

Effects of General and Broad Cognitive Abilities on Mathematics Achievement

Gordon E. Taub
University of Central Florida

Randy G. Floyd
The University of Memphis

Timothy Z. Keith
The University of Texas, Austin

Kevin S. McGrew
University of Minnesota

This study investigated the direct and indirect effects of general intelligence and 7 broad cognitive abilities on mathematics achievement. Structural equation modeling was used to investigate the simultaneous effects of both general and broad cognitive abilities on students' mathematics achievement. A hierarchical model of intelligence derived from the Cattell–Horn–Carroll (CHC) taxonomy of intelligence was used for all analyses. The participants consisted of 4 age-differentiated subsamples (ranging from ages 5 to 19) from the standardization sample of the Woodcock–Johnson III (WJ III; Woodcock, McGrew, & Mather, 2001). Data from each of the 4 age-differentiated subsamples were divided into 2 data sets. At each age level, one data set was used for model testing and modification, and a second data set was used for model validation. The following CHC broad cognitive ability factors demonstrated statistically significant direct effects on the mathematics achievement variables: Fluid Reasoning, Crystallized Intelligence, and Processing Speed. In contrast, across all age levels, the general intelligence factor demonstrated indirect effects on the mathematics achievement variable.

Keywords: CHC, cognitive, mathematics, cognition, SEM

During the 20th century, research on the relation between cognitive ability and academic achievement focused predominately on the predictive utility of measures of general intelligence (g) and to a lesser extent on specific cognitive abilities that may account for additional achievement variance beyond g . The conclusions from most of this research were that measures of general intelligence are the most powerful predictors of academic achievement

and that g accounts for more variance than any other variable (or combination of variables exclusive of g) in the prediction of success across a wide array of outcomes (Cronbach & Snow, 1977; Hunter & Hunter, 1984; Jensen, 1984, 1998). Based, in part, on this body of research, many scholars adopted a “just say no” stance to the interpretation of specific measures of cognitive abilities (clusters, indexes, or individual subtest scores) from intelligence test batteries (McDermott, Fantuzzo, & Glutting, 1990). McNemar's (1964) presidential address to the American Psychological Association is often cited as the key event leading to the termination of attempts to develop multifactor test batteries that could differentially predict academic achievement.

A number of researchers have suggested that the “just say no” mantra needs a critical reexamination in the light of recent developments in intelligence theory, measurement, and research methodology (e.g., McGrew, Flanagan, Keith, & Vanderwood, 1997). Collectively, a series of recent studies based on the Cattell–Horn–

Gordon E. Taub, Department of Child, Family, & Community Sciences, University of Central Florida; Randy G. Floyd, Department of Psychology, The University of Memphis; Timothy Z. Keith, Department of Educational Psychology, The University of Texas, Austin; Kevin S. McGrew, Department of Educational Psychology, University of Minnesota.

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Correspondence concerning this article should be addressed to Gordon E. Taub, The University of Central Florida, Department of Child, Family, & Community Sciences, Orlando, FL 32816. E-mail: gtaub@mail.ucf.edu

Carroll (CHC) theory of intelligence have suggested that specific cognitive abilities may make important contributions to understanding academic achievement above and beyond the effect of *g* (Floyd, Keith, Taub, & McGrew, 2007; Hale, Fiorello, Kavanagh, Hoepfner, & Gaither, 2001; McGrew et al., 1997; Vanderveld, McGrew, Flanagan, & Keith, 2002). Collectively, these “just say maybe” studies indicated that specific cognitive abilities may be important in understanding and explaining academic achievement, above and beyond *g*, and that specific cognitive abilities may help to explain why some students experience difficulties in specific areas of the academic curriculum.

Under girding the optimism of the just say maybe contingent of researchers are advances in research methodology. Much of the extant just say no research used multiple-regression analysis (MR). A significant limitation of MR is that composite scores (*g*) and components that compose the composite scores (clusters, indexes, or individual subtest scores) should not be simultaneously analyzed in a MR equation (Thorndike, Hagen, & Sattler, 1986) because multicollinearity and the likelihood of singular matrices would threaten conclusions from that research. Because MR will not allow the prediction of a criterion from both a composite score and the components that contribute to the composite score, creative uses of MR have been employed to conduct most of the just say no analyses. However, use of MR has not allowed for the direct comparison of the effects of general abilities and specific abilities in a single model (McGrew et al., 1997). In addition, most of the research employing MR has focused exclusively on partitioning sources of explained variance, a practice that can mask the relative importance of different variables on a criterion and that may result in the underestimation of important specific effects of one variable on another (e.g., Abelson, 1995; Keith, 2006; Pedhazur, 1997).

According to most just say maybe researchers, the other reason for optimism about the benefits of measures of specific cognitive abilities is the recent convergence of theories yielding CHC theory. This taxonomy of human cognitive abilities is a synthesis of two models of intelligence based on more than a half a century of factor analytic, developmental, heritability, external outcome validity,

and neurocognitive research evidence (McGrew, 2005; McGrew & Flanagan, 1998). The first model is Cattell and Horn’s extended Gf–Gc theory (Horn & Blankson, 2005). The second model is Carroll’s three-stratum theory of cognitive abilities (Carroll, 1993). Both the Cattell–Horn model and the Carroll model are hierarchical in nature—with one fundamental difference. Carroll’s model includes three levels. At the top of the Carroll hierarchy is *g* (Stratum III), at the second level are broad cognitive abilities (Stratum II), and at the first level are over 70 narrow cognitive abilities (Stratum I). In contrast, the Cattell–Horn model contains only two hierarchical levels, the level containing the broad cognitive abilities (Stratum II) and the level containing the narrow cognitive abilities (Stratum I). The general factor of intelligence, *g*, is excluded in the Cattell–Horn model.

The Cattell–Horn model and the Carroll model also differ in the categorization and placement of a small number of narrow cognitive abilities under the broad cognitive ability domains, and they occasionally use different ability terminology. In an attempt to provide a common nomenclature for these models, McGrew (1997) integrated them into a single model (see McGrew, 2005). According to the integrated CHC model, there appears to be some consensus that there are 10 broad cognitive abilities. These abilities include Fluid Reasoning, Crystallized Intelligence, Short-Term Memory, Visual Processing, Auditory Processing, Long-Term Storage and Retrieval, Processing Speed, Reading and Writing Ability, Quantitative Knowledge, and Reaction Time/Decision Speed. These CHC broad cognitive abilities subsume approximately 70 narrow cognitive abilities. Based on the accumulated evidence supporting the Cattell–Horn model and the Carroll model, as well as the integrated CHC model from recent research and test validation, the CHC theory provides a well-supported theoretical framework for research examining the cognitive influences on the development and maintenance of academic skills.

Mathematics and Cattell–Horn–Carroll-Based Research

Using CHC theory, McGrew et al. (1997) investigated the relative contribution of seven

CHC broad cognitive abilities and g on mathematics achievement. McGrew et al. identified the CHC broad cognitive abilities Crystallized Intelligence, Fluid Reasoning, and Processing Speed as influences on mathematics reasoning beyond the effects of g . Keith (1999) also reported mathematics reasoning was strongly influenced by g and a number of other CHC broad ability factors including Crystallized Intelligence, Fluid Reasoning, and Processing Speed.

Both McGrew et al. (1997) and Keith (1999) used structural equation modeling (SEM) to investigate the relative contribution of individual CHC broad cognitive abilities, beyond g , on mathematics achievement. The benefits of using SEM to identify the relative contribution of broad cognitive abilities and g on academic achievement and mathematics achievement were demonstrated in earlier research (Gustafsson & Balke, 1993). Together, these studies provided a foundation for understanding the influence of CHC broad cognitive ability factors on mathematics achievement beyond the acknowledged large effect of g . However, these studies do not explain fully how cognitive abilities affect mathematics achievement because the broad cognitive abilities were not specified to “compete” equally with g in explaining mathematics achievement (Floyd et al., in press).

Purpose of the Study

The purpose of this study was twofold. The first purpose was to identify which factors representing g and CHC broad cognitive abilities explain mathematics achievement from early kindergarten through high school. The second purpose was to identify how these effects change during this period of academic development. To extend the research in this area, a hierarchical model was developed using the CHC theory as a blueprint and tested using SEM. This model included a second-order general factor and seven CHC broad cognitive ability first-order factors as influences on mathematics achievement. Although previous research indicated the CHC broad ability factors Crystallized Intelligence, Fluid Reasoning, and Processing Speed were statistically significant influences on mathematics achievement (e.g., McGrew et al., 1997; Keith, 1999), four addi-

tional broad ability factors were also specified in the explanatory models.

Method

Participants

The participants for this investigation were a subsample of the nationally representative WJ III standardization sample used by McGrew and Woodcock (2001) in their structural analyses. The standardization sample was stratified according to race, sex, geographic region, education, and age to ensure that the sample mirrored the population characteristics of children, adolescents, and adults in the United States, as described by the United States Census projections for the year 2000. Participants in the current study consisted of that portion of the standardization sample between 5 years old and 19 years old. Four age-differentiated subsamples included children ages 5 to 6 ($n = 639$), 7 to 8 ($n = 720$), 9 to 13 ($n = 1,995$), and 14 to 19 ($n = 1,615$). Each age-based sample was randomly split into a calibration sample and validation sample.

Instruments

This study used 18 tests from the WJ III Tests of Cognitive Abilities (WJ COG), 4 tests from the WJ III Tests of Achievement (WJ ACH), and 5 tests and 1 special composite from the WJ III Diagnostic Supplement (Woodcock, McGrew, Mather, & Schrank, 2003) as indicators of CHC cognitive abilities. The special composite was Numerical Reasoning, which represents a combination of Number Series and Number Matrices tests. Together, these 28 measures provided indicators of the seven CHC broad ability cognitive factors. Table 1 contains a description of each of the 28 cognitive tests and the broad ability and narrow abilities associated with each test.

Mathematics achievement was measured by two tests from the WJ III ACH (Mather & Woodcock, 2001), Applied Problems and Calculation. The Applied Problems test required comprehending the nature of a problem, identifying relevant information, performing calculations, and stating solutions. The Calculation test required calculation of problems ranging from simple addition facts to calculus. These two

Table 1
Descriptions of Woodcock Johnson III Tests Included in the Cognitive Ability Measurement Models

Test	Broad Ability	Narrow Ability	Test Description
Numerical Reasoning	<i>Gf</i>	Quantitative reasoning	Examinees must determine numerical sequences and determine a two-dimensional numerical pattern
Concept Formation	<i>Gf</i>	Induction	Examinees must identify rules governing the organization of colored geometric figures when shown instances and noninstances of concepts
Analysis-Synthesis	<i>Gf</i>	General sequential reasoning	Examinees must analyze the components of an incomplete logic puzzle and to determine the missing components
Block Rotation	<i>Gv</i>	Visualization	Examinees must identify geometric designs that match a target design but have been rotated to a different visual perspective
Spatial Relations	<i>Gv</i>	Spatial relations	Examinees must select the component parts of whole shape
Picture Recognition	<i>Gv</i>	Visual memory	Examinees must study images for 5 s and identify images within a larger array after the initial images have been removed
Visual Matching	<i>Gs</i>	Perceptual speed	Examinees must quickly locate and circle the two identical numbers in a row of six numbers during a 3-minute period
Decision Speed	<i>Gs</i>	Mental comparison speed	Examinees must rapidly scan a row of images and mark two images are the most closely related during a 3-minute period
Cross Out	<i>Gs</i>	Perceptual speed, rate-of-test-taking	Examinees must mark drawings that are identical to the first drawing in the row during a 3-minute period
Rapid Picture Naming	<i>Gs</i>	Naming facility	Examinees must quickly name a series of stimulus pictures
Retrieval Fluency	<i>Glr</i>	Ideational fluency	Examinees must state as many words from specified categories as possible in 1 minute
Visual-Auditory Learning: Delayed	<i>Glr</i>	Associative memory	Examinees must recall and relearn (after a 30-minute to 8-day delay) symbols presented in Visual-Auditory Learning
Visual-Auditory Learning	<i>Glr</i>	Associative memory	Examinees must associate new visual symbols with orally presented words in order to translate the series of symbols
Memory for Names	<i>Glr</i>	Associative memory	Examinees must remember an increasingly large number of names of novel cartoon characters
Memory for Names: Delayed	<i>Glr</i>	Associative memory	Examinees must recall and relearn (after a 30-minute to 8-day delay) names of novel cartoon characters presented in Memory for Names
Sound Blending	<i>Ga</i>	Phonetic coding: Synthesis	Examinees must listen to a series of individual syllables, individual phonemes, or both that form words and name the complete words
Incomplete Words	<i>Ga</i>	Phonetic coding: Analysis	Examinees must listen to words with one or more phonemes missing and name the complete words
Sound Patterns	<i>Ga</i>	Speech-sound discrimination	Examinees must indicate whether pairs of complex sound patterns are the same or different. The patterns may differ in pitch, rhythm, or sound content
Auditory Working Memory	<i>Gsm</i>	Working memory	Examinees must listen to a mixed series of words and digits and then to rearrange them by first saying the words in order and then the numbers
Numbers Reversed	<i>Gsm</i>	Working memory	Examinees must repeat a series of random numbers backward
Memory for Words	<i>Gsm</i>	Memory span	Examinees must repeat lists of unrelated words in the correct sequence
Memory for Sentences	<i>Gsm</i>	Memory span	Examinees must repeat complete sentences

Table 1 (continued)

Test	Broad Ability	Narrow Ability	Test Description
Picture Vocabulary	<i>Gc</i>	Lexical knowledge	Examinees must name familiar and unfamiliar pictured objects
Verbal Comprehension	<i>Gc</i>	Language development, lexical knowledge	Examinees must name familiar and unfamiliar pictured objects, say words similar in meaning to word presented, say words that are opposites in meaning to the word presented, and complete phrases with words that complete analogies
General Information	<i>Gc</i>	General information	Examinees must provide common or typical characteristics of objects by responding to questions, such as “Where you would find . . .?” and “What you would do with . . .?”
Academic Knowledge	<i>Gc</i>	General information	Examinees must provide information about the biological and physical sciences; history, geography, government, and economics; and art, music, and literature.
Oral Comprehension	<i>Gc</i>	Listening ability	Examinees must listen to a short passage and orally supply the word missing at the end of the passage
Story Recall	<i>Gc</i>	Listening ability	Examinees must listen to a short passage and describe the gist of what they heard

Note. *Gf* = Fluid Reasoning; *Gv* = Visual-Spatial Thinking; *Gs* = Processing Speed; *Gltr* = Long-Term Retrieval; *Ga* = Auditory Processing; *Gsm* = Short-Term Memory; *Gc* = Comprehension-Knowledge.

tests provided indicators of the Quantitative Knowledge factor, the mathematics achievement dependent variable.

Analysis

All analyses were conducted using SEM via the Amos program (Arbuckle & Wothke, 1999). Correlations and standard deviations for both the calibration sample and the validation sample were converted into covariance matrices for all analyses. (Matrices are available from the first author, by request.)

Models

Figure 1 presents the CHC-based measurement and structural model. This model is hierarchical in nature and contains a second-order general factor of intelligence (*g*) at the apex. The left side of Figure 1 presents the measured variables—scores on the WJ III tests—that are represented by rectangles. The seven broad cognitive ability factors are represented as ellipses to the immediate right of their measured variables. The arrows leading from each factor to the measured variables represent the first-order factor structure of the WJ tests. The circles to the right of each rectangle represent the unique and error variance associated with each

test. As can be seen in Figure 1, each CHC broad cognitive factor was measured by at least three tests, and each factor measures a number of narrow abilities subsumed by the broad cognitive factor (see McGrew & Woodcock, 2001).¹ Support for portions of the measurement model has been previously published (Floyd et al., in press; Keith, Kranzler, & Flanagan, 2001; McGrew & Woodcock, 2001; Taub & McGrew, 2004). The second-order *g* factor is represented by a single

¹ The WJ III includes many additional tests that could have been specified to be additional indicators of each latent CHC cognitive factor. A decision to limit the number of manifest indicators for each CHC cognitive factor was based on two primary considerations. First, to ensure the purest possible latent factors, an attempt was made to minimize the use of WJ III tests that had previously been demonstrated to be factorally complex (loading on more than one factor). Second, to ensure that each latent CHC cognitive factor was “broad” in breadth of coverage, tests were selected that are considered to be measures of different narrow CHC abilities (see McGrew & Woodcock, 2001). Finally, as a test of this “less (3 indicators)-rather-than-more (4 to 5 indicators)” per CHC latent factor approach, a series of preliminary analyses were run on the measurement model. The magnitude of the intercorrelations among the seven CHC cognitive latent factors, when defined by 3 versus 4 or 5 indicators, indicated that the magnitude and pattern of latent factor intercorrelations for the 3-indicator factors were indistinguishable from 4 and 5 indicator factors.

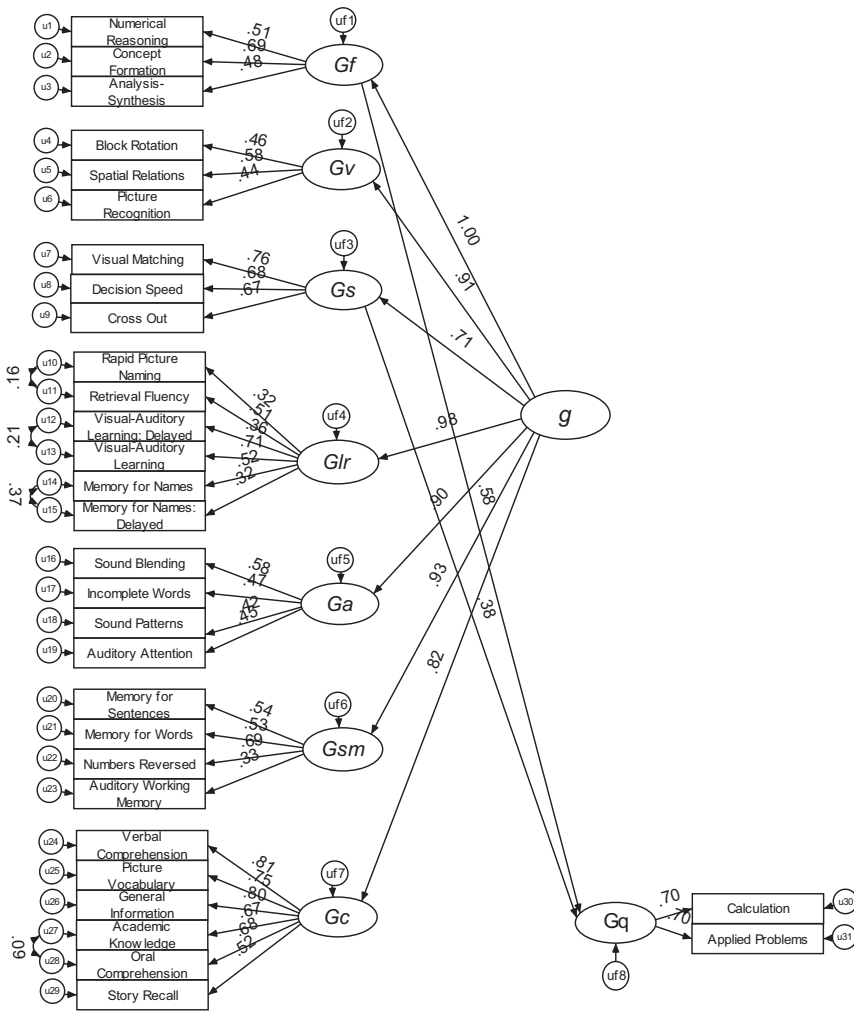


Figure 1. The validation model for the 5- to 6-year-old age group, which accounts for the indirect effects of the general factor and identifies the first-order CHC broad cognitive ability factors that have a direct effect on Quantitative Knowledge. *Gf* = Fluid Reasoning, *Gv* = Visual Processing, *Gs* = Processing Speed, *Glr* = Long-Term Storage and Retrieval, *Ga* = Auditory Processing, *Gsm* = Short-Term Memory, *Gc* = Crystallized Intelligence, *g* = General Intelligence, and *Gq* = Quantitative Knowledge.

ellipse near the middle of the figure. The bottom right side of the figure presents the measurement model for the Quantitative Knowledge factor with the same interpretation associated with rectangles, circles, and ellipses. The structural portions of the models are represented by single-headed arrows between the factors (ellipses).

Analysis

The purpose of the first analyses was to identify which CHC general or broad cognitive abil-

ity factors displayed significant effects on mathematics achievement. Analyses were conducted in phases. In the first phase (Phase 1), a model was specified that included the direct effect of all seven broad cognitive ability factors and *g* on the Quantitative Knowledge factor using the calibration sample for each age group. After a model was estimated, the highest negative path was removed, the model was re-estimated until all negative paths were eliminated. Following the elimination of negative paths, nonsignificant

Table 2
Fit Statistics for Final Models Across Four Age Groups

Sample Ages	χ^2	<i>df</i>	TLI	CFI	RMSEA (90% Interval)	SRMR
Ages 5 to 6	725.07	422	.899	.908	.046 (.04–.051)	.050
Ages 7 to 8	912.15	423	.872	.884	.054 (.05–.059)	.055
Ages 9 to 13	1323.50	421	.887	.898	.052 (.049–.056)	.046
Ages 14 to 19	1328.45	422	.873	.885	.059 (.055–.062)	.052

Note. TLI = Tucker Lewis Index; CFI = comparative fit index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

paths (paths with critical values less than 1.96, $p \geq .05$) were eliminated from the model one at a time, and the model was re-estimated until all nonsignificant paths were eliminated. Finally, modification indexes were examined to determine if any eliminated paths should be added. The same procedures were followed for all age groups. Through this process of iterative model generation, the final models were those containing only significant positive structural paths at each age group.

The purpose of Phase 2 was to validate the final models developed during Phase 1. This phase involved estimating the final model from the calibration sample using the data from independent validation sample at each age level (MacCallum et al., 1994). Structural paths that were not statistically significant were deleted, and modification indexes were examined to determine if deleted structural paths should be added. Results from the final models using the validation samples are reported.

Results

The structural paths contained within the final models derived from the calibration samples were all statistically significant when tested using the data from the validation samples. No structural paths were added based on modification indexes. An example of the final model for ages five to six is provided in Figure 1. Table 2 presents the goodness-of-fit indexes for the final models for the validation samples for the four age groups. The Tucker-Lewis index (TLI), comparative fit index (CFI), root mean square error of approximation (RMSEA), and the standardized root mean square residual (SRMR) were used to evaluate each model’s fit to the data. The RMSEA and SRMR served as the primary fit indexes to evaluate the fit of single

models at each age level. Current rules of thumb and empirical research indicate that RMSEA values below .06 and SRMR values below .08 suggest a good fit of the model to the data (Hu & Bentler, 1998, 1999). Of the two indexes, the SRMR may be the preferred index because it is easily interpreted as the average difference between the correlation matrix used to estimate the model and the matrix implied by the model. As can be seen in the Table 2, all fit indexes suggest the models provided a good fit the observed data across all age groups. The TLI and CFI are also reported; however, these indexes tend to worsen when models contain many different variables, as in the current study (Kenny & McCoach, 2003).

Table 3 presents the standardized path coefficients between the CHC cognitive ability factors and the Quantitative Knowledge factor for all age groups. One of the most thought-provoking findings from the study was that general intelligence, *g*, had large but only indirect effects on the Quantitative Knowledge factor

Table 3
Standardized Indirect Effects of g and Standardized Direct Effects of CHC Broad Cognitive Abilities on Quantitative Knowledge Across Four Age Groups

Standardized Effects	Age Groups			
	5 to 6	7 to 8	9 to 13	14 to 19
From <i>g</i>	.85	.73	.68	.72
From <i>Gc</i>	—	—	<u>.17</u>	<u>.43</u>
From <i>Gs</i>	<u>.38</u>	—	<u>.15</u>	—
From <i>Gf</i>	<u>.58</u>	.75	<u>.49</u>	<u>.37</u>

Note. Direct effects are underlined. *g* = general intelligence; *Gf* = Fluid Reasoning; *Gs* = Processing Speed; *Gc* = Crystallized Intelligence; *Glr* = Long-Term Storage and Retrieval.

across all age groups.² This set of findings means that between ages 5 and 19, *g* had a direct effect on the broad CHC factors, which in turn, had a direct effect on the mathematics dependent variable, Quantitative Knowledge.

Results presented in Table 3 demonstrate how the direct effects of the CHC broad cognitive ability factors on Quantitative Knowledge change as a function of age. Fluid Reasoning demonstrated large direct effects on Quantitative Knowledge across all ages. Crystallized Intelligence also demonstrated moderate effects on Quantitative Knowledge for ages 9 to 13 and strong effects for ages 14 to 19. Processing Speed demonstrated strong effects on Quantitative Knowledge for ages 5 to 6 and moderate effects for ages 9 to 13. Thus, Fluid Reasoning was the only broad cognitive ability factor to have a statistically significant direct effect on Quantitative Knowledge for ages 7 to 8. The indirect effect of *g* on Quantitative Knowledge was primarily via *Gf*; *g* had a large direct effect on *Gf* at every age level.

Discussion

During much of the 20th century research in cognitive abilities and academic achievement focused on the predictive utility of general intelligence, *g*, on achievement. The results from much of this research found that the inclusion of specific cognitive abilities in the prediction of achievement violated the rule of parsimony because they tended to add very little predictive variance. More recent advances in intellectual theory (CHC theory) and data analysis methods (SEM) have assisted researchers in quantifying the portion of variance accounted for by specific cognitive factors on academic achievement. The purpose of this study was to employ SEM to identify the influences of general intelligence and seven CHC broad cognitive abilities on mathematics achievement. To accomplish this goal, a hierarchical model based on contemporary CHC theory was analyzed. The CHC model contained two levels of cognitive factors. At the apex of the model was a second-order general factor of intelligence. At the first-order factor level were seven CHC broad cognitive ability factors.

Direct Effects

The results from this study revealed that first-order CHC broad cognitive factors do in fact have statistically significant direct effects on mathematics achievement across all four age groups. These factors included Fluid Reasoning, Crystallized Intelligence, and Processing Speed. Fluid Reasoning demonstrated consistent large direct effects on the Quantitative Knowledge dependent variable across all four age-differentiated samples included in the analysis. The finding of a significant direct effect of Fluid Reasoning on mathematics achievement was not unexpected. The robust effect of Fluid Reasoning was consistent with earlier CHC-based studies that investigated the relations between measures of Fluid Reasoning and mathematics achievement (e.g., Floyd, Evans, & McGrew, 2003; Keith, 1999; McGrew et al., 1997; McGrew & Hessler, 1995; Proctor, Floyd, & Shaver, 2005; Williams, McCallum, & Reed, 1996) as well as other research (Fuchs et al., 2005, 2006; Rourke, 1993; Swanson & Beebe-Frankenberger, 2004). Fluid Reasoning seems to account for some of the prominent problem-solving constructs and strategies implicated in mathematics performance (Cummins, 1991; Fuchs et al., 2006; Lemaire & Siegler, 1995; Swanson & Beebe-Frankenberger, 2004).

Crystallized Intelligence demonstrated moderate to large direct effects on Quantitative Knowledge with two age groups. Previous research investigating the role of CHC cognitive abilities in mathematics achievement also found a strong and consistent relation between Crystallized Intelligence and mathematics achievement (e.g., Floyd et al., 2003; Hale et al., 2001; Keith, 1999; McGrew et al., 1997; McGrew & Hessler, 1995; Proctor et al., 2005; Williams et al., 1996). Fuchs et al. (2006) also found that a similar broad ability factor was a significant predictor of completion of arithmetic word problems. It is logical that completion of arithmetic word problems, like those from the WJ III Applied Problems test, would require receptive

² Standardized coefficient effect sizes of .05 and above can be considered small effects, effect sizes around .15 can be considered moderate effects, and effect sizes above .25 can be considered *large* effects (cf. Keith, 1999, 2006; Pedhazur, 1997).

language abilities and that completion of math-related tasks require declarative and procedural knowledge learned from formal schooling. However, other researchers have not demonstrated significant effects for similar measures in explaining mathematics performance (Fuchs et al., 2005; Swanson & Beebe-Frankenberger, 2004).

Processing Speed was significantly related to Quantitative Knowledge at the earliest age level and for ages 9 to 13. These cross-age effects corroborate the findings of a number of studies focusing on CHC theory (e.g., Floyd et al., 2003; Keith, 1999; McGrew et al., 1997; McGrew & Hessler, 1995), as well as others studies (Bull & Johnston, 1997; Fuchs et al., 2006; Kirby & Becker, 1988) that suggest that the ability to process and make decisions quickly about visual stimuli (without verbalizations) is related to the ability to complete mathematics computations and other early academic tasks (Fry & Hale, 2001).

In contrast to previous research that included a higher order g factor in SEM models predicting mathematics performance that were guided by CHC theory (Keith, 1999; McGrew et al., 1997) and supporting the factor structure of a test battery (Oh, Glutting, Watkins, Youngstrom, & McDermott, 2004), general intelligence did not have a direct effect on the Quantitative Knowledge dependent variable. This finding means that g had indirect, rather than direct, effects on mathematics achievement, through the CHC broad cognitive ability factors. The indirect effects of g on mathematics achievement is at odds with previous research (Keith, 1999; McGrew et al., 1997; Oh et al., 2004) because it is likely that the path from g to the math achievement dependent variable was purposefully retained in prior studies, whereas in the present investigation, g was allowed to compete equally with the specific abilities in the prediction of mathematics achievement. It is also the case, however, that there was a large and consistent effect of g on fluid reasoning across all four age groups in the present research. The path coefficients between these two constructs from the youngest to the oldest age group were .999, .980, .913, and .948, respectively. Thus, although g affected mathematics achievement via Gf , it was not always possible to separate g and Gf . Nevertheless, it is also important to note that this finding (indirect

rather than direct effects for g) is consistent with a recent study using the same CHC model in which general intelligence and the seven CHC broad ability factors influenced reading decoding skills (Floyd et al., in press). In this study, general intelligence did not have a direct effect on reading decoding skills between the ages of 5 through 39.

Other Abilities

Based on much previous research, it was not unexpected that Auditory Processing did not demonstrate significant effects on mathematics performance (e.g., Floyd et al., 2003; Swanson & Beebe-Frankenberger, 2004, cf. Fuchs et al., 2005). It was also not unexpected that Visual Processing did not demonstrate significant effects, although perhaps Visual Processing contributes to the earliest stages of mathematics skill development (Geary, 1993). Such effects have not been consistently demonstrated in research guided by CHC theory or in other research (see Friedman, 1995). However, in some instances in which measures of Visual Processing were combined with measures of Fluid Reasoning, the resulting composite score demonstrated significant relations with measures of mathematics (e.g., Fuchs et al., 2005).

It was unexpected to find the absence of significant effects for Short-Term Memory on mathematics achievement. This result is at odds with some previous research guided by CHC theory (e.g., Floyd et al., 2003; McGrew & Hessler, 1995) and with a rather large body of research indicating the importance of the construct system dealing with the limited capacity and management of information in immediate memory (i.e., working memory) during mathematics performance in general, and especially during completion of mathematics word problems (Fuchs et al., 2005; Passolunghi & Seigel, 2001; Swanson & Beebe-Frankenberger, 2004). However, like evident in Fuchs et al. (2006), it is possible that most of the variance attributable to working memory may be accounted for by cognitive abilities measures simultaneously entered in the model.

Limitations

The interpretation of these findings should be tempered by some limitations. First, all mea-

asures used in this research came from a single battery of tests completed within a brief period of time. Future research should determine if similar effects are found using other measures and across time. Second, this study used CHC theory as the blueprint for all analyses. It is possible that future research employing a different theoretical framework would yield different results with these same data. Third, mathematics achievement latent variable represents a purposely general measure of mathematics skills, so it may not accurately reflect the mathematics performance across the varied areas of mathematics achievement, such as arithmetic, algorithmic computation, arithmetic word problems, and geometry (Fuchs et al., 2006).

There were also several advantages to the present study. This research used a calibration-validation approach to model development and testing. Models were developed using data from one sample and validated on a second sample. This method of calibration and validation provides more stable findings, which should be more easily replicated when compared to using a single data set that do not guard against the dangers of specification searches. Finally, the mathematics achievement dependent variable, the seven CHC broad cognitive ability factors, and in turn the general factor of intelligence were measured by tests from a well-validated instrument that was standardized on a nationally represented sample of children.

Implications

The implications from the results of this study are threefold. First, in contrast to earlier studies, general intelligence appears to have an indirect effect on mathematics achievement. Thus, the large but indirect effect of g on mathematics achievement can be observed through its direct effect on the CHC broad cognitive ability factors. Second, there is a direct effect of the CHC broad cognitive ability factors Fluid Reasoning, Crystallized Intelligence, and Processing Speed on mathematics achievement. For practicing school psychologists, these findings indicate that measures of these factors should be included when conducting cognitive ability testing with an individual experiencing difficulty in mathematics. These results also are important for future research in understanding the cognitive influences on mathematic achieve-

ment. Specifically, explanatory models that do not include measures of these three cognitive factors may not provide a comprehensive explanation of the cognitive components responsible for mathematics achievement. Finally, the results provide support for the “just say maybe” mantra by providing evidence that there is a need to look beyond g when explaining children’s mathematics achievement. These results may lead us to recognize that underdeveloped broad cognitive abilities may interfere with an individual’s ability to acquire academic skills in mathematics problem-solving and accurate numerical calculation and that well-developed broad cognitive abilities may facilitate advanced performance in these academic areas.

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