The higher-stratum structure of cognitive abilities:

Current evidence supports *g* and about ten broad factors

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Abstract

(Not for publication; Nyborg wants it for his own reference in order to write a summary chapter for the edited volume.]

Three somewhat different views about the higher-stratum structure of cognitive abilities are considered: (a) The classic view of Spearman, Thurstone (in his later years), Jensen, Carroll, and many others that a general factor of intelligence ***g*** exists and can be confirmed, along with a series of about 10 broad second-stratum factors, including factors called ***Gf*** and ***Gc*** defined by specifiable types of variables; (b) the view of Gustafsson, Undheim, and some others that a general factor of intelligence exists and can be confirmed (along with various second-stratum factors), but that it is highly or even perfectly correlated with a second-stratum factor ***Gf*** as proposed by others, and (c) the view of Horn and some others that there is "no such thing" as a general factor of intelligence, because it cannot be properly conceived or experimentally demonstrated, but that the factors ***Gf*** and ***Gc*** exist as second-stratum factors (along with about 8 others) and can be confirmed. Relevant datasets assembled and studied by McGrew, Werder, and Woodcock are analysed by both exploratory and confirmatory factoring methods to investigate the statistical hypotheses implied in the three views that have been mentioned. In response, the evidence from reanalyses of these datasets suggests the following conclusions:

(a) Classical hypotheses claiming a general factor ***g*** and, orthogonal to it and to each other, two or more second-stratum factors, can be confirmed even when the second-stratum factors include ***Gf***. The existence of ***Gf*** as a second-stratum factor separate from ***g*** is no longer doubtful, but there may be problems in validly measuring it.

(b) The notion (favored by Gustafsson) that there exists a general factor of intelligence ***g*** in addition to broad second-stratum factors, including a factor ***Gf*** with which it is highly or perfectly correlated, can be accepted, with the provision that in some datasets, ***Gf*** can be clearly distinguished from ***g***.

(c) The notion (favored by Horn) that factor ***g*** does not exist cannot be accepted. It ignores the fact that with the use of confirmatory analysis techniques in which a general intelligence factor ***g*** is postulated, such a factor can easily be confirmed, even when ***Gf*** and ***Gc*** factors independent of ***g*** can be shown to be present. If suitable test variables are present in the test battery analyzed, factor ***g*** shows significant loadings on a great variety of mental tests (though not necessarily all such tests).

It remains to verify these conclusions by analyses of further datasets.

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As I proposed in my volume Human Cognitive Abilities (Carroll, 1993), cognitive abilities may be assumed to exist at three principal levels or strata: a first, lower-order stratum comprising some 50 to 60 or more narrow abilities that are linearly independent of each other (that is, possibly intercorrelated but with clearly separated vectors in the factorial space); a second stratum comprising approximately 8 to 10 or more broad abilities, also linearly independent of each other; and a third still higher stratum containing only a single, general intellectual ability commonly termed ***g.*** (For consistency with the custom observed in the literature, I use only the lower case letter to refer to the supposed factor ***g*** occupying the highest level.) The present chapter is concerned with the structure of the two higher levels, that is, with questions about whether there actually exist two higher-order levels, and with what factors can be shown to occupy each of these levels.

The title of the present volume of essays written in tribute to Arthur Jensen, as well as the title of Jensen's most recent masterwork, The G Factor (Jensen, 1998), would appear to guarantee definitively that there exists a unitary factor of cognitive ability (or "intelligence") that can be termed ***g.*** Indeed, ever since the publication of Charles Spearman's seminal writings on intelligence (1904, 1923, 1927) the almost universally accepted assumption among many psychologists, educators, and even popular writers has been that there does indeed exist a single general factor of intelligence, possibly along with other, more specialized dimensions of ability (Eysenck, 1994). The assertion that a general factor of intelligence exists would be the principal statement to make about the higher-stratum structure of cognitive abilities.

At the time of writing Human Cognitive Abilities, I did not think it reasonable to question the existence of a general factor because I tended to accord with the widespread acceptance of such a factor in the psychometric research community. Nevertheless, even in the several decades after the publication of Spearman's writings about a general factor in 1904 and later, concern (reviewed by Burt, 1949a,b) about the genuine existence of such a factor began to appear, particularly in England. However, considerable evidence for a general factor was accumulated in the early years of the twentieth century, and Holzinger (1936) and even Thurstone and Thurstone (1941), who had endorsed the existence of first-order "primary" factors, published evidence supporting a ***g*** factor. In his book on technical considerations in performing multiple factor analyses, Thurstone (1947, p. 421) included a chapter on the computation of second-order factors, one of which, he thought, might be a general factor similar to that espoused by Spearman.

In recent times the most pertinent developments concerning the higher-order structure of cognitive abilities have been based on technical advances in factor-analytic methodology, in particular the development of confirmatory factor analysis as opposed to the exploratory factor analysis techniques formulated earlier by Thurstone (1938, 1947) and many others. In confirmatory factor analysis (Jöreskog & Sörbom, 1989), it has become possible readily to apply statistical significance tests to factorial results in order to confer on them a greater degree of scientific plausibility. Using structural equation models involved in confirmatory factor analysis, Gustafsson and others (Gustafsson, 1984, 1989, 2001; Gustafsson & Balke, 1993; Gustafsson & Undheim, 1996) have consistently found what they interpret as a general factor ***g***, but they also have found and confirmed two factors similar to those proposed by Cattell as early as 1941 (Cattell, 1941) and later studied by Cattell (1971) and Cattell and Horn (1978; see also Horn, 1965), namely, a ***Gf*** (fluid intelligence) factor and a ***Gc*** (crystallized intelligence) factor. Gustafsson and his colleagues report that their ***Gf*** factor tends to be highly or even perfectly correlated with ***g*** but Cattell and Horn have tended to reject the notion of a general factor at a third stratum in the hierarchy of abilities. Further, Gustafsson and colleagues found and confirmed what they call a cultural knowledge factor ***Gc*** as previously described by Cattell (1971) and Cattell and Horn (1978). Such findings at least raise questions about the interpretation and even the existence of a general factor.

In the early writings of Cattell and Horn, there was already an insistence that the type of ***g*** factor identified by Spearman and many others, including Thurstone, might be suspect and not confirmable. For example, Chapter 5 in Cattell's (1971) treatise on cognitive abilities was an extensive attempt to discredit the Spearman ***g*** factor, chiefly because, in Cattell's opinion, it could not be supported within the theory of rotated factors. Cattell and Horn's rejection of a Spearman-type ***g*** factor has been carried forward to recent articles by Horn (1991, 1998; Horn & Noll, 1994, 1997), which summarize Horn's current views.

In the present chapter, then, we may consider three somewhat different views about the higher-order structure of cognitive abilities:

(1) The classic view of Spearman (1927, or particularly in a book published posthumously, Spearman & Wynn Jones, 1950), Thurstone and Thurstone (1941), Jensen (1998), and many others, that one can accept the existence of a general factor and of a series of nongeneral "broad" factors that together contribute variance to a wide variety of mental performances, but not necessarily to all of them. This is the view adopted by the present author (Carroll, 1993) in proposing that all human cognitive abilities can be classified as occupying one of three hierarchical strata, as mentioned previously. I call this the "standard multifactorial" view of cognitive abilities.

(2) The view of Gustafsson and others (Gustafsson, 1984, 1989, 200l; Gustafsson & Balke, 1993; Gustafsson & Undheim, 1996) is that a general factor exists but is essentially identical to, or highly correlated with, a second-order fluid intelligence factor ***Gf***, but is linearly independent of a second-order crystallized intelligence factor ***Gc*** and other possible second-order factors. For present purposes, I call this the "limited structural analysis view."

(3) The view of Horn and some others (Cattell, 1971, p. 87; Horn, 1998; Horn & Noll, 1994) that there is "no such thing" (as Horn likes to phrase it) as a general factor, but that non-zero intercorrelations among lower-stratum factors can be explained by accepting the existence of two or more second-stratum factors, mainly ***Gf*** and ***Gc***. Because this view denies the existence of a third stratum, it may be termed the second-stratum multiplicity view.

In this relatively brief chapter, I assemble and analyze sample data and arguments favoring or disfavoring each of these alternative views, eventually to permit drawing conclusions as to which of them is most nearly correct in terms of logical reasonableness and closeness of fit to a wide range of empirical data. Some of these data and arguments appeared in previous publications (Carroll, 1993, 1995, 1997a,b), which may be consulted for more detailed information. It is, however, my present belief that although currently available evidence tends to favor the standard multifactorial view, we still do not have enough objective evidence to permit making any final decision on which view merits acceptance over the others.

How decide among these views?

Each of the three views cited above can be expressed as a series of statements that could be put to the test by statistical analyses of appropriate datasets either newly designed for the purpose or drawn from the literature on human cognitive abilities--datasets containing measurements of a large number of people on a suitable variety of tests or other measurements of these abilities. The people for whom measurements are obtained should be a respectable sample of some defined population--preferably one that could be regarded as representative of an important segment of the general population of, say, English-speaking inhabitants of the United States of America or of some other country, within some defined age range. Furthermore, the tests should be reliable and valid measurements of the main types of known cognitive abilities at the first stratum--that is, tests designed to measure narrow abilities as opposed to broader or more general abilities, which show up only in correlations among different types of narrow abilities. Any dataset chosen for analysis should contain a table of the actual or estimated population intercorrelations of all the tests used in the dataset. It is desirable that there be at least three somewhat different tests of each narrow ability that the dataset was designed to measure, in order to insure that the ability is adequately defined for logical and statistical analysis.

A dataset that would be truly adequate for studying the higher-stratum structure of cognitive abilities would probably be too large, in terms of its number of test variables, to include in this necessarily brief chapter to illustrate a possible solution to the problem posed. There is a real question whether any such dataset yet exists in the literature. In its place, it may be sufficient, for drawing conclusions about the higher-stratum structure of cognitive abilities, to portray the application of factorial methods to two relatively small datasets developed and analyzed by McGrew, Werder, and Woodcock (1991). It is a dataset that was designed to test factorial structure only at a second or higher stratum, as suggested by Carroll (1993, p. 579), in that it has sufficient test variables to define several second-stratum factors, as well as the single third-stratum factor, but not necessarily any first-stratum factors. Table X-1 shows the intercorrelations among the 16 variables used in the first of these datasets, derived from data from the administration of the Woodcock-Johnson Psycho-Educational Battery--Revised (McGrew, Werder, & Woodcock, 1991, Appendix I, Table I-1) to 2261 persons of both sexes from kindergarten to adult educational levels. The developers of this battery had the specific objective of preparing a series of tests that would reliably and validly measure the more important general and special abilities that had been discovered in the past fifty or sixty years of research on cognitive abilities. I therefore consider the data shown in Table X-1 to be appropriate for analysis in order to draw at least tentative conclusions about the structure of higher-stratum cognitive abilities by testing the statistical hypotheses implied by the different views of that structure that I have been considering.

Table X-1

Pearsonian Intercorrelation Matrix, Combined Kindergarten to Adult Sample (decimals omitted). 16 Variables from the Woodcock-Johnson Psycho-Educational Battery--Revised, N = 2261, Correlations Corrected for Age

Variable:		1	2	3	4	5	6	7	8
Memory for Names	1	1000							
Memory for Sentences . .	2	310	1000						
Visual Matching	3	271	269	1000					
Incomplete Words	4	246	347	295	1000				
Visual Closure	5	239	160	310	291	1000			
Picture Vocabulary . . .	6	396	413	331	381	378	1000		
Analysis-Synthesis . . .	7	331	369	351	321	276	396	1000	
Visual-Auditory Learning	8	561	349	316	290	338	424	464	1000
Memory for Words	9	235	562	244	298	126	258	286	297
Cross Out	10	248	271	671	293	354	365	364	336
Sound Blending	11	283	372	326	486	268	433	356	410
Picture Recognition . .	12	334	239	297	248	338	320	327	377
Oral Vocabulary	13	400	523	406	410	350	706	502	479
Concept Formation . . .	14	367	417	354	324	289	404	541	445
Calculation	15	292	341	496	241	246	453	464	403
Applied Problems . . .	16	353	433	456	300	264	548	502	447

Variable:		9	10	11	12	13	14	15	16
Memory for Words	9	1000							
Cross Out	10	221	1000						
Sound Blending	11	358	355	1000					
Picture Recognition . .	12	205	321	269	1000				
Oral Vocabulary	13	387	407	492	347	1000			
Concept Formation . . .	14	283	370	356	330	516	1000		
Calculation	15	278	433	380	264	586	460	1000	
Applied Problems	16	342	424	432	304	672	537	702	1000

Note: Reprinted with permission, McGrew, K. S., Werder, J. K., & Woodcock, R. W., WJ-R Technical Manual, p. 345. (C) 1991, DLM, Allen, TX.

My intention is to perform what I consider to be appropriate exploratory and/or confirmatory analyses of the correlations shown in Table X-1 (and later, the correlations from a related dataset shown in Table X-4). From the results, it should be possible to reach the desired conclusions.

All analyses presented here assume that the factors obtained can be represented as completely orthogonal to each other; this is true not only for analyses made by the exploratory Schmid and Leiman (1957) technique described below but also for analyses made by the confirmatory LISREL 7 factor analysis program (Jöreskog & Sörbom, 1989), which can provide factor loadings on orthogonal factors by allowing the user to specify that the matrix of correlations among factors at the highest level of analysis is to be an identity matrix (a square matrix containing 1's in all diagonal entries, 0's elsewhere; or, in the language of the program, PH=ID). This method of representing factors is theoretically sound even though it is impossible to compute, from empirical data, error-free and uncorrelated estimates of factor scores on orthogonal factors. It has the advantage that conceptualizing results does not require consideration of possible correlations among factors at different levels of analysis. Also, it is mathematically equivalent to other procedures of analysis in that all these procedures seek to predict the matrix of empirical correlations found for a dataset. That matrix can be predicted from either orthogonal or oblique factor matrices, but if orthogonal matrices are used, it is not necessary to take account of correlations among the factors.

For exploratory factor analysis, Schmid and Leiman (1957) developed a procedure whereby data on different orders (or strata) of factors could be represented in a single matrix showing the predicted loadings of tests on orthogonal factors at different strata. Typically, just one factor would be found at the highest stratum, and such a factor might be considered to be a "general factor" when the tests or variables which had substantial loadings on it were sufficiently diverse, and preferably, identical or similar to tests or variables having loadings on general factors in other datasets. I used the exploratory factor analysis Schmid/Leiman procedure, incorporated in a factor analysis program I developed (Carroll, 1989), to produce Table X-2, which shows factor loadings of the 16 tests on a third-stratum g factor and eight second-stratum factors. All tests have substantial loadings on factor 1, which can be regarded as a general factor because the 16 tests that have substantial loadings on it are quite diverse in terms of test content and required mental operations. The eight second-stratum factors generally have substantial loadings on only two tests each, indicating that they cover restricted types of content and mental operation. Other loadings on these factors approach zero except for a few values that are still strikingly different from zero, though not as high as the two values that mainly define a given factor. From this, it is clear that the results support the classical or standard multifactorial view that postulates a general factor ***g*** at the third stratum and a series of broad abilities at the second stratum.

Table X-2

<u>Orthogonal Hierarchical Exploratory Factor Matrix for 16 variables and 9 factors:</u> Factor Loadings (decimals omitted)

Stratum :	3	2	2	2	2	2	2	2	2	
Factor:	g	Glr	Gsm	Gs	Ga	Gv	Gc	Gf	Gq	h^2
Factor No.:	1	2	3	4	5	6	7	8	9	
Var.										
1 MEMNAM	**511**	**585**	-012	-001	002	003	014	002	-014	604
2 MEMSEN	**544**	-014	**466**	027	-020	-174	**275**	**166**	016	649
3 VISMAT	**595**	016	**004**	**779**	-012	-000	002	001	-004	961
4 INCWDS	**481**	-061	109	087	**238**	022	193	132	-117	380
5 VISCLO	**434**	002	-015	-002	-013	**531**	020	001	-002	471
6 PICTVO	**597**	027	-001	003	007	011	**639**	001	**381**	912
7 ANLSYN	**632**	023	001	-016	016	058	-023	**310**	025	501
8 VISAUD	**625**	**434**	008	-092	076	193	-135	051	031	652
9 MEMWDS	**451**	016	**670**	-015	014	000	004	001	-010	653
10 CRSOUT	**564**	-025	-002	**410**	030	164	-001	055	019	518
11 SNDBND	**612**	004	001	003	**732**	-006	002	001	-003	910
12 PICREC	**460**	152	033	020	-005	**264**	-044	073	-049	316
13 ORALVO	**710**	-020	103	-013	008	-023	**490**	085	**407**	930
14 CNCPTF	**662**	-001	-012	003	-008	-016	025	**403**	-028	602
15 CALCUL	**626**	000	-019	029	-004	097	-002	003	**566**	723
16 APLPRB	**689**	-001	026	-016	-004	015	149	078	**524**	779
SMSQ	5394	561	692	794	600	460	806	328	926	10561
%CCV:	51.07	5.31	6.55	7.51	5.68	4.35	7.63	3.10	8.76	100.00

<u>Note</u>: Based on the correlation matrix of Table X-1, which see for full names of variables. Salient loadings of variables on common factors are shown in **bold**. Factor Names: g: General Intellectual Ability; Glr: Long-Term Retrieval; Gsm: Short-Term Memory; Gs: Processing Speed; Ga: Auditory Processing; Gv: Visual-Spatial Thinking; Gc: Comprehension-Knowledge; Gf: Fluid Reasoning; Gq: Mathematics. $\underline{h}^2$: Communality. SMSQ: Sums of Squares. %CCV: Percentages of Common Covariance.

It may help the reader unfamiliar with factor analysis to realize that "cross-multiplying" any two rows of the factor-loading table (ordinarily termed a "factor matrix") should yield a fairly good estimate of the correlation between the two variables involved. (Cross-multiplying means finding the sum, over the number of entries in each row, of the products of the two values in a given column. Cross-multiplying a row by itself yields the value of the "communality" (h^2) or amount of common factor variance associated with a given variable.) The last two rows of the table (labeled SMSQ and %CCV) provide information on the relative weight of the factors in determining the predicted correlations. The entries labeled SMSQ are the sums of squared values in a given column, indicating the amount of variance contributed by a given factor, and the entries labeled %CCV are percents of common factor variance for that factor. The SMSQ values are highest for the factor ***g*** and the percent of common covariance is 51.07 for that factor, in contrast to the relatively low values for each of the second-stratum factors. At the same time, sizable loadings of particular tests on second-stratum factors are seen. For example, note the loadings of the Analysis-Synthesis Test and the Concept Formation Test on the so-called ***Gf,*** Fluid Reasoning factor, .310 and .403, respectively. In factor analysis, it is often shown that tests have loadings on more than one factor, indicating that their scores are to be regarded as functions of two or more latent abilities. Thus, it is not at all surprising to find, in Table X-2, tests with substantial loadings on two or more factors. The "substantial" loadings are printed in **bold**. In exploratory factor analysis, loadings are customarily computed for all entries in matrices of results; many of these, of course, are very close to zero, confirming the "simple structure" principle that is one of the bases of exploratory factor analysis. It is common to find that values close to zero are simply not shown in tables of factor-analytic results in the literature, but they are, of course, shown in Table X-2.

Literally hundreds of tables of exploratory factor-analytic results based on the classical view of factor structure were published in the 20th century from about 1940 to about 1990. The present author (Carroll, 1993) reported re-analyses of about 450 of the datasets that generated these tables. The main advantage of the author's re-analyses was that they were based on a largely uniform exploratory methodology that was believed to be the best available at the time these analyses were conducted (Carroll, 1995). The chief disadvantage of this methodology was that it suffered from a lack of adequate procedures for establishing the statistical significance of findings. Nevertheless, it is probable that most of the many ***g*** factors found in Carroll's analyses would be found statistically significant if the corresponding data could be submitted to appropriate analyses.

When the correlation matrix shown in Table X-1 was submitted to confirmatory analysis with the Jöreskog and Sörbom (1989) LISREL 7 program, the resulting maximum likelihood estimates of the factor loadings are shown in Table X-3, the general pattern of which is comparable in many ways to that of Table X-2. The confirmatory analysis procedure requires the researcher to specify the general form of the model to be tested (for example, the number of factors to be assumed and whether the factors are to be uncorrelated or correlated), and exactly which variables are hypothesized to measure each factor (either positively or negatively); the researcher is not required to specify a value or even a range of possible values for factor loadings. The actual determination of each specified value is accomplished by iterative computational procedures leading to an approximation of the empirically observed correlation matrix as predicted from the LISREL estimates. Occasionally the iterations cannot be completed after a programmed number of them have taken place, because of improperly selected hypotheses about the structure. For further understanding of the procedure used here, see useful articles by Keith (1997) and Gustafsson (2001).

In the present case, the model that was tested assumed (or hypothesized) that the correlations could be explained by loadings (weights) on a single third-stratum general factor ***g*** plus a series of eight second-stratum factors, each of which was to be defined by 2 or 3 tests that previous research with these tests suggested would load on a given factor. It was also specified that the factors were to be orthogonal to each other. In the hope of attaining a complete solution for all factors, an attempt was made to find values of loadings on at least 3 variables for each of the second-stratum factors, but it was found not easily possible to do this for all of these factors, most likely because the battery was in fact not designed to include tests such that there would be more than two significant loadings on each second-order factors. For each factor found not easily possible to define in this way, the loadings for each of the two variables known (from previous research) to define the factor were set equal to each other (as indicated by asterisks placed next to certain values in the table). The principal difference from Table X-2 was that information on the statistical significance of the results was provided. All non-zero loadings presented in Table X-3 (i.e., those not shown as "---") were statistically significant at p <= .001, and the goodness of fit indices (noted at the end of Table X-3) for the whole model were at levels deemed to indicate satisfactory fit.

Table X-3

LISREL Estimates of Orthogonal Factor Loadings for 16 Variables on 9 Factors, Decimals Omitted.

Stratum :	3	2	2	2	2	2	2	2	2	
Factor:	g	Glr	Gsm	Gs	Ga	Gv	Gc	Gf	Gq	h^2
Factor No.:	1	2	3	4	5	6	7	8	9	
1 Memnam	**504**	**488***	---	---	---	---	---	---	---	492
2 Memsen	**566**	---	**551***	---	---	---	**136**	---	---	642
3 Vismat	**551**	---	---	**496**	---	---	---	---	---	550
4 Incwds	**485**	---	---	---	**445***	---	---	---	---	433
5 Visclo	**418**	---	---	**152**	---	**402**	---	---	---	359
6 Pictvo	**676**	---	---	---	---	---	**371**	---	---	595
7 Anlsyn	**646**	---	---	---	---	---	---	**336***	---	530
8 Visaud	**623**	**488***	---	---	---	172	---	---	---	656
9 Memwds	**457**	---	**551***	---	---	---	---	---	---	512
10 Crsout	**549**	---	---	**743**	---	---	---	---	---	853
11 Sndbnd	**594**	---	---	---	**445***	---	---	---	---	551
12 Picrec	**461**	---	---	---	---	**334**	---	---	---	316
13 Oralvo	**812**	---	---	---	---	---	**425**	---	---	840
14 Cncptf	**663**	---	---	---	---	---	---	**336***	---	602
15 Calcul	**684**	---	---	---	---	---	---	---	**415***	640
16 Aplprb	**774**	---	---	---	---	---	---	---	**415***	771
SMSQ	5762	476	607	821	396	303	337	226	344	9302
%ccv:	61.94	5.11	6.52	8.82	4.25	3.25	3.62	2.42	3.69	100.00

MEASURES Of GOODNESS OF FIT FOR THE WHOLE MODEL :
CHI-SQUARE WITH 90 DEGREES Of FREEDOM = 798.90 (P = .000)
GOODNESS Of FIT iNDEX = .959
ADJUSTED GOODNESS OF FiT INDEX = .938
ROOT MEAN SQUARE RESIDUAL = .035

Note: Based on the correlation matrix of Table X-1, which see for full names of variables, and Table X-2 for full names of factors. h^2: Communality or Squared Multiple Correlation. SMSQ: Sums of Squares. %CCV: Percents of Common Factor Covariance. "*": Loadings equated within factor.

As found with Table X-2, all variables (tests) had very substantial loadings on the third-order ***g*** factor, leaving only so much room, as it were, for high loadings on second-order factors (because the sum of squares of the 2 loadings for a given variable could not exceed 1.000). Overall, however, the second-order loadings conformed to expectations, and the results suggested that the hypothesized model for the dataset was valid, confirming the classical view of higher-stratum factorial structure.

The results show that there is indeed a factor ***Gf*** (Fluid Reasoning) that is significantly separate and different from factor ***g***, tending to disconfirm any view that ***Gf*** is identical to ***g***. If provisions for factor loadings of ***g*** are removed from the hypothesized pattern of factor loadings (leaving only 8 orthogonal factors), the goodness of fit of the model deteriorates significantly. Specifically, in this case the value of chi-square increases to 7108.02 with 106 degrees of freedom, and the Goodness of Fit Index decreases to .601, casting doubt on the validity of any view that denies the existence of a factor ***g***. Because the model without ***g*** is nested within the model that includes ***g***, the statistical significance of this finding is captured in the change of 6309.12 in the chi-square value, associated with a change of 16 in the number of degrees of freedom, making it clearly significant at $p < .0005$. Alternatively, the model tested could be specified as PH=ST, meaning that the second-order factors could be estimated as being correlated. In this case, chi-square with 78 degrees of freedom becomes 479.02, with a Goodness of Fit Index of .974. The change in the chi-square value is 319.88 with 12 degrees of freedom, clearly significant with a virtually zero probability. With correlated factors, it would be possible to assume at least one third-order factor. But it has already been demonstrated, above, that the existence of a third-order general factor in the first-order correlations can be accepted; it is unnecessary to perform further calculations unless one is interested in further third-order factors.

There is still a problem with ***Gf***, namely, that it appears to be a rather weak, poorly defined factor, at least in the dataset examined here. Note the relatively small factor loadings for the two tests indicated as measuring ***Gf*** in both Table X-2 and Table X-3, also the relatively low values of %CCV for ***Gf*** in these tables (3.1% and 2.42%, respectively). In view of the undoubtedly careful and persistent efforts that were made in constructing these tests at the time the battery was being developed, the low ***Gf*** factor loadings most likely indicate that factor ***Gf*** is inherently difficult to measure reliably independently of its dependence on ***g*** (as indicated by the high ***g*** loadings for these tests). This may account for the finding by Gustafsson (1989, 2001; see also Gustafsson & Balke, 1993, and Gustafsson & Undheim, 1996) that it is often difficult to distinguish ***Gf*** from ***g***. Thus, Gustafsson's views on factor structure may be affected more by characteristics of tests than by factor structure as such. I have been tempted to suggest, on the basis of these and similar findings in other studies, that the reality of a Fluid Reasoning factor independent of ***g*** is at least questionable, and that Horn's (1998) support for a ***Gf*** factor can possibly be conceived of as support for a ***g*** factor (when no other factor interpretable as ***g*** is present in a given dataset).

Another Dataset

To provide information illustrating the generality of the higher-stratum structure found with the analysis of the correlation matrix shown in Table X-1, a correlation matrix from a related dataset is presented in Table X-4 and its confirmatory factor analysis is shown in Table X-5. The 29-variable correlation matrix and a confirmatory factor analysis of it were published by McGrew, Werder, and Woodcock (1991) in a Technical Manual pertaining to the 1989 version of the Woodcock-Johnson Psycho-Educational Battery--Revised. The confirmatory factor analysis presented here was run by the present author from the original correlations (Table X-4) with a slightly different model from that employed by McGrew et al., with computations to parallel the analysis presented in Table X-2; specifically, there was provision for determining the factor loadings of all 29 variables on a general factor ***g*** and for computing data on the relative importance of the factors.

Table X-4

Pearsonian Intercorrelation Matrix, Combined Kindergarten to Adult Sample (decimals omitted). 29 Variables from the Woodcock-Johnson Psycho-Educational Battery--Revised, *N*=1425, Correlations Corrected for Age

Variable:		1	2	3	4	5	6	7	8
Memory for Names	1	1000							
Memory for Sentences . .	2	279	1000						
Visual Matching	3	213	254	1000					
Incomplete Words	4	167	255	191	1000				
Visual Closure	5	148	103	178	176	1000			
Picture Vocabulary . . .	6	404	403	202	267	229	1000		
Analysis-Synthesis . . .	7	275	324	280	205	161	323	1000	
Visual-Auditory Learning	8	542	343	267	192	205	382	376	1000
Memory for Words	9	208	559	221	245	046	225	215	246
Cross Out	10	170	241	621	168	241	242	291	265
Sound Blending	11	245	323	245	367	133	323	265	332
Picture Recognition . .	12	293	216	212	123	234	256	233	299
Oral Vocabulary	13	388	534	310	319	234	632	419	405
Concept Formation . . .	14	306	382	306	236	206	325	484	376
Memory for Names-- (Delayed Recall). . .	15	721	236	155	168	129	383	269	460
Visual-Auditory Learning-- (Delayed Recall) . . .	16	345	164	162	120	192	255	269	460
Numbers Reversed . . .	17	259	416	384	227	129	255	368	321
Sound Patterns	18	233	257	204	221	109	269	271	259
Spatial Relations . . .	19	280	266	278	158	265	317	389	369
Listening Comprehension	20	331	469	266	334	204	576	349	344
Verbal Analogies	21	379	454	334	228	242	522	455	445
Calculation	22	256	331	435	142	132	299	423	347
Applied Problems	23	337	416	419	206	175	439	470	388
Science	24	380	437	260	285	233	633	368	364
Social Studies	25	371	477	298	262	200	626	386	374
Humanities	26	390	447	308	281	252	622	343	414
Word Attack	27	281	370	356	263	119	316	303	366
Quantitative Concepts .	28	342	427	408	205	162	497	437	416
Writing Fluency	29	225	350	494	193	123	260	309	347

Note: Reprinted with permission, McGrew, K. S., Werder, J. K., & Woodcock, R. W., WJ-R Technical Manual, p. 345. (C) 1991, DLM, Allen, TX.

Table X-4 (continued)

	9	10	11	12	13	14	15	16	17	18	19	20
9	1000											
i0	203	1000										
11	335	246	1000									
12	155	257	212	1000								
13	364	315	389	304	1000							
14	227	305	275	269	458	1000						
15	173	123	242	236	359	284	1000					
16	110	168	192	275	271	323	446	1000				
17	401	309	316	206	396	354	225	182	1000			
18	243	229	294	168	331	299	222	214	282	1000		
19	189	343	225	288	388	404	240	289	311	294	1000	
20	279	263	351	256	642	375	294	221	308	274	320	1000
21	310	344	355	322	639	496	377	330	403	304	465	526
22	252	358	293	208	471	401	249	242	413	257	376	374
23	312	388	360	273	603	489	313	268	438	315	486	524
24	246	280	323	246	658	389	362	270	336	260	385	619
25	270	278	323	255	693	411	348	245	332	256	344	638
26	297	284	355	283	665	359	368	283	326	282	340	572
27	322	255	484	202	468	329	269	228	398	316	312	326
28	309	361	320	244	615	413	337	280	433	299	440	513
29	266	410	358	196	398	335	197	194	365	229	276	285

	21	22	23	24	25	26	27	28	29
21	1000								
22	483	1000							
23	631	655	1000						
24	544	440	570	1000					
25	595	508	617	702	1000				
26	598	427	536	633	672	1000			
27	415	422	450	346	354	398	1000		
28	624	656	728	602	637	576	471	1000	
29	394	420	426	293	336	409	488	434	1000

Table X-5

<u>LISREL Estimates of Orthogonal Factor Loadings for 29 Variables on 10 Factors</u>, Decimals omitted.

Stratum :	3	2	2	2	2	2	2	2	2	2	
Factor:	g	Glr	Gsm	Gs	Ga	Gv	Gc	Gf	Gq	Lang	h^2
Factor No.:	1	2	3	4	5	6	7	8	9	10	
01MEMNAM	**478**	**695**	---	---	---	---	---	---	---	---	712
02MEMSEN	**587**	---	**396**	---	---	---	---	---	---	---	501
03VISMAT	**499**	---	---	**709**	---	---	---	---	---	---	752
04INCWDS	**340**	---	---	---	**308**	---	---	---	---	---	210
05VISCLO	**279**	---	---	---	---	**472**	---	---	---	---	301
06PICVOC	**566**	---	---	---	---	---	**531**	---	---	---	602
07ANLSYN	**591**	---	---	---	---	---	---	**213**	---	---	395
08VISAUD	**579**	**343**	---	---	---	---	---	---	---	---	453
09MEMWDS	**424**	---	**782**	---	---	---	---	---	---	---	791
10CRSOUT	**478**	---	---	**539**	---	---	---	---	---	---	519
11SNDBND	**490**	---	---	---	**642**	---	---	---	---	---	652
12PICREC	**398**	---	---	---	---	**260**	---	---	---	---	226
13ORALVO	**749**	---	---	---	---	---	**377**	---	---	---	703
14CNCPTF	**623**	---	---	---	---	---	---	**543**	---	---	683
15MMNADR	**439**	**729**	---	---	---	---	---	---	---	---	724
16VSAUDR	**404**	**320**	---	---	---	---	---	---	---	---	266
17NMRVRS	**571**	---	**203**	---	---	---	---	---	---	---	367
18SNDPAT	**436**	---	---	---	**144**	---	---	---	---	---	211
19SPAREL	**580**	---	---	---	---	**219**	---	---	---	---	384
20LISCMP	**619**	---	---	---	---	---	**424**	---	---	---	563
21VBLANL	**761**	---	---	---	---	---	**162**	**052**	---	---	608
22CALCUL	**652**	---	---	---	---	---	---	---	**432**	---	612
23APLPRB	**783**	---	---	---	---	---	---	---	**335**	---	725
24SCIENC	**651**	---	---	---	---	---	**491**	---	---	---	665
25SOCSTU	**686**	---	---	---	---	---	**488**	---	---	---	709
26HUMANI	**661**	---	---	---	---	---	**448**	---	---	**107**	649
27WDATCK	**587**	---	---	---	**273**	---	---	---	---	**197**	458
28QUANCN	**743**	---	---	---	---	---	**177**	---	**400**	---	743
29WRIFLU	**549**	---	---	**286**	---	---	---	---	---	**685**	852
SMSQ:	9515	1235	810	875	602	338	1341	343	459	519	16037
%CCV:	59.33	7.70	5.05	5.45	3.75	2.10	8.36	2.13	2.23	3.23	100.00

MEASURES OF GOODNESS OF FIT FOR THE WHOLE MODEL :
CHI-SQUARE WITH 343 DEGREES OF FREEDOM = 1488.60 (P = .000)
GOODNESS OF FIT INDEX = .931; ADJUSTED GOODNESS OF FIT INDEX = .912
ROOT MEAN SQUARE RESIDUAL = .039

Note: Analysis of the correlation matrix of Table X-4, which see for full names of variables. Factor Names (as given by McGrew, Werder, & Woodcock, 1991): g: General Intellectual Ability; Glr: Long-Term Retrieval; Gsm: Short-Term Memory; Gs: Processing Speed; Ga: Auditory Processing; Gv: Visual-Spatial Thinking; Gc: Comprehension-Knowledge; Gf: Fluid Reasoning: Gq: Mathematics; Lang: Language. h^2: Communality or Squared Multiple Correlation; SMSQ: Sums of Squares; %CCV: Percents of Common Factor Covariance

The results in Table X-5 confirm the classical, standard multifactorial model of the higher-stratum structure even more clearly than the results in Table X-2. That is, the presence and generality of a ***g*** factor for all 29 tests is supported, and the existence, separate from ***g***, of nine second-stratum factors (including ***Gf*** and ***Gc***) is shown, each with from 3 to 8 significant loadings on a specific group of tests, which help to define the nature of the factor. One of these factors, labeled "Lang" (Language), was not present in the previous dataset; its presence here invites further research on the nature of this factor.

At the same time, the results in Table X-5 tend to disconfirm other views on the higher-stratum structure of cognitive abilities. They deny the view that a factor ***g*** does not exist, and some doubt is cast on the view that emphasizes the importance of a ***Gf*** factor, in view of the relatively low factor loadings of some tests (numbered 07 and 21) on this factor. Thus, these data tend to discredit the limited structural analysis view and the second-stratum multiplicity view.

Discussion

There is need for consideration of certain important features of recent publications--tests, test manuals, and writings concerned with the higher-stratum structure of cognitive abilities.

In some of the literature that has been cited, one finds a consistent bias against full acknowledgment of the existence of a general factor and its role in commonly used tests of cognitive ability. Consider, for example, the technical manual of the 1989 version of the Woodcock-Johnson cognitive test battery--the WJ-R (McGrew, Werder, and Woodcock, 1991), from which the correlational data of Tables X-1 and X-4 were taken and further analyzed in the present chapter. Over many of its pages, this manual reveals a studious neglect of the role of any kind of general factor in the WJ-R. Confirmatory factor analyses of correlational data either are based on orthogonal structures (e.g., Table 6-27, p. 166) or assume oblique structure, with correlations between factors presented (e.g., Table 6-28, p. 167), but there is no mention of a possible general factor, despite the fact that appropriate analysis would have revealed an important role of what I would call a third-order general factor (as shown in Table X-3 of the present chapter). On page 170 is found Figure 6-5, a path diagram, which on page 169 is described as providing "a highly restricted and parsimonious representation of the factor structure of the WJ-R", but Figure 6-5 contains no representation of a general factor. Only on page 171 could one find a statement about the possibility that a hierarchical ***g*** factor might influence what were (erroneously) called first-order factors, but with a comparison of fit statistics said to be obtained with a hierarchical ***g*** model versus a "***Gf-Gc*** model" (that did not include ***g***) the reader would be left with the impression that the ***Gf-Gc*** model was in every way superior to a "hierarchical ***g*** model." This impression would have been reinforced by Horn's (1991) essay, constituting Chapter 7 of the manual, that reviewed theory on the measurement of intellectual capabilities and emphasized the presumed superiority of the ***Gf-Gc*** theory. Thus for some ten years before a further revision of the WJ-R became available, users of the WJ-R were left largely uninformed of the fact that scores on the subtests in the battery were likely to be heavily influenced by a general factor of ability.

Fortunately, this situation changed in 2001 when a new version of the test, its scoring, and its technical manual became available (McGrew & Woodcock, 2001), introducing a so-called CHC

(Cattell-Horn-Carroll) theory of cognitive abilities that supplemented Horn's ***Gf-Gc*** theory with essentially a three-stratum theory similar to that proposed by the present writer (Carroll, 1993).

Even though I was to some extent involved in this change (as an occasional consultant to the authors and publisher), I am still not quite sure what caused or motivated it. For a number of years, Horn had espoused the so-called ***Gf-Gc*** theory and had written a number of papers that included criticisms of a hierarchical ***g*** theory (Horn & Noll, 1994, 1997). For example, as he reported:

> Carroll (1993) identified a general factor at the third stratum in 33 separate analyses in his reanalysis of the data of 461 studies. These factors are general in the sense that they are defined at the highest order in higher-order analyses, and each is defined by non-chance correlations with many different cognitive tests. The problem for theory of general intelligences is that the factors are not the same from one study to another. For example, in one case (an analysis labeled ARNO0l) the factor is defined by lexical knowledge, spatial relations, memory span, general interest, and an unidentified first-order factor, whereas in another case (analysis DENT0l) the factor is defined by reasoning, number, word fluency, short-term memory, and perceptual speed. The different general factors do not meet the requirements for the weakest form of invariance (Horn & McArdle, 1992) or satisfy the conditions of the Spearman model. The general factors represent different mixture measures, not one general intelligence (Horn & Noll, 1997, p. 68).

In response, I will not speak to technical problems relating to invariance or the conditions of the Spearman model. I merely point out that it is not the case that the ***g*** factor in the ARNO0l analysis was "defined by lexical knowledge, spatial relations.. ," ... as Horn claimed; it was defined by whatever was common to a series of tests or factors at a lower stratum. Nor was the ***g*** factor in the DENT0l study "defined by reasoning, number, ..."; it was defined by whatever was common to a series of factors (or tests) at a lower stratum, namely, loadings on a ***g*** factor. All the ***g*** factors that I studied and characterized as general factors can be considered to be the same if they indeed measure a single factor, which they do, according to the Schmid and Leiman (1957) procedure by which, generally, they were computed. Perhaps this conclusion can be better understood if one looks at either Table X-2 or Table X-3 in the present chapter. In either case, one notes that all the tests are shown as measuring a single factor ***g*** along with a variety of second-order factors. We can doubtless agree that this factor ***g*** is the same for all 16 tests. Now, if we conduct two new analyses (either exploratory or confirmatory) of these data, one of them using tests 1-4 and 8-11 (i.e., using tests with high loadings on factors ***Glr, Gsm, Gs,*** and ***Ga***) and the other using tests 5-7 and 12-16 (i.e., tests with high loadings on factors ***Gv, Gc, Gf***, and ***Gq***), both analyses would yield a factor ***g***--the "same" ***g*** in each case. It would be difficult to argue that the ***g*** factors yielded by the two analyses are different, even though they involve different second-order factors. Horn's comment suggests that he conveniently forgets a fundamental principle on which factor analysis is based (a principle of which he is undoubtedly aware)--that the nature of a single factor discovered to account for a table of intercorrelations does not necessarily relate to special characteristics of the variables involved in the correlation matrix; it relates only to characteristics or underlying measurements (latent variables) that are common to those variables. I cannot regard Horn's comment as a sound basis for denying the existence of a factor ***g***, yet he succeeded in persuading himself and many others to do exactly this for an extended period of years.

I believe I have covered the more important theoretical problems in describing the higher-stratum structure of cognitive abilities. Researchers who are concerned with this structure in one way or another, like Burns and Nettelbeck (in press), Case, Demetriou, Platsidou, and Kazi (2001), Deary (2000), Garlick (in press), Jensen (1998), and Plomin (1999) can be assured that a general factor ***g*** exists, along with a series of second-order factors that measure broad special abilities. Special facts that should be considered, however, are that more and better tests of factor ***Gf*** are needed to establish this factor as linearly independent of factor ***g*** if indeed this is possible, and that factor ***Gc***, as a factor in certain kinds of tests of general knowledge, is also a factor that can strongly influence certain tests intended to measure factor ***g***; it is suggested that this influence needs to be statistically controlled. Indeed, it may be recommended that estimates of scores on factor ***g*** should be based on multiple regression formulas for determining factor scores (Gorsuch, 1983) rather than simple weighted sums of scores like those recommended in the WJ-III Technical Manual (McGrew & Woodcock, 2001). Further research is needed on the best tests and procedures to use in estimating scores on all higher-stratum factors of cognitive ability, and continued psychological and even philosophical examination of the nature of factor ***g*** is a must.

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