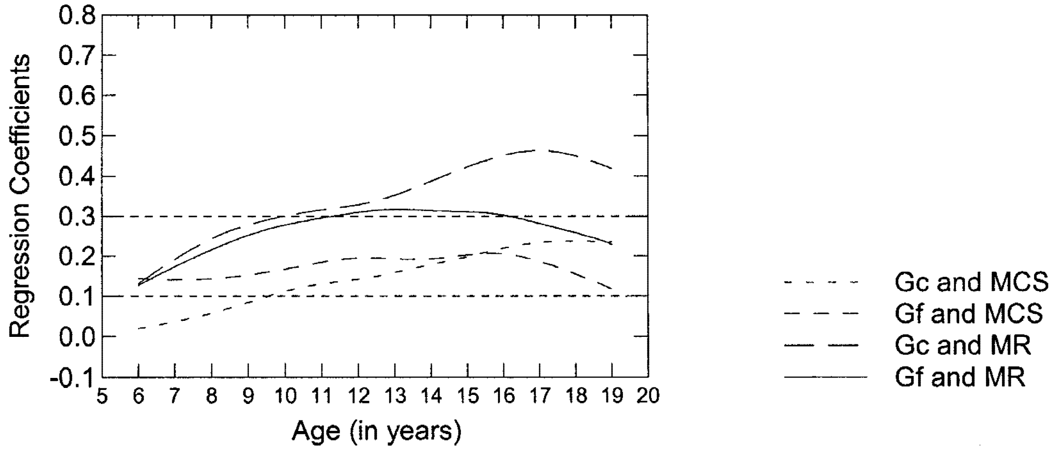

FIGURE 1. Standardized regression coefficients as a function of age for Comprehension-Knowledge (Gc) and for Fluid Reasoning (Gf) with Math Calculation Skills (MCS) and Math Reasoning (MR).

RESULTS

Figures 1 through 4 present the results of the analyses. Each figure displays four lines representing the smoothed standardized regression coefficient values for two cognitive clusters that were used as predictors of Math Calculation Skills and Math Reasoning. Two parallel dashed lines that correspond to standardized regression coefficients of .10 and .30 are also presented in each figure. The lines are guides for interpreting the significance of the smoothed regression coefficient values and correspond to previously established rules-of-thumb (McGrew, 1993; McGrew & Hessler, 1995; McGrew & Knopik, 1993). These rules operationally define statistical and practical significance to be associated with standardized regression coefficients of .10 or above. Coefficients ranging from .10 to .29 are classified as representing moderate relations, whereas those .30 or above are classified as strong relations.[4]

CHC Factor Clusters

The smoothed standardized regression coefficients reveal the relations between the cognitive and mathematics clusters across childhood and adolescence (see Figures 1 through 4). Comprehension-Knowledge (Gc) demonstrated perhaps the strongest relations with both Math Calculation Skills and Math Reasoning (see Figure 1). Gc demonstrated moderate relations with Math Calculation Skills after age 9. In contrast, its relations with Math Reasoning were moderate until age 10 and strong throughout the remaining age groups. Fluid Reasoning (Gf) demonstrated moderate relations with Math Calculation Skills and moderate to strong relations with Math Reasoning throughout childhood and adolescence (see Figure 1). Short-term Memory (Gsm) displayed moderate relations with Math Calculation Skills after age 7, and its relations with Math Reasoning were moderate until age 17 (see Figure 2). Processing Speed (Gs) demonstrated moderate to strong relations with Math

[4]These rules-of-thumb are generally similar to other rules-of-thumb for interpreting the effect sizes for manipulable influences on achievement or learning. According to Keith (1999), effect sizes that are less than .05 are not meaningful, effect sizes of .05 and above are small effects, effect sizes above .10 or .15 are moderate effects, and effect sizes above .25 are large effects.

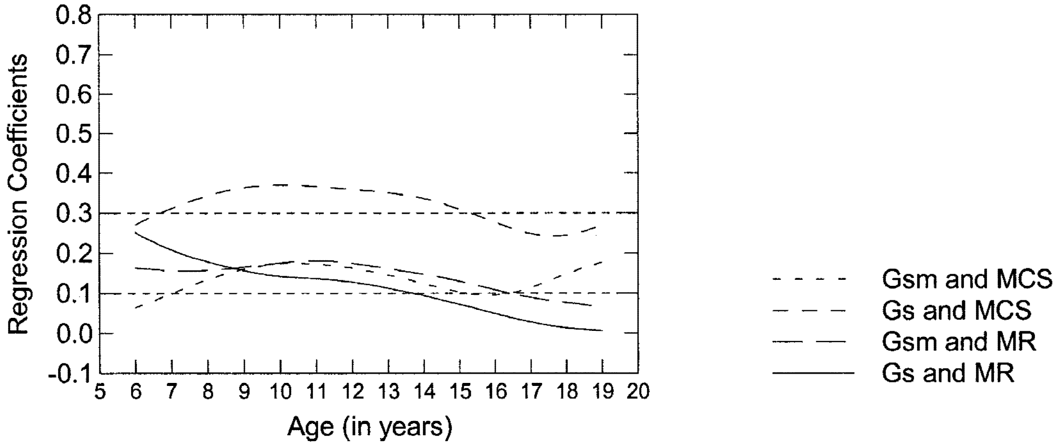

FIGURE 2. Standardized regression coefficients as a function of age for Short-term Memory (Gsm) and Processing Speed (Gs) with Math Calculation Skills (MCS) and Math Reasoning (MR).

Calculation Skills (see Figure 2). However, Gs demonstrated moderate relations with Math Reasoning only until age 14.

Two CHC factor clusters demonstrated significant relations with at least one of the mathematics clusters only during the early elementary school years. Long-term Retrieval (Glr) demonstrated moderate relations with Math Calculation Skills and Math Reasoning from age 6 through approximately age 8 (see Figure 3). Similarly, Auditory Processing (Ga) demonstrated moderate relations with Math Calculation Skills only during the earliest ages of analysis. However, Ga generally demonstrated nonsignificant relations with Math Reasoning (see Figure 3). Visual-Spatial Thinking (Gv) also generally demonstrated nonsignificant relations with Math Calculation Skills and Math Reasoning (see Figure 4).

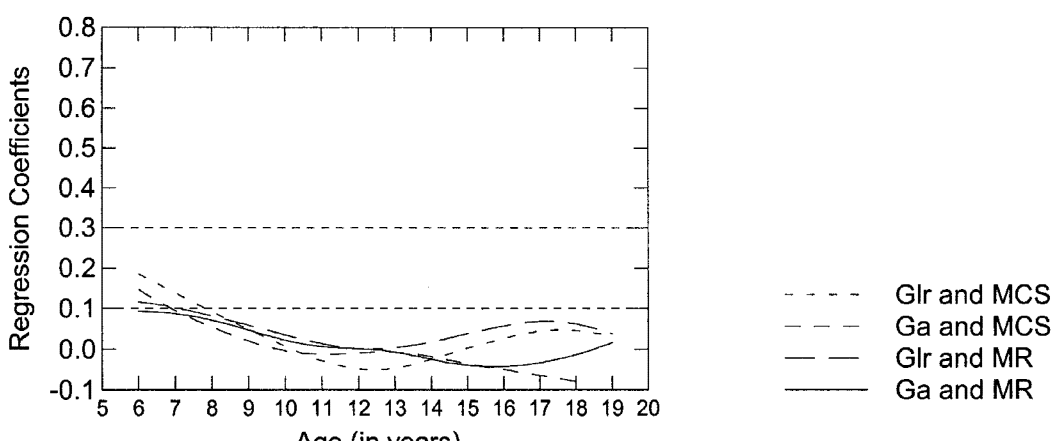

FIGURE 3. Standardized regression coefficients as a function of age for Long-term Retrieval (Glr) and for Auditory Processing (Ga) with Math Calculation Skills (MCS) and Math Reasoning (MR).

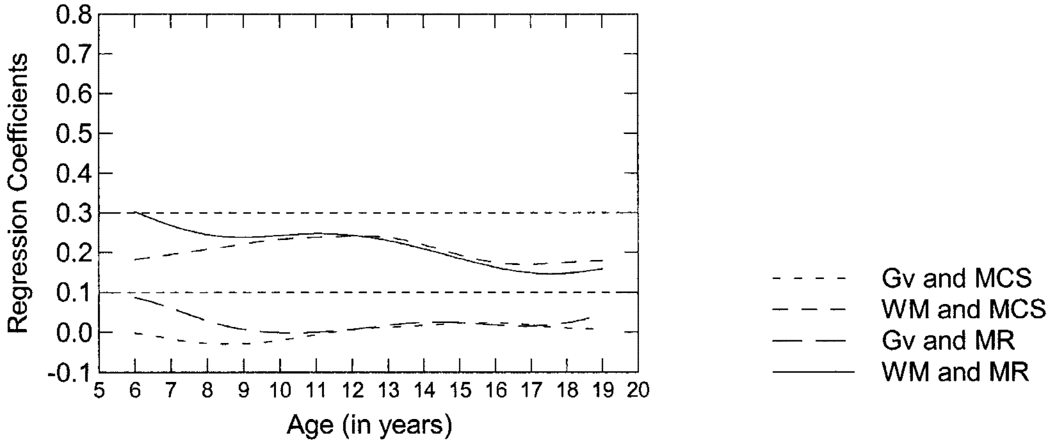

FIGURE 4. Standardized regression coefficients as a function of age for Visual-Spatial Thinking (Gv) and for Working Memory (WM) with Math Calculation Skills (MCS) and Math Reasoning (MR).

Clinical Clusters

When Working Memory was substituted for Short-term Memory (Gsm) in the regression analyses, its patterns of relations with Math Calculation Skills and Math Reasoning were generally stronger in magnitude than those of Gsm. Working Memory displayed moderate relations with both mathematics clusters throughout childhood and adolescence (see Figure 4).

DISCUSSION

The results of this study replicated and extended several findings from the CHC-organized mathematics research. When integrated with prior research, the current study contributes to an emerging body of knowledge regarding the relations between CHC broad and narrow cognitive abilities and mathematics achievement. In this context, the results of the current study have potentially important implications for both research examining mathematics achievement and related assessment practices.

Gc and Mathematics

Given the established and strong link between Comprehension-Knowledge (Gc) and mathematics achievement (e.g., Hale et al., 2001; Keith, 1999; McGrew et al., 1997; Williams et al., 1996), it was not surprising that the Gc cluster was generally the strongest predictor of mathematics achievement throughout the school-age years. General cultural knowledge and knowledge of mathematics concepts, facts, and the procedures to conduct arithmetic stem largely from the acquisition and modification of declarative and procedural knowledge structures (Woodcock, 1993, 1998). In fact, both abilities may be considered types of academic achievement (Anastasi, 1988; Flanagan et al., 2002; Kaufman, 1994; Woodcock, 1990) or developing expertise (Sternberg, 1998). Because of these similarities, the increasing strength of relations between Gc and Math Reasoning are logical because knowledge of mathematics, rather than more fundamental cognitive processes, likely contributes significantly to further mathematics skill development after basic mathematics skills (e.g., simple addition and multiplication) are established (Geary, 1994). Furthermore, because Gc subsumes narrow cognitive abilities associated with listening and speaking, the moderate to strong relations between Gc and the mathematics clusters are likely due to the influence of language-based cognitive processes (e.g., self-directed speech and listening abilities) on mathematics performance. For instance, the stronger relations between Gc and Math Reasoning than those between Gc and Math Calculation Skills are likely due, in part, to the linguistic demands of tests in the Math Reasoning cluster (Cummins, 1991; Geary, 1994).

Gf and Mathematics

The consistent moderate to strong relations between Fluid Reasoning (Gf) and the mathematics clusters indicate that domain-general problem-solving and reasoning abilities are strong influences on mathematics achievement. This finding is also not surprising for a number of reasons. First, Carroll's (1993) analysis and comprehensive review of human cognitive abilities indicated that quantitative reasoning abilities could be included under the stratum II ability, Fluid Intelligence (cf. Carroll, in press). Second, Gf appears to represent some of the prominent constructs in studies of mathematics skill development, such as problem-solving schemata, strategy use, and strategic change (Cummins, 1991; Lemaire & Siegler, 1995; Vaughn & Wilson, 1994). Third, research that has included instruments that appear to measure Gf abilities (e.g., the Wisconsin Card Sort Test and the Halstead Category Test) has indicated that such abilities are significant correlates of mathematics achievement (Bull, Johnston, & Roy, 1999; Rourke, 1993; Shute & Huertas, 1990; Strang & Rourke, 1983).

Gsm, Working Memory, and Mathematics

Most contemporary mathematics research postulates the importance of a construct system dealing with the limited capacity and management of immediate memory during arithmetic performance. The relations between this construct system and the abilities described in CHC theory are important to discuss. The current study and the WJ III measurement system distinguishes (a) between active and more passive operations of this system and (b) between immediate memory for auditory and visual stimuli (Flanagan, McGrew, & Ortiz, 2000; McGrew & Woodcock, 2001). In the CHC framework, the narrow cognitive ability of Working Memory refers to the ability to temporarily store and perform a set of cognitive operations on information after a single presentation while managing the limited capacity of immediate memory. In contrast, the narrow cognitive ability Memory Span refers to the ability to attend to, temporarily store, and immediately recall information (without active manipulation) after a single presentation. Both Working Memory and Memory Span are subsumed by the broad cognitive ability Short-term Memory (Gsm). Measures of Gsm typically reflect performance on tests of immediate memory that include phonological or auditory stimuli (e.g., words or numbers) as content. In contrast, tests of immediate memory that focus on visual stimuli (e.g., visual images or patterns) are thought to measure the narrow cognitive ability Visual Memory, which is subsumed by Visual-Spatial Thinking (Gv; Carroll, 1993; McGhee & Lieberman, 1994; McGrew & Woodcock, 2001).

In this context, both Gsm and Working Memory demonstrated moderate relations with mathematics achievement. However, the relations between Working Memory and both mathematics clusters appear to be stronger and more consistent than those for Gsm. Thus, the cluster formed from two tests that measure the narrow cognitive ability of Working Memory generally displayed stronger and more consistent relations with mathematics achievement than the cluster demonstrating broader construct representation by measuring both Working Memory and Memory Span (i.e., the Gsm cluster). This finding is consistent with research indicating the importance of processes associated with the central executive, an attentional control system that mediates cognitive operations, such as arithmetic calculations, performed in immediate memory (Baddeley, 1996, 2001; Baddeley & Hitch, 1994). Although a number of studies have demonstrated that measures of Memory Span are significant predictors of mathematics achievement (Ashcraft, 1995; Furst &

Hitch, 2000; Hitch & McAuley, 1991; McLean & Hitch, 1999; Swanson, 1993), this finding is not always confirmed (see DeRammelaere, Stuyven & Vandierendonck, 2001). Conversely, research is more consistent in suggesting that the ability to divide attention, to manage limited memory resources, and to manipulate information in immediate memory is significantly related to mathematics performance (Geary, in press; Geary, Hoard, & Hamson, 1999; Lehto, 1995; Wilson & Swanson, 2001).

Gs and Mathematics

The relations between Processing Speed (Gs) and more domain-specific processing of mathematical information corroborate the results of a number of studies that have demonstrated a strong influence of speed of processing (a.k.a., operational efficiency) during mathematics performance (e.g., Bull & Johnston, 1997; Kirby & Becker, 1988; Royer, Tronsky, Chan, Jackson, & Marchant, 1999). These findings also are consistent with a wide array of research that indicates that speed of processing is important during the early stages of acquiring most cognitive and academic skills (Fry & Hale, 2001; Kail, 1991; Kail, Hall, & Caskey, 1999; Necka, 1999; Rindermann & Neubauer, 2000; Snow & Swanson, 1992; Weiler et al., 2000).

Glr and Mathematics

Some researchers have asserted that difficulties in the ability to retrieve information, including words and arithmetic facts, from long-term memory represent a specific cognitive deficit that results in severe delays in the early development of mathematics skills (Geary, 1993, in press; Geary et al., 1999). The results of this study indicate that Long-term Retrieval (Glr) abilities are important to early mathematics calculation skill development. Significant relations between Glr and the mathematics clusters were generally evident from age 6 through approximately age 8. It is possible that the predictive power of Glr during this period is due to the observation that rote recall of mathematics facts from declarative memory (and not more complex cognitive operations representing procedural memory) is required to complete simple math problems (Ashcraft, 1995). However, when considered within the context of other CHC broad cognitive abilities, the current results do not indicate significant and consistent relations between Glr and mathematics achievement beyond age 9. It is logical to suggest that previous findings of mathematical retrieval deficits represent either domain-specific memory deficits or insufficient representation of mathematics knowledge stored in memory.

Ga and Mathematics

Although Auditory Processing (Ga) demonstrated significant relations with Math Calculation Skills during the very earliest ages of the analysis, these relations quickly declined into nonsignificance. In a manner similar to previous research drawing upon CHC theory (e.g., McGrew & Hessler, 1995), Ga generally demonstrated consistent nonsignificant relations with mathematics achievement throughout childhood and adolescence.

Gv and Mathematics

The extant research examining the relations between Visual-Spatial Thinking (Gv) and mathematics achievement has produced mixed findings. Some evidence suggests that visual-spatial abilities and visual memory span are associated with the development of certain mathematics skills (e.g, Geary & Burlington-Dubree, 1989; Geary, 1994; Geary, Saults, Liu & Hoard, 2000; McLean & Hitch, 1999; Rourke, 1993). However, a number of more recent studies indicate that visual processing abilities play a small to negligible role in arithmetic computational performance and other mathematics skills (Bull & Johnston, 1999; Bull et al., 1999; Butterworth, Cipolotti, &

Warrington, 1996; Geary et al., 2000; Hegarty & Kozhevnikov, 1999). In fact, a meta-analysis of the correlations between mathematics achievement and visual-spatial abilities indicate that visual-spatial abilities "are no better related, and are often less well related, to mathematical ones than are other skills—some of which seem irrelevant to mathematics [sports information and reading comprehension]" (Friedman, 1995, p. 41). The results of the current study support the latter position. Although perhaps Gv abilities contribute to the earliest stages of mathematics skill development (Geary, 1993; Geary & Burlington-Dubree, 1989), the current results suggest consistently weak relations between Gv and mathematics achievement from age 6 throughout adolescence. It is possible that the relations between visual-spatial abilities and measured mathematics achievement depend on the content of items included in the math tests, but at present, additional research is needed to support the relative importance of Gv abilities in predicting specific mathematics competencies.

Limitations

Several limitations of this study should be considered. First, because the analysis relied on simultaneous multiple regression in which all relevant cognitive variables were included as predictors, cognitive abilities contributing to mathematics performance may have been judged to be unimportant when grouped with highly correlated measures that account for a greater proportion of the mathematics criterion variance. Second, although recent research has focused on the importance of reading ability in predicting mathematics achievement (Bull & Johnston, 1997; Geary, 1993, in press), these abilities were not included as predictors in the this analysis. Third, several predictor and criterion variables in this analysis may share latent variables and be factorially complex. For example, confirmatory factor analyses using the WJ III indicate that the Math Fluency test, which is included in the Math Calculation Skills cluster, measures both the CHC broad cognitive abilities Quantitative Knowledge (Gq) and Processing Speed (Gs; McGrew & Woodcock, 2001). Such analyses also have indicated that the Quantitative Reasoning test, which is included in the Math Reasoning cluster, measures both the CHC broad cognitive abilities Quantitative Knowledge (Gq) and Fluid Reasoning (Gf). This predictor-criterion contamination may have spuriously increased the amount of predictive power that Gs and Gf demonstrated in this study. Finally, although the four mathematics tests from the WJ III require knowledge of numerous mathematics concepts and assess a variety of mathematics operations and domains, only two mathematics clusters were included in the current analysis. These clusters represent comprehensive and purposefully general measures of mathematics skills. Consequentially, the results of this study may not accurately describe the relations between the CHC cognitive abilities and specific mathematics skills, such as geometry and calculus (Geary, in press; Ginsburg, 1997).

Implications for Assessment Practices

The results of this study may have practical implications for the selection and use of WJ III cognitive clusters during mathematics-related assessments and for other assessment practices guided by CHC theory (e.g., the CHC Cross-Battery approach; Flanagan et al., 2000; Flanagan & Ortiz, 2001; Flanagan et al., 2002; McGrew & Flanagan, 1998). To guide the selection of cognitive clusters that are significant predictors of mathematics achievement, Table 3 depicts the degree of relations between the cognitive and mathematics clusters across the school-age years.

Summary

The results indicate that a focus in mathematics research on only competencies related to circumscribed numerical and arithmetical domains and more general competencies describing the abilities associated with the functioning of immediate phonological or working memory, the

Table 3
Relations between WJ III Clusters and Mathematics Achievement for Each Age Group

WJ III Cluster and Mathematics Ability		Age Group 6	7	8	9	10	11	12	13	14	15	16	17	18	19
Comprehension-Knowledge (*Gc*)	MC	White	White	White	White	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray
	MR	Gray	Gray	Gray	Gray	Black	Black	Black	Black	Black	Black	Black	Black	Black	Black
Long-term Retrieval (*Glr*)	MC	Gray	Gray	Gray	White	White	White	White	White	White	White	White	White	White	White
	MR	Gray	White	White	White	White	White	White	White	White	White	White	White	White	White
Visual-Spatial Thinking (*Gv*)	MC	White	White	White	White	White	White	White	White	White	White	White	White	White	White
	MR	White	White	White	White	White	White	White	White	White	White	White	White	White	White
Auditory Processing (*Ga*)	MC	Gray	Gray	White	White	White	White	White	White	White	White	White	White	White	White
	MR	White	White	White	White	White	White	White	White	White	White	White	White	White	White
Fluid Reasoning (*Gf*)	MC	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray
	MR	Gray	Gray	Gray	Gray	Gray	Gray	Black	Black	Black	Black	Black	Gray	Gray	Gray
Processing Speed (*Gs*)	MC	Gray	Black	Black	Black	Black	Black	Black	Black	Black	Black	Gray	Gray	Gray	Gray
	MR	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	White	White	White	White	White	White
Short-term Memory (Gsm)	MC	White	White	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray
	MR	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	White	White
Working Memory	MC	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray
	MR	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray	Gray

Note. MC = Math Calculations cluster, MR = Math Reasoning cluster, White = no significant relations, Gray = moderately significant relations, Black = strong significant relations.

storage and retrieval of information in long-term memory, and the processing of visual-spatial information is probably a premature restriction or specification that may lead to hardening of the cognitive construct research categories. It appears that some of these previously identified aptitudes for mathematics achievement interact with a number of other aptitudes to facilitate mathematics performance. These additional aptitudes include language- and knowledge-based abilities, reasoning abilities, and processing speed. The current results indicate that the breadth of investigation of mathematics skill development and failure should be widened to include operational measures of these aptitudes. Furthermore, these results suggest that prior research may have focused too heavily on abilities that are inconsistent predictors of mathematics achievement (viz., visual-spatial processing and visual short-term memory). Organizing future research based on the body of evidence indicating the importance of broad and narrow cognitive abilities specified in CHC theory may offer a remedy to the errors in specification seen in the extant mathematics research.

Acknowledgments

We thank David Geary, Dawn Flanagan, Maria Leforgee, Jill Bose, and Claudia Votino for their reviews and comments on drafts of this manuscript. We also wish to thank Richard Woodcock and Measurement/Learning/Consultants for making data from the WJ III standardization sample available for this research.

References

Anastasi, A. (1988). Psychological testing. New York: Macmillan.

Ashcraft, M.H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. Mathematical Cognition, 1, 3–34.

Baddeley, A.D. (1996). Exploring the central executive. The Quarterly Journal of Experimental Psychology, 49A, 5–28.

Baddeley, A.D. (2001). Is working memory still working? American Psychologist, 56, 851–864.

Baddeley, A.D., & Hitch, G.J. (1994). Development in the concept of working memory. Neuropsychology, 8, 485–493.

Bryant, B.R., & Rivera, D.P. (1997). Educational assessment of mathematics skills and abilities. Journal of Learning Disabilities, 30, 57–68.

Bull, R., & Johnston, R.S. (1997). Children's arithmetic difficulties: Contributions from processing speed, item identification, and short-term memory. Journal of Experimental Child Psychology, 65, 1–24.

Bull, R., Johnston, R.S., & Roy, J.A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. Developmental Neuropsychology, 15, 421–442.

Butterworth, B., Cipolotti, L., & Warrington, E.K. (1996). Short-term memory impairments and arithmetic ability. Quarterly Journal of Experimental Psychology, 49A, 251–262.

Carnine, D. (1991). Reforming mathematics instruction: The role of curriculum materials. Journal of Behavioral Education, 1, 37–57.

Carroll, J.B. (1993). Human cognitive abilities: A survey of factor analytic studies. New York: Cambridge University.

Carroll, J.B. (1997). The three-stratum theory of cognitive abilities. In D.P. Flanagan, J.L. Genshaft, & P.L. Harrison (Eds.), Contemporary intellectual assessment: Theories, tests, and issues (pp. 122–130). New York: Guilford.

Carroll, J.B. (in press). The higher stratum structure of cognitive abilities: Current evidence supports *g* and about ten broad factors. In H. Nyborg (Ed.), The scientific study of general intelligence: Tribute to Arthur R. Jensen. New York: Pergamon.

Cummins, D.D. (1991). Children's interpretations of arithmetic word problems. Cognition and Instruction, 8, 261–289.

Dark, V.J., & Benbow, C.P. (1990). Enhanced problem translation and short term memory: Components of mathematical talent. Journal of Educational Psychology, 82, 420–429.

Dark, V.J., & Benbow, C.P. (1991). Differential enhancement of working memory with mathematical versus verbal precocity. Journal of Educational Psychology, 83, 48–60.

DeRammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? Memory and Cognition, 29, 267–273.

Flanagan, D.P., McGrew, K.S., & Ortiz, S.O. (2000). The Wechsler intelligence scales and *Gf-Gc* theory: A contemporary approach to interpretation. Boston: Allyn & Bacon.

Flanagan, D.P., & Ortiz, S.O. (2001). Essentials of Cross-Battery assessment. New York: John Wiley & Sons, Inc.

Flanagan, D.P., Ortiz, S.O., Alfonso, V.C., & Mascolo, J.T. (2002). The achievement test desk references (ATDR): A comprehensive framework for LD determination. Boston: Allyn & Bacon.

Fleischner, J.E. (1994). Diagnosis and assessment of mathematics learning disabilities. In G.R. Lyon (Ed.), Frames of reference for the assessment of learning disabilities: New views on measurement issues (pp. 441–458). Baltimore, MD: Paul H. Brookes.

Floyd, R.G., Shaver, R.B., & McGrew, K.S. (in press). Interpretation of the Woodcock-Johnson III Tests of Cognitive Abilities: Acting on evidence. In F.A. Schrank & D.P. Flanagan (Eds.), WJ III clinical use and interpretation. New York: Academic Press.

Friedman, L. (1995). The space factor in mathematics: Gender differences. Review of Educational Research, 65, 22–50.

Fry, A.F., & Hale, S. (2001). Relationships among processing speed, working memory, and fluid intelligence in children. Biological Psychology, 54(1–3), 1–34.

Furst, A.J., & Hitch, G.J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. Memory and Cognition, 28, 774–782.

Geary, D.C. (1993). Mathematics disabilities: Cognitive, neuropsychological, and genetic components. Psychological Bulletin, 114, 345–362.

Geary, D.C. (1994). Children's mathematical development: Research and practical applications. Washington, DC: American Psychological Association.

Geary, D.C. (in press). Learning disabilities in arithmetic: Problem solving differences and cognitive deficits. In H.L. Swanson, K. Harris, & S. Graham (Eds.), Handbook of learning disabilities. New York: Guilford.

Geary, D.C., & Burlington-Dubree, M. (1989). External validation of strategy choice model for addition. Journal of Experimental Child Psychology, 47, 175–192.

Geary, D.C., & Hoard, M.K. (2001). Numerical and arithmetic deficits in learning-disabled children: Relation to dyscalculia and dyslexia. Aphasiology, 15, 635–647.

Geary, D.C., Hoard, M.K., & Hamson, C.O. (1999). Numerical and arithmetic cognition: Patterns of functions and deficits in children at risk for a mathematic disability. Journal of Experimental Child Psychology, 74, 213–239.

Geary, D.C., Saults, S.J., Liu, F., & Hoard, M.K. (2000). Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. Journal of Experimental Child Psychology, 77, 337–353.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. Journal of Special Education, 33, 18–28.

Ginsburg, H.P. (1997). Mathematics learning disabilities: A view from developmental psychology. Journal of Learning Disabilities, 30, 20–33.

Hale, J.B., Fiorello, C.A., Kavanagh, J.A., Hoeppner, J.B., & Gaither, R.A. (2001). WISC-III predictors of academic achievement for children with learning disabilities: Are global and factor scores comparable? School Psychology Quarterly, 16, 31–55.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. Journal of Educational Psychology, 91, 684–689.

Hitch, G., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. British Journal of Psychology, 82, 375–398.

Hoard, M.K., Geary, D.C., & Hamson, C.O. (1999). Numerical and arithmetic cognition. Mathematical Cognition, 5, 65–91.

Horn, J.L. (1991). Measurement of intellectual capabilities: A review of theory. In K.S. McGrew, J.K. Werder, & R.W. Woodcock (Eds.), WJ-R technical manual (pp. 197–232). Itasca, IL: Riverside Publishing.

Horn, J.L., & Noll, J. (1997). Human cognitive capabilities: Gf-Gc theory. In D.P. Flanagan, J.L. Genshaft, & P.L. Harrison (Eds.), Contemporary intellectual assessment: Theories, tests, and issues (pp. 53–93). New York: Guilford.

Hulme, C., & Roodenrys, S. (1995). Verbal working memory development and its disorders. Journal of Child Psychology and Psychiatry, 36, 373–398.

Kail, R. (1991). Development of processing speed in childhood and adolescence. In H.W. Reese (Ed.), Advances in child development and behavior (Vol. 23, pp. 151–185). San Diego, CA: Academic Press.

Kail, R., Hall, L.K., & Caskey, B.J. (1999). Processing speed, exposure to print, and naming speed. Applied Psycholinguistics, 20, 303–314.

Kaufman, A.S. (1994). Intelligent testing with the WISC-III. New York: John Wiley & Sons, Inc.

Kaufman, A.S., & Kaufman, N.L. (1985). The Kaufman Test of Educational Achievement. Circle Pines, MN: American Guidance Service.

Keith, T.Z. (1999). Effects of general and specific abilities on student achievement: Similarities and differences across ethnic groups. School Psychology Quarterly, 14, 239–262.

Kirby, J.R., & Becker, L.D. (1988). Cognitive components of learning problems in arithmetic. Remedial and Special Education, 9, 7–27.

Lehto, J. (1995). Working memory and school achievement in the ninth form. Educational Psychology, 15, 271–281.

Lemaire, P., & Siegler, R.S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. Journal of Experimental Psychology: General, 124, 83–97.

Licht, M.H. (1995). Multiple regression and correlation. In L.G. Grimm & P.R. Yarnold (Eds.), Reading and understanding multivariate statistics (pp. 19–64). Washington, DC: American Psychological Association.

Lyon, G.R., Vaasen, M., & Toomey, F. (1989). Teachers' perceptions of their undergraduate and graduate preparation. Teacher Education and Special Education, 12, 164–169.

McGhee, R.L., & Lieberman, L. (1994). Gf-Gc theory and human cognition: Differentiation of short-term memory and visual memory factors. Psychology in the Schools, 21, 297–304.

McGrew, K.S. (1993). The relationship between the Woodcock-Johnson Psycho-Educational Assessment Battery—Revised Gf-Gc cognitive clusters and reading achievement across the life-span. Journal of Psychoeducational Assessment [Monograph Series: Woodcock-Johnson Psycho-Educational Assessment Battery—Revised], 39–53. Cordova, TN: Psychoeducational Corporation.

McGrew, K.S. (1997). Analysis of the major intelligence batteries according to a proposed comprehensive Gf-Gc framework. In D.P. Flanagan, J.L. Genshaft, & P.L. Harrison (Eds.), Contemporary intellectual assessment: Theories, tests, and issues (pp. 131–150). New York: Guilford.

McGrew, K.S., & Flanagan, D.P. (1998). The intelligence test desk reference (ITDR): *Gf-Gc* Cross-Battery assessment. Boston: Allyn & Bacon.

McGrew, K.S., Flanagan, D.P., Keith, T.Z., & Vanderwood, M. (1997). Beyond g: The impact of Gf-Gc specific cognitive abilities research on the future use and interpretation of intelligence tests in the schools. School Psychology Review, 26, 189–210.

McGrew, K.S., & Hessler, G.L. (1995). The relationship between the WJ-R Gf-Gc cognitive clusters and mathematics achievement across the life-span. Journal of Psychoeducational Assessment, 13, 21–38.

McGrew, K.S., & Knopik, S.N. (1993). The relationship between the WJ-R Gf-Gc cognitive clusters and writing achievement across the life-span. School Psychology Review, 22, 687–695.

McGrew, K.S., & Woodcock, R.W. (2001). Technical manual. Woodcock-Johnson III. Itasca, IL: Riverside Publishing.

McGrew, K.S., & Wrightson, W. (1997). The calculation of new and improved WISC-III subtest reliability, uniqueness, and general factor characteristic information through the use of data smoothing procedures. Psychology in the Schools, 34, 181–195.

McLean, J.F., & Hitch, G.J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. Journal of Experimental Child Psychology, 74, 240–260.

Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., O'Connor, K.M., Chrostowski, S.J., Gregory, K.D., Garden, R.A., & Smith, T.A. (2001). Mathematics benchmarking report: The Third International Math and Science Study—Eighth Grade. Boston, MA: Boston College International Study Center.

Necka, E. (1999). Learning, automaticity, and attention: An individual-differences approach. In P.L. Ackerman, P.C. Kyllonen, & R.D. Roberts (Eds.), Learning and individual differences (Vol. 7, pp. 161–184). Washington, DC: American Psychological Association.

Noel, M.P., Desert, M., Aubrun, A., & Seron, X. (2001). Involvement of short-term memory in complex mental calculation. Memory and Cognition, 29, 34–42.

Pedhazur, E.J., & Schmelkin, L.P. (1991). Measurement, design, and analysis: An integrated approach. Hillsdale, NJ: Lawrence Erlbaum.

Rindermann, H., & Neubauer, A.C. (2000). Speed of information processing and success at school: Do basal measures of intelligence have predictive validity? Diagnostica, 46(1), 8–17.

Rivera-Batiz, F.L. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. Journal of Human Resources, 27, 313–328.

Robinson, N., Abbott, R., Berninger, V.W., & Busse, J. (1996). The structure of abilities in math-precocious young children: Gender similarities and differences. Journal of Educational Psychology, 88, 341–352.

Rourke, B.P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. Journal of Learning Disabilities, 26, 214–266.

Rourke, B.P., & Conway, J.A. (1997). Disabilities of arithmetic and mathematical reasoning: Perspectives from neurology and neuropsychology. Journal of Learning Disabilities, 30, 34–46.

Royer, J.M., Tronsky, L.N., Chan, Y., Jackson, S.J., & Marchant, H. III. (1999). Math-fact retrieval as the cognitive mechanism underlying gender differences in math test performance. Contemporary Educational Psychology, 24, 181–266.

Russell, R., & Ginsburg, H.P. (1984). Cognitive analysis of children's mathematical difficulties. Cognition and Instruction, 1, 217–247.

Shute, G.E., & Huertas, V. (1990). Developmental variability in frontal lobe function. Developmental Neuropsychology, 6, 1–11

Snow, R., & Swanson, J. (1992). Instructional psychology: Aptitude, adaptation, and assessment. Annual Review of Psychology, 43, 583–626.

Sternberg, R.J. (1998). Abilities as developing forms of expertise. Educational Researcher, 27(3), 11–20.

Strang, J.D., & Rourke, B.P. (1983). Concept-formation/non-verbal reasoning abilities in children who exhibit specific academic problems in arithmetic. Journal of Clinical Child Psychology, 12, 33–39.

Swanson, H.L. (1993). Working memory in learning disability subgroups. Journal of Experimental Child Psychology, 56, 87–114.

Vaughn, S., & Wilson, C. (1994). Mathematics assessment for students with learning disabilities. In G.R. Lyon (Ed.), Frames of reference for the assessment of learning disabilities: New views on measurement issues (pp. 459–472). Baltimore, MD: Paul H. Brookes.

Wechsler, D. (1991). Wechsler Intelligence Scale for Children—Third Edition. San Antonio, TX: The Psychological Corporation.

Wechsler, D. (1992). Wechsler Individual Achievement Test. San Antonio, TX: The Psychological Corporation.

Weiler, M.D., Harris, N.S., Marcus, D.J., Bellinger, D., Kosslyn, S.M., & Waber, D.P. (2000). Speed of information processing in children referred for learning problems: Performance on a visual filtering test. Journal of Learning Disabilities, 33, 538–550.

Wilkinson, L. (1990). SYGRAPH: The system for graphics. Evanston, IL: SYSTAT.

Williams, P.C., McCallum, R.S., & Reed, M.T. (1996). Predictive validity of the Cattell-Horn Gf-Gc constructs to achievement. Journal of Psychoeducational Assessment, 3, 43–51.

Wilson, K.M., & Swanson, H.L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? Journal of Learning Disabilities, 34, 237–248.

Woodcock, R.W. (1978). Development and standardization of the Woodcock-Johnson Psycho-Educational Battery. Allen, TX: DLM Teaching Resources.

Woodcock, R.W. (1990). Theoretical foundations of the WJ-R measures of cognitive ability. Journal of Psychoeducational Assessment, 8, 231–258.

Woodcock, R.W. (1993). An information processing view of the Gf-Gc theory. Journal of Psychoeducational Assessment [Monograph Series: Advances in Psychoeducational Assessment: Woodcock-Johnson Psycho-Educational Battery—Revised], 80–102. Cordova, TN: Psychoeducational Corporation.

Woodcock, R.W. (1998). Extending Gf-Gc theory into practice. In J.J. McArdle & R.W. Woodcock (Eds.), Human cognitive abilities in theory and practice (pp. 137–156). Mahwah, NJ: Lawrence Erlbaum.

Woodcock, R.W., & Dahl, M.N. (1971). A common scale for the measurement of person ability and test items difficulty (AGS Paper No. 10). Circle Pines, MN: American Guidance Service.

Woodcock, R.W., & Johnson, M.B. (1989). Woodcock-Johnson Psycho-Educational Battery—Revised. Chicago: Riverside Publishing.

Woodcock, R.W., & Mather, N. (2001). Examiner's Manual. Woodcock-Johnson III Tests of Achievement. Itasca, IL: Riverside Publishing.

Woodcock, R.W., McGrew, K.S., & Mather, N. (2001). Woodcock Johnson Psychoeducational Battery-Third Edition. Chicago: Riverside Publishing.