Multivariate Behavioral Research, *39* (4), 595-624

Estimating Cognitive Profiles Using Profile Analysis via Multidimensional Scaling (PAMS)

Se-Kang Kim
Harcourt Assessment, Inc., San Antonio, TX

Craig L. Frisby
University of Missouri-Columbia

Mark L. Davison
University of Minnesota-Twin Cities

Two of the most popular methods of profile analysis, cluster analysis and modal profile analysis, have limitations. First, neither technique is adequate when the sample size is large. Second, neither method will necessarily provide profile information in terms of both level *and* pattern. A new method of profile analysis, called Profile Analysis via Multidimensional Scaling (PAMS; Davison, 1996), is introduced to meet the challenge. PAMS extends the use of simple multidimensional scaling methods to identify latent profiles in a multi-test battery. Application of PAMS to profile analysis is described. The PAMS model is then used to identify latent profiles from a subgroup (N = 357) within the sample of the *Woodcock-Johnson Psychoeducational Battery—Revised* (WJ-R; McGrew, Werder, & Woodcock, 1991; Woodcock & Johnson, 1989), followed by a discussion of procedures for interpreting participants' observed score profiles from the latent PAMS profiles. Finally, advantages and limitations of the PAMS technique are discussed.

Introduction and Background

In psychology and education, the most frequently used commercial standardized test batteries typically provide users with a variety of subtest scores in addition to global index scores. "Profile analysis" is a generic term used to describe the practice of distinguishing between groups of test-takers based on their unique configuration, or pattern, of subtest scores (e.g., Stanton & Reynolds, 2000). In the area of cognitive testing, there is a rich history of debate over the clinical utility of results from profile analysis; in the

---

Correspondence concerning the manuscript should be addressed to Se-Kang Kim, Harcourt Assessment, Inc., 19500 Bulverde Road, San Antonio, TX 78259. Electronic mail may be sent via Internet to se-kang_kim@harcourt.com. We sincerely thank Roger Millsap and two anonymous reviewers for their valuable comments to improve the current study and also Kevin McGrew and Richard Woodcock for providing the data analyzed in this study.

ability to make differential diagnoses and to design appropriate interventions based on the individual's profile. It also has been argued that profile analysis is only useful when recommendations are made under the assumption that probabilities of producing outcomes are known (Carroll, 2000). According to these criteria, however, the previous literature does not contain impressive support for the utility of profile analysis for individual test-takers (Watkins, 2000). In the cognitive testing domain, research may not have yielded fruitful results about differential diagnoses for clinical utility because the substantial *g* factor (or general ability) is confounded with group factors. At this juncture, a test's predictive validity for differential diagnoses is directly related to the test's *g* factor loadings (see Jensen, 1998), but the *g* factor cannot function adequately in making differential diagnoses for clinical utility because it is an overall measure of ability, not a specific one.

Ipsatizing scores (subtracting the average scaled-score from each subtest score) has been used as one approach to profile analysis. The individual's ipsatized score yields a profile of the person's strengths and weaknesses in each subtest. However, McDermott and his colleagues have shown that ipsatized scores on cognitive ability tests are degraded psychometrically (i.e., cannot be analyzed using parametric statistical procedures) and are largely ineffective both in discriminating between clinical groups and in predicting academic success (McDermott, Fantuzzo, & Glutting, 1990; McDermott, Fantuzzo, Glutting, Watkins, & Baggaley, 1992; McDermott & Glutting, 1997).

In addition to these problems, an individual's unique profile does not provide information about how similar/dissimilar the individual's profile is relative to the profile of a larger, more representative group. For this reason, researchers have turned their attention to the identification of "core" profiles, which represent a limited number of normative profiles reflective of the prominent profile patterns in a data set (McDermott et al., 1990).

Different methods can be used to identify and classify meaningful core (or latent) subtest profiles obtained from the administration of a multi-test battery to a sample of test-takers. At the simplest level, test developers will identify discrepancies of a given size between pairs of subtests or factor scales, and will calculate their frequency of occurrence in the standardization group (Kaufman, 1994). When researchers seek to identify meaningful latent profiles based on more than two subtests or factor scales, however, more complex decision-making is required.

Empirical methods for the identification of latent profiles begin with the selection of a measure that reflects the degree of similarity/dissimilarity between individuals' obtained profiles. In selecting similarity measures, profile analysis researchers must determine the extent to which a given measure is or is not sensitive to differences in a person's profile shape (i.e., the pattern of peaks and valleys across subtest or factor scores), scatter (i.e., the degree of dispersion of scores around their average), and/or elevation (i.e., the average value for all subtest or factor scores for an individual). Some similarity measures (such as the Pearson correlation coefficient) are sensitive only to differences in profile shape, while other measures (i.e., distance measures; see Aldenderfer & Blashfield, 1984) are more sensitive to elevation and scatter differences.

Among the most popular methods for profile analysis are cluster analysis and Modal Profile Analysis (MPA). Cluster analysis provides an alternative for identifying latent profiles in a data set (Aldenderfer & Blashfield, 1984). The essence of a cluster analysis is to classify objects into meaningful clusters or groups, where the objects within each cluster are more similar to each other than to the objects in other clusters and each cluster is relatively independent. The mean of each subtest score for all people within the cluster is used to describe the profile characteristics of the cluster. MPA is a mixture of cluster analysis and Q-factor analysis (Cattell, 1967). MPA yields clusters that vary in terms of profile shape. MPA identifies the most frequently occurring profile patterns in a dataset, which are then compiled to create the "modal" profiles (Pritchard, Livingston, Reynolds, & Moses, 2000; Skinner & Lei, 1980).

Limitations of MPA and Cluster Analysis for Profile Analysis

According to Davison and Kuang (2000), the Q-factor approach on which MPA is based relies on ipsatized scores that have been standardized to have variance 1.00. This procedure permits an individual's profile to reflect the shape of the original scores but not the level (average of all subtest scores) of the profile. Note that the profile level represents general or overall ability in the cognitive testing area. For researchers who value information provided by a profile's level parameter, the MPA profile analysis method may not be the wisest choice. On the other hand, although the clustering method used by Konold, Glutting, McDermott, Kush, and Watkins, (1999) forms groups based on both level and pattern information, the resulting profile groups from general cognitive ability test data tend to differ primarily in profile level. Thus, the clusters have largely described individual differences in overall profile level or general intelligence, rather than individual differences in profile pattern. Researchers who are primarily interested in profile pattern may not wish to employ the cluster methods. Given this situation, it would be more desirable to identify methods that put more emphasis on profile pattern (provided that differences in profile pattern are associated with external variables which are included to estimate differential validity).

In addition, Davison and Kuang (2000) note that both MPA and cluster analysis can be difficult to apply when sample sizes are large because the matrix to be factored (or clustered) is a persons × persons matrix. If the sample size is large, that matrix may be too complex for analysis. For example, in the proposed study, the sample size is 357. As a result, the matrix would include 357 rows and 357 columns for analysis (by either MPA or cluster analysis). In past research, the sample size was often so large that the sample was divided into sub-sets. Then each group was analyzed separately and the "group" results were combined to form generalizations for the sample as a whole (Moses & Pritchard, 1995).

Given the limitations of MPA and Cluster approaches, it would be more desirable to find a method that can include both profile level (which indexes general ability) and pattern information (which is useful for differential diagnosis or validity in relation to external variables, such as achievement scores) and that can easily handle large samples. Davison and his colleagues developed a profile analysis called Profile Analysis via Multidimensional Scaling, which includes both profile level and pattern information and efficiently analyzes samples of any size (Davison, 1996; Davison, Kuang, & Kim, 1999).

Purpose of Study

As discussed in the previous subsection, two of the most popular profile analysis methods, cluster analysis and MPA, have limitations when either the sample size is large or when examination of both level and pattern of the profiles is desired. An alternative method of profile analysis, Profile Analysis via Multidimensional Scaling (hereafter referred to as PAMS), is introduced to address these limitations. The purpose of this study is to illustrate how PAMS can be used for profile analysis, especially for large sample sizes and utilizing simultaneous level and pattern analysis. First, PAMS is used to identify latent profile patterns. Second, each individual's score profile, or array of observed scores, is interpreted in relation to the PAMS latent profiles.

PAMS is based on a generalization of the Q-factor model and uses Multidimensional Scaling (MDS) techniques to identify latent profiles in a dataset. Just as the analysis of variance model separates main effects and interactions by representing them with distinct parameters, the MDS approach separates individual differences in profile level from individual differences in profile pattern (shape combined with scatter). One parameter set corresponds to MDS dimension scale-values, and represents individual differences in profile pattern. The other parameter set indexes individual differences in profile level. Since an MDS analysis uses a tests × tests proximity matrix (rather than a persons × persons matrix), the matrix to be analyzed does not grow as sample size increases, and the analysis can be performed on samples of any size. Just as each factor in Q-factor analysis defines two profile shapes that are mirror images of each other, each dimension in a MDS analysis defines two mirror-image profile shapes.

We will first explain how PAMS is applied to profile analysis. Next, we describe Davison's (1996) PAMS model and examine the nature of profile patterns in a subgroup within the standardization sample of the *Woodcock-Johnson Psychoeducational Battery—Revised* (WJ-R; Mcgrew, Werder, & Woodcock, 1991; Woodcock & Johnson, 1989). We then demonstrate a procedure for interpreting participants' actual score profiles based upon the latent profiles identified from a PAMS approach.

Application of The PAMS Model to Profile Analysis

Whereas the factor analytic model assumes that the observed variables (subtest scores) can be accounted for by a linear combination of a smaller number of latent variables (factors), a simple MDS analysis does not allow for such modeling of test scores. For this reason, Davison (1996) developed PAMS[1] as a technique to apply MDS methodology to the modeling of person profiles.

The PAMS procedure begins with a data matrix of rows representing persons and columns representing their scores on each variable (i.e., a persons × items/subtests data matrix). While most factor analyses of multi-test batteries typically identify latent factors among the column variables (items or subtests), a PAMS analysis identifies latent profiles among the row variables (profiles of individuals). In solutions from a simple MDS (as illustrated in the Kline, Guilmette, Snyder, & Mastellanos, 1992, example), a dimension (axes on a visual map) represents a continuous bipolar characteristic of individual subtests. In solutions from a PAMS analysis, a dimension represents a continuous bi-directional *latent profile*. The relationship between a persons × subtests data matrix, latent factors, and latent profiles is graphically shown in Figure 1.

Figure 1 shows a persons × subtests data matrix, in which each column represents an observed variable (i.e., subtest) and each row represents a person. Each data point (m_{pt}, see Equation 1, to follow) is an element of the data matrix. The basic assumption of the factor model is that a small set of latent variables (i.e., latent factors) can be posited, represented by columns

[1] SPSS (SPSS, Inc., 1999) and SAS (SAS Institute, Inc., 1990) modules to implement PAMS are available from Mark L. Davison. Electronic mail may be sent via Internet to mld@umn.edu.

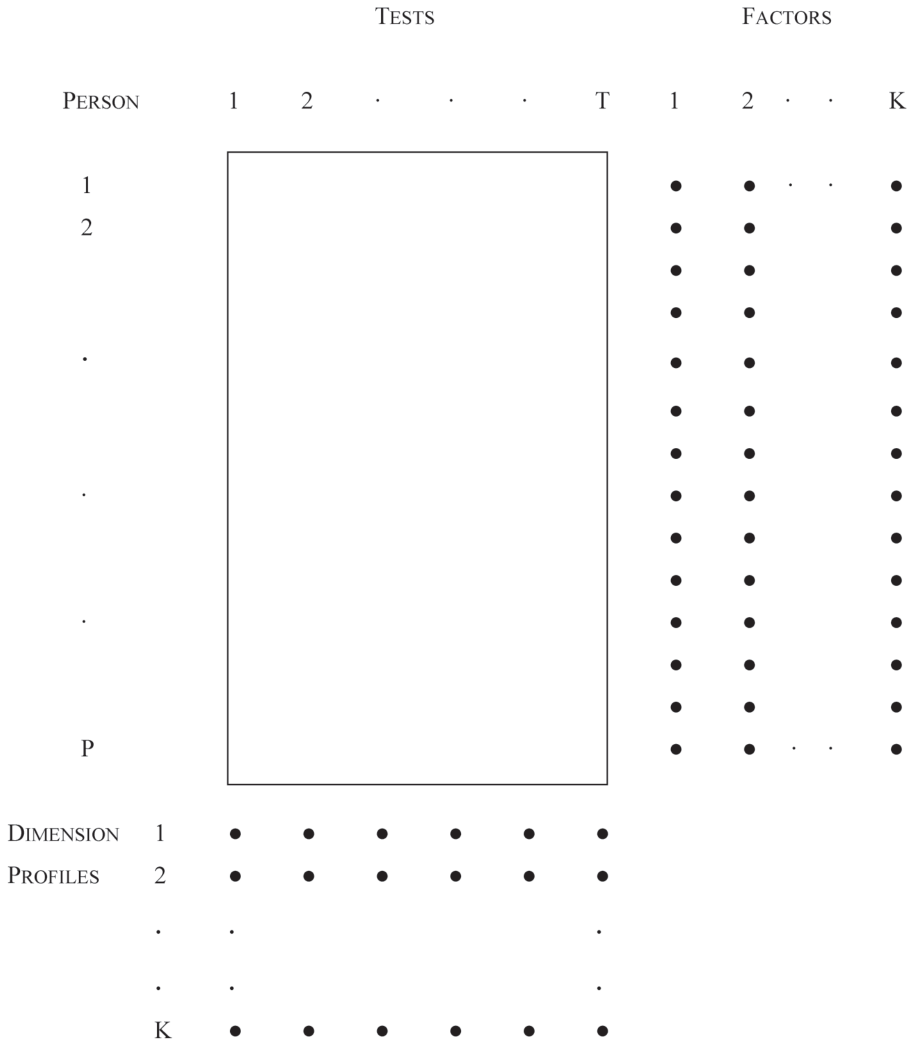


Figure 1
Person × Subtest Data Matrix with Latent Factors and Latent Dimension Profiles
Adapted from Davison, Kuang, and Kim (1999).

of dots (variables) in the figure, so that the observed variables can be explained as linear combinations of the latent factors. Similarly, a small set of latent variables (i.e., latent profiles) can be posited, however, represented by rows of dots (persons), so that the observed variables can be explained as linear combinations of the latent profiles, rather than latent factors.

The latent profile represents an important source of variation in a data set from a multi-test battery. Conceptually, one end of a latent dimension from a PAMS analysis would represent a prototypical profile shape, and the opposite end would represent its mirror image. Once latent profiles are identified from a PAMS analysis, each person's observed profiles can be interpreted as a linear combination of these latent profiles.

Compared to cluster analyses, a PAMS analysis offers three important advantages. First, PAMS uses a variables × variables data matrix for analysis and the size of the data matrix is independent of sample sizes. However, cluster analyses use similarity measures for all individuals in a data set and the construction of a persons × persons similarity (or correlation) matrix for large samples becomes computationally unwieldy. Second, different clustering methods can produce different results when applied to the same data (Aldenderfer & Blashfield, 1984). In some approaches to cluster analysis (e.g., divisive *k*-means or certain approaches to complete- or average-linking agglomerative clustering), subgroups of persons with similar profiles are defined and the average across attributes serves as a prototypical profile. Therefore, the prototypical profile may not be consistent when different clustering methods are applied to the same data. However, a PAMS approach can provide a more consistent prototypical profile when the same data are analyzed. PAMS takes the reverse approach by first defining the prominent profiles in a data set and then determining the extent to which each person resembles a prototypical profile (Davison, Gasser, & Ding, 1996). Third, although most cluster analyses begin with a model of proximity data, they do not provide an explicit model of test scores (Davison, 1996).

Modeling Test Scores using PAMS

The PAMS model begins with the following equation

$$m_{pt} = c_p + \sum_{k=1}^{K} \omega_{pk} \cdot x_{tk} + \varepsilon_{pt} \qquad (1)$$

where m_{pt} is the observed score of person *p* on test *t*, which represents an element of a profile data matrix shown in Figure 1 where each row represents a person (*p*) and each column represents a test score (*t*); c_p is the level parameter, which indexes the overall height of person *p*'s observed profile, and it is obtained by calculating the unweighted average of all test scores for a person (*p*),

$$c_p = m_{p\bullet} = \left(1/T\right)\sum_{t=1}^{T} m_{pt} \;;$$

ω_{pk} is a weight for person *p* on latent dimension *k* in which the "person weight" indexes the degree of correspondence between the actual (observed) test scores of person p and the tests' coordinates on a latent dimension (k) and this correspondence index is estimated by regressing the person's observed test scores onto the scale-values with the unweighted least squares method; x_{tk} is the test parameter, which equals the scale-value (coordinate) of test t on latent dimension k; and ε_{pt} is the error term, representing residuals from the model.

As shown in Equation 1, the PAMS model is based on a decomposition of an individual's observed test scores into two parts. The first part is a profile *level*, c_p. The level parameter c_p represents person p's average score on T tests, and determines the level of person p's profile. The PAMS model uses this level parameter to identify individual differences in observed profile heights. The second part is the profile *pattern* defined as the deviations of the tests about the profile level; that is, a deviation T-length vector = (m_{pt} – c_p). Individual differences in these deviations are represented by

$$\left(m_{pt} - c_p\right) = \sum\nolimits_{k=1}^{K} \omega_{pk} \cdot x_{tk}$$

in Equation 1. When the deviations of all tests around the profile level are considered together, this represents a person's profile pattern.

Assumptions and Restrictions. Some assumptions and restrictions must be added to uniquely define the parameters of the PAMS model.

(2) $$\sum_{t=1}^{T} x_{tk} = 0.0 \text{ for all } k$$

(3) $$E\left(\omega_{pk}^2\right) = 1.0 \text{ for all } k$$

(4) $$E\left(\omega_{pk} \cdot \omega_{pk'}\right) = 0.0 \text{ for all } \left(k, k'\right)$$

(5) $$E\left(\varepsilon_{pt}\right) = 0.0 \text{ for all } t$$

(6) $$Var\left(\varepsilon_{pt}\right) = \sigma^2 \text{ for all } t$$

(7) $$E\left(\omega_{pk} \cdot \varepsilon_{pt}\right) = 0.0 \text{ for all } \left(k, t\right)$$

Equations 2-7 are standard assumptions and restrictions for the PAMS model. It should be noted that Equation 2 implies that each dimension (or dimension profile) k is ipsative so that the mean of the scores in each dimension profile equals zero. Consequently, dimension profiles will reproduce observed score profile patterns, but not the level of observed scores profiles that is accounted for by the level parameter, c_p. Equation 3 states that the expectation of the squared correspondence weights is assumed to be one. Equation 6 implies that the error variances are equal for all tests. This is an extremely strong assumption, but the equal variance assumption seems necessary to justify the most common scaling analyses (Kruskal, 1964a, 1964b; Ramsay, 1977; Shepard, 1962a, 1962b; Takane, Young, & de Leeuw, 1977) available in existing statistical packages. Equation 7 states that the expectation of the cross product between the correspondence weight, ω_{pk} and error, ε_{pt} equals zero.

In this study, each participant had seven WJ-R cognitive observed variable scores, and for participant p (p = 1, ..., 357) seven observed variable scores can be reproduced by the PAMS model, such as $\left(\hat{m}_{p1}, \hat{m}_{p2}, ..., \hat{m}_{p7}\right)$. The model-reproduced scores of participant p on seven observed variables are

$$\hat{m}_{p1} = c_p + \sum_{k=1}^{2} \hat{\omega}_{pk} \cdot x_{1k}, \hat{m}_{p2} = c_p + \sum_{k=1}^{2} \hat{\omega}_{pk} \cdot x_{2k}, \ldots, \hat{m}_{p7} = c_p + \sum_{k=1}^{2} \hat{\omega}_{pk} \cdot x_{7k}.$$

The error term in Equation 1 is the difference between the observed variable score (m_{pt}) and the model reproduced variable score ($\hat{m}_{pt}$), that is, $m_{pt} - \hat{m}_{pt} = \varepsilon_{pt}$.

Method

Sample

Participants. A subset of participants from the *Woodcock-Johnson Psychoeducational Battery—Revised* (WJ-R; McGrew et al., 1991; Woodcock & Johnson, 1989) standardization sample was used to illustrate the application of the PAMS procedure. The overall standardization sample included 6359 participants ranging from 2 to 95 years of age, and were drawn from over 100 geographically diverse U.S. communities between September 1986 and August 1988. All subjects completed all subtests of the WJ-R, including Achievement scales. Among them, only three hundred and fifty seven participants (N = 357) who ranged from 15 to 19 years old were included in this study. The original sample (N = 6359) included 48.7% males, 51.3% females, 78.6% Whites, 16.9% Blacks, 4.5% Other, 90.7% Non Hispanic Origin, and 9.3% of Hispanic Origin. In the current subsample (N = 357), these percentages were 47.3%, 52.7%, 89.6%, 6.4%, 3.9%, 89.9%, and 10.1% respectively.

Woodcock-Johnson Psychoeducational Battery—Revised (WJ-R). The WJ-R includes both cognitive ability and academic achievement scales. The cognitive ability battery includes 21 subtests. Of these 21 subtests, the

Standard Battery includes 7 subtests, and the Supplemental battery includes 14 additional subtests. However, scores from 7 subtests in the Standard Battery and 7 subtests in the Supplemental Battery (i.e., 14 subtests) can be combined to yield seven factor scores comprised of two subtests per factor. The Horn-Cattell theory of intellectual processing (Horn, 1988; Horn & Cattell, 1966) provides the theoretical foundation on which the seven factors are organized. The ability factors, and the two associated subtests for each factor, are as follows: Long-term Retrieval (Memory for Names, Visual-Auditory Learning), Short-term Memory (Memory for Sentences, Memory for Words), Processing Speed (Visual Matching, Cross Out), Auditory Processing (Incomplete Words, Sound Blending), Visual Processing (Visual Closure, Picture Recognition), Comprehension-Knowledge (Picture Vocabulary, Oral Vocabulary), and Fluid Reasoning (Analysis-Synthesis, Concept Formation). For this study, participant scores on the 7 cognitive factors (which had already been given in the original data set) were considered as observed variable scores that were used as raw data for analysis. The internal consistency (Cronbach alpha) among the 7 cognitive observed variables was .84.

The achievement portion of the WJ-R includes 14 subtests arranged into four “clusters” (Broad Reading, Broad Mathematics, Broad Written Language, and Broad Knowledge). The Broad Reading cluster consists of four subtests (Letter-Word Identification, Passage Comprehension, Word Attack, and Reading Vocabulary), the Broad Mathematics cluster consists of three subtests (Calculation, Applied Problems, and Quantitative Concepts), the Broad Written Language cluster consists of four subtests (Dictation, Writing Samples, Proofing, and Writing Fluency), and the Broad Knowledge cluster consists of three subtests (Science, Social Studies, and Humanities). In the second phase of the study, the data from the PAMS analysis of the cognitive portion was correlated with data from the achievement portion.

Procedures

Step #1: Simple MDS. The first step in a PAMS procedure is to conduct a simple MDS on proximity data, which are Euclidian distances computed from subtests across all people in the data set. In the first stage, PAMS uses a nonmetric scaling procedure, ALSCAL (alternating least squares scaling) in our analyses, which estimates scale-values (or dimension coordinates). The original person × test (357 persons × 7 observed variable scores) matrix as shown in Figure 1 was entered in PAMS, and this matrix was analyzed in the first stage of the PAMS procedure. The ALSCAL program (Takane et al., 1977) is available in the SPSS statistical package (SPSS, Inc., 1999).

ALSCAL calculates a "distance" between all possible pairs of seven WJ-R cognitive observed variable scores across all subjects (N = 357) using a common formula discussed in Davison (1983, p. 55): The dissimilarity measure ($\delta_{tt'}$)between two observed variables for person p is computed by summing squared differences between the two observed variable scores across all 357 participants, such that

$$\delta_{tt'} = \left[\sum_{p=1}^{357} \left(m_{pt} - m_{pt'} \right)^2 \right]^{1/2} .$$

This proximity measure is a Euclidean distance between two subtests computed across participants. Next, the proximity values between all pairs of WJ-R observed variables are entered into a 7 observed variables × 7 observed variables data matrix, where smaller values reflect greater similarity between observed variables. Note that since the MDS starts by analyzing a 7 WJ-R observed variables × 7 WJ-R observed variables dissimilarity matrix (rather than a persons × persons matrix), the matrix to be analyzed does not grow as sample size increases. A simple MDS was performed on this matrix, and scale-values (dimension coordinates) for all observed variables (for the entire group of subjects) were computed on two extracted latent dimensions. Here, each of the seven WJ-R observed variables has 2 coordinates (one for each of the 2 extracted latent dimensions).

Step #2: Estimating Person Parameters. Individual differences in cognitive profile patterns are represented by estimates of person parameters. To estimate person parameters in the PAMS model, the original observed variable scores of person p (that serve as dependent variables) are regressed onto the observed variable dimension coordinate values (that serve as independent variables). To minimize discrepancies between the original scores and predicted scores (by the PAMS model), the Least Squares method is used as in ordinary regression. To understand this procedure, the PAMS model is expressed in matrix form in terms of estimated parameters as follows:

$$(8) \qquad \begin{aligned} \vec{\mathbf{M}}_p &= \left(m_{p1}, \ldots, m_{p7} \right)^t \\ &= c_p \cdot \underset{7\times1}{\vec{\mathbf{1}}} + \underset{7\times2}{\hat{\mathbf{X}}} \cdot \underset{2\times1}{\hat{\mathbf{W}}} + \underset{7\times1}{\vec{\mathbf{E}}} \end{aligned}$$

Here, $\vec{\mathbf{M}}_p = \left(m_{p1},...,m_{p7}\right)^t$, which are the original observed variable scores of person p; $c_p \cdot \underset{7\times1}{\vec{\mathbf{1}}} = \left(c_p,...,c_p\right)^t$, which is a level parameter vector; $\underset{7\times2}{\hat{\mathbf{X}}}$ is a 7 × 2 MDS scale-value matrix computed from the observed variable-score dissimilarity matrix, in which 7 rows represent 7 WJ-R observed variables and 2 columns represent 2 MDS dimensions; $\underset{2\times1}{\hat{\mathbf{W}}} = \left(\hat{\omega}_{p1}, \hat{\omega}_{p2}\right)^t$ which are transposed person p's weights estimated by regressing the seven observed variable scores onto the observed variable dimension coordinate values; and $\underset{7\times1}{\vec{\mathbf{E}}}$ is an error vector (= ε_{p1}, ..., $\varepsilon_{p7})^t$ for person p, which are deviations between observed and model reproduced scores.

These person weights are similar to unstandardized regression coefficients and index the degree of correspondence between the observed score profile of person p and the dimension profiles as identified by PAMS. However, these weights were adjusted for dimension variation since person weights are affected by dimension variances.

If the dimension scale-values are orthogonal, the form of the correspondence weight is as follows:

$$\hat{\omega}_{pk} = \frac{r_{pk} \cdot s_p}{s_k} \tag{9}$$

where $\hat{\omega}_{pk}$ is the correspondence weight along dimension k, r_{pk} is the Pearson Product-Moment correlation between the observed profile of person p and dimension profile k, s_p is the standard deviation of person p's observed profile (or scores), and s_k is the standard deviation of dimension profile k. For an observed profile of person p, s_p is a constant across dimensions, but s_k varies among multiple dimension profiles. Therefore, the importance of the correspondence weights across dimensions within a person is influenced not only by r_{pk} but also by s_k. For example, given the same correlations across dimensions with an observed profile, a large standard deviation of a dimension provides a smaller value of correspondence weights compared to that of another dimension that may have a smaller standard deviation. To make it possible to compare the relative importance of dimensions for a given person, the s_k component in $\hat{\omega}_{pk}$, should be fixed or removed, and then the magnitude of correspondence weights are determined only by the correlation (r_{pk}) between the observed profile and the dimensions. For this reason, Kuang (1998) suggested an adjustment as follows:

$$\hat{\nu}_{pk} = \hat{\omega}_{pk} \cdot s_k = \frac{\left(r_{pk} \cdot s_p\right)}{s_k} \cdot s_k = r_{pk} \cdot s_p\,, \tag{10}$$

where the adjusted weight, $\hat{\nu}_{pk}$, removes the effect of standard deviation for a given dimension (the size of the standard deviation masks the relative importance of dimensions for a given observed profile).

Step #3: Estimating Standard Errors of Scale-values through Bootstrapping. The ALSCAL nonmetric scaling procedure does not provide standard errors of estimate for dimension coordinates, which leaves an open question as to whether or not estimated scale values are significantly different from zero. If some scale-value estimates for subtests are not statistically significant from zero, then the interpretation of PAMS profiles will be misleading. For this reason, a resampling technique, called "bootstrapping", was introduced to estimate standard errors of the estimates. The bootstrap method essentially re-creates a distribution of scale-values (dimension coordinates), from which a standard error can be computed. This standard error is then used to evaluate the original coordinate (that is estimated from the simple MDS performed on the WJ-R sample before bootstrapping) for statistical significance.

Using the PAMS model (Davison, 1996), Kim (1999) examined the validity of bootstrap standard error estimation. In their Monte Carlo study, Kim explored the accuracy rate of standard error estimation for the bootstrapping method by simulating dimensionality, number of subtests, and error levels. Across all simulated conditions, the accuracy rate of estimating "true" standard error reached, on average, 80%. In addition, Kim compared accuracy rates between bootstrap and maximum likelihood methods, and the bootstrap procedure performed significantly better in estimating standard error than did the maximum likelihood method.

The bootstrapping procedure was applied to the original WJ-R sample (N = 357). The procedure begins by selecting at random one case from the original WJ-R sample, documents the seven WJ-R observed variable scores for this case, returns the case to the sample, randomly selects another case, documents its seven WJ-R observed variable scores, returns the case to the sample, and so on. These steps are repeated until the size of the first bootstrap sample reached N = 357, which is the same size as the original WJ-R sample. Since Efron and Tibshirani (1993) recommend generating no more than 200 bootstrap samples for estimating standard errors (p. 52), 200 bootstrap samples were generated (with each sample size N = 357) to estimate scale-value standard errors for the current study.

The ALSCAL nonmetric scaling procedure was applied to the 357 persons × 7 observed variables matrix from each bootstrap sample, using the same simple MDS procedure as described earlier (and yielding a 2-dimensional solution). Each of the 200 simple MDS analyses yields 2 dimension coordinates for each of the 7 WJ-R cognitive observed variables. For each analysis, there is one 7 × 2 MDS scale-value matrix. Since there are 200 bootstrap samples, there are two hundred 7 × 2 MDS matrices of bootstrapped scale-values. In other words, there are 200 replicates of a single scale-value of observed variable t on dimension k, where $t = 1, ..., 7$ and $k = 1, 2$.

Step #4: Determine Statistical Significance of Scale-values. From these 200 replicates, a sampling distribution of a single scale-value for observed variable t on dimension k is generated. Each point in the distribution can be given as $\hat{x}_{tk}^{b}$, where the superscript b refers to the number of the bootstrap sample (1 to 200). Each sampling distribution can be given as ($\hat{x}_{tk}^{1}$, $\hat{x}_{tk}^{2}$, ..., $\hat{x}_{tk}^{200}$), from which a mean and standard deviation can be computed. This standard deviation is in fact a bootstrap standard error. This standard error is used as a denominator when the original scale-value (dimension coordinate) is evaluated for statistical significance, stating the null hypothesis that the coordinate value is equal to "0" against the alternative hypothesis that the coordinate value is not equal to "0". The coordinate value minus "0" is divided by the bootstrap standard error estimate and the result is compared to the corresponding z-value (e.g., 1.96 at $\alpha = 0.05$). The zero value is considered the population parameter in the numerator for evaluation (assuming that the null hypothesis is true) since we evaluate whether the coordinate is statistically different from the null value "0."

The original scale-values (estimated from the simple MDS performed on the WJ-R sample before bootstrapping) and the estimated standard errors of these values (derived from application of the bootstrapping procedure) are shown in Table 1. The scale-values for each of the 7 WJ-R observed variables on two dimensions are shown, where the rows represent 7 WJ-R cognitive ability observed variables and the columns represent the scale-values. The columns then become the two PAMS core profiles. With the bootstrap standard error estimates, z-tests (at $\alpha = 0.05$) are used to determine statistical significance of scale-values within each PAMS profile.

Only statistically significant scale-value estimates are labeled in bold print. Recall that without the standard error estimates, significance tests of estimated scale values cannot be performed. In this sense, therefore, the standard error estimates play a crucial role in determining which scale-values are included to interpret the PAMS profiles.

Table 1
Scale-values and Standard Errors Estimated from 200 Bootstrap Replicated Samples

Observed Variables	Dimension 1	Dimension 2
LTR	**-1.44** (.35)	.17 (.09)
STM	.01 (.14)	**-1.43** (.37)
SPR	**1.04** (.27)	**.51** (.19)
APR	**-1.10** (.27)	.06 (.09)
VPR	**-1.01** (.25)	**.46** (.13)
CKW	**2.46** (.60)	.08 (.09)
FRE	.03 (.09)	.15 (.12)

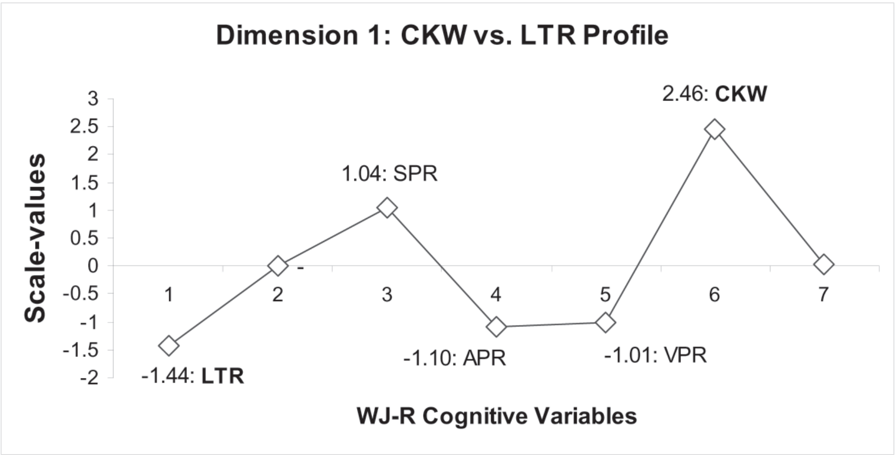
Note. Statistically significant scale-value estimates at $\alpha = 0.05$ are in bold print. LTR = Long-term Retrieval; STM = Short-term Memory; SPR = Speed of Processing; APR = Auditory Processing; VPR = Visual Processing; CKW = Comprehension-Knowledge; FRE = Fluid Reasoning.

Results

Significance Test for Scale-Values

The patterns for the two latent profiles (corresponding to Dimension 1 and Dimension 2) are graphically displayed in Figure 2. The 2-dimensional solution was chosen because of fit and interpretability. The model fits for the 2-dimesional solution were that Stress = .01 and RSQ = 1.00, while the fits for the unidimensional model were that Stress = .17 and RSQ = .91. RSQ values are the proportions of variance of the scaled data that were accounted for by their corresponding distances computed from the scale-values. Stress values here were Kruskal's stress formula 1 (see pp. 87-89, Davison, 1983). Among the scale-values, only statistically significant values (at $\alpha = 0.05$) were labeled. On Dimension 1, there were significant peaks ($p < .01$) for Speed of Processing and Comprehension Knowledge. A significant lower point ($p < .01$) in the profile occurred for Long-term Retrieval, Auditory Processing, and Visual Processing observed variables. Given the highest elevation for the Comprehension Knowledge observed variable, coupled with the lowest elevation for the Long-term Retrieval observed variable, the first latent profile was called a *High Comprehension Knowledge vs. Low Long-term Retrieval* profile.

On Dimension 2, there were significant elevations ($p < .01$) for Speed of Processing and Visual Processing observed variables. A significant lower elevation ($p < .01$) occurred for the Short-term Memory observed variable. Given the highest elevation for the Speed of Processing observed variable, coupled with the lowest elevation for the Short-term Memory observed variable, the second latent profile was called a *High Speed & Visual Processing vs. Low Short-term Memory* profile.



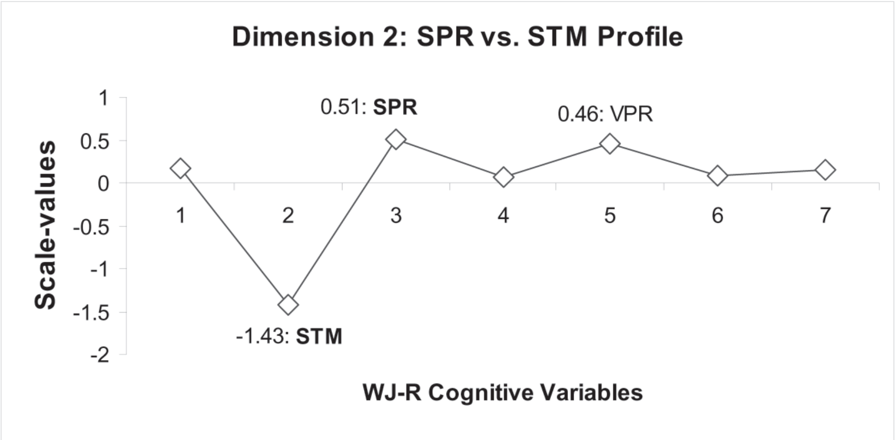


Figure 2
WJ-R Latent Dimension Profile Patterns
Note. LTR = Long-term Retrieval; STM = Short-term Memory; CKW = Comprehension Knowledge; VPR = Visual Processing; SPR = Speed of Processing; and APR = Auditory Processing.

The Meaning of Person Parameter, Level Parameter, and R-Squared

Recall that each participant was assigned person weights on dimensions (or latent profiles), which reflected the degree of correspondence between their observed score profiles and each of the two PAMS latent profiles. To illustrate the meaning of these weights (called person parameters), a small group of subjects was selected from the 357 participants on which PAMS latent profiles were obtained (see Table 2). The person weights for the Dimension 1 profile pattern (*High Comprehension Knowledge vs. Low Long-term Retrieval)* are displayed in Column 2 in the table. Those with substantial positive person weights on this profile but trivial weights on the other dimension (e.g., Participant #30) would have an observed profile pattern similar to the Dimension 1 profile pattern.

Participant #30 $\left[\hat{\nu}_{30(1)} = 140.44, \hat{\nu}_{30(2)} = -4.87\right]$ had a substantially high positive weight on Dimension 1, but a trivial weight on Dimensions 2. Note that adjusted weights, $\hat{\nu}_{pk}$, not $\hat{\omega}_{pk}$, were illustrated here to compare the magnitude of weights between dimensions. The person's observed profile displayed a pattern that was similar to the *High Comprehension Knowledge vs. Low Long-term Retrieval* profile pattern in Figure 2. Participant #30's profile was well accounted by the two latent dimensions (R^2 = .83) and this person's observed profile was a bit elevated (as reflected

Table 2
Person Parameter, Level Parameter, and *R*-Squared Estimated by PAMS

	High Comprehension Knowledge vs. Low Long-term Retrieval: Dim 1 Profile	High Speed & Visual Processing vs. Low Short-term Memory: Dim 2 Profile	Standardized Level Parameter	*R*-Squared
Participant	$\nu_{p1}\left(\omega_{p1}\right)$	$\nu_{p2}\left(\omega_{p2}\right)$	C_p	R^2
#30	140.44 (101.71)	–4.87 (–7.46)	.68	.83
#144	–6.60 (–4.78)	144.07 (220.45)	–2.40	.79
#145	–5.87 (–4.25)	5.92 (9.06)	2.00	.06
#171	118.33 (85.70)	130.43 (199.59)	–2.22	.94
#309	24.70 (17.89)	–110.65 (–169.31)	2.00	.63

by the level parameter estimate, c_{30} = .68), meaning that Participant #30 scored above average overall. Note that in order to make graphical comparisons between observed score profiles and latent profiles shown in Figures 3 and 4, observed scores for persons illustrated here were standardized to *z*-scores (mean zero and standard deviation one) to be comparable with units of latent profiles in terms of coordinates and heights (whose heights were all zeros). A level parameter estimate above/below 0.00 indicated an above/below average overall profile level, which is the latent profile level.

The person weights for the Dimension 2 profile pattern (*High Speed & Visual Processing vs. Low Short-term Memory)* are displayed in Column 3. Participant #144 $\left[\hat{v}_{144(1)} = -6.60, \hat{v}_{144(2)} = 144.07\right]$ had a substantially high positive weight on Dimension 2, but a trivial weight on Dimensions 1. The observed profile for this participant displayed a shape that was similar to the *High Speed & Visual Processing vs. Low Short-term Memory* profile pattern in Figure 3. Participant #144's profile was well accounted for by the two latent dimensions (R^2 = .79) and this person's observed profile was depressed (as reflected by the level parameter estimate, c_{144} = –2.40), meaning that Participant #144 scored much below average overall.

In contrast, Participant #309 $\left[\hat{v}_{309(1)} = 24.70, \hat{v}_{309(2)} = -110.65\right]$ had a substantially negative weight on Dimension 2, but a trivial weight on Dimension 1. This person's observed profile would display a trend that is the "mirror image" of the Dimension 2 profile pattern as shown in the first graph of Figure 4. The original pattern of the Dimension 2 profile was *High Speed & Visual Processing* and *Low Short-term Memory*, but the mirror image of it was *Low Speed & Visual Processing* and *High Short-term Memory*. 63% of Participant #309's observed profile was accounted for by the two latent dimensions (R^2 = .63) and this person's observed profile was elevated (as reflected by the level parameter estimate, c_{309} = 2.00), meaning that Participant #309 scored far above average overall.

Here the R^2s are similar to coefficients of determination in a multiple regression. Considering two latent profiles as predictors (independent variables) and each observed profile as a criterion (dependent variable) in the regression, the value of R^2 represents the proportion of each dependent variable (observed profile) accounted for by two independent variables (latent profiles). As shown in Table 2, those who had high R^2s also had substantial weights (absolute values) on one or both dimensions. For example, the weights of Participant #171 (who had .94 for R^2) on Dimension 1 and Dimension 2 were 118.33 and 130.43, respectively, whereas the weights of Participant #145 who had almost zero *R*-squared (R^2 = .06) on Dimensions 1 and 2 were –5.87 and 5.92, respectively.

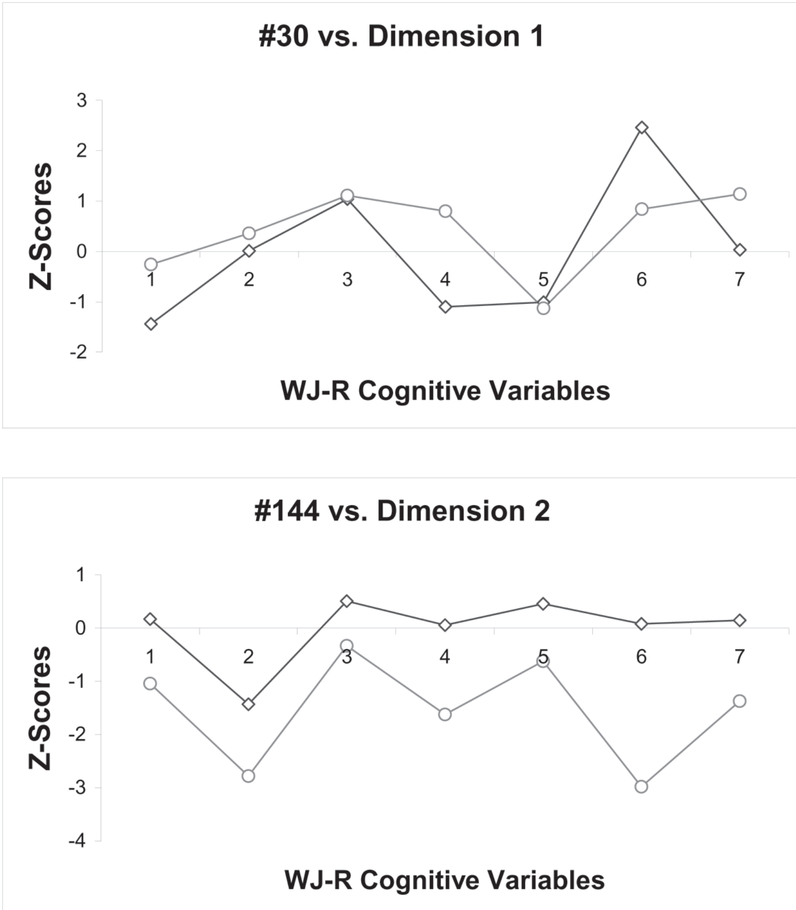
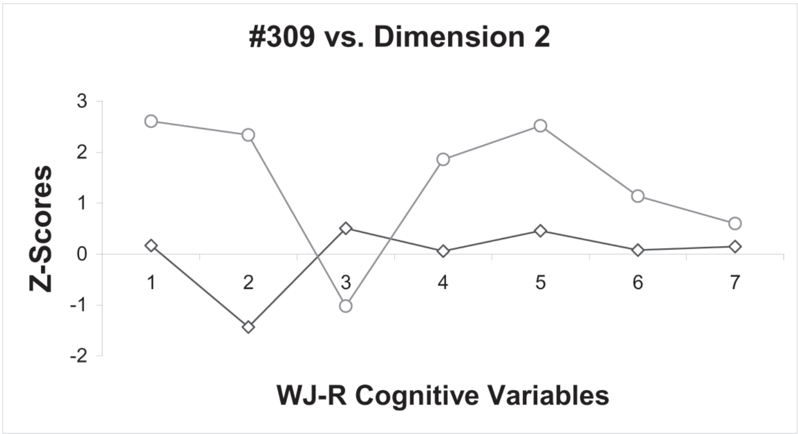


Figure 3
Observed Profile Patterns (Circles) Superimposed on Latent Dimension Profiles (Diamonds)

The last example illustrates that an observed profile can be represented as a linear combination of both dimensions. The dimension weights for Participant #171 were fairly large for both dimensions and the observed profile was accounted for quite well (R^2 = .94). This suggests that the observed profile of Participant #171 should resemble a linear combinations of the Dimension 1 and Dimension 2 profiles. The circles in Figure 4 show the observed profile of Participant #171 and the diamonds represent the sum of the two latent dimensions, Dimension 1 + Dimension 2. This observed profile is depressed, as reflected by the negative level parameter estimate (c_{171} = –2.22),



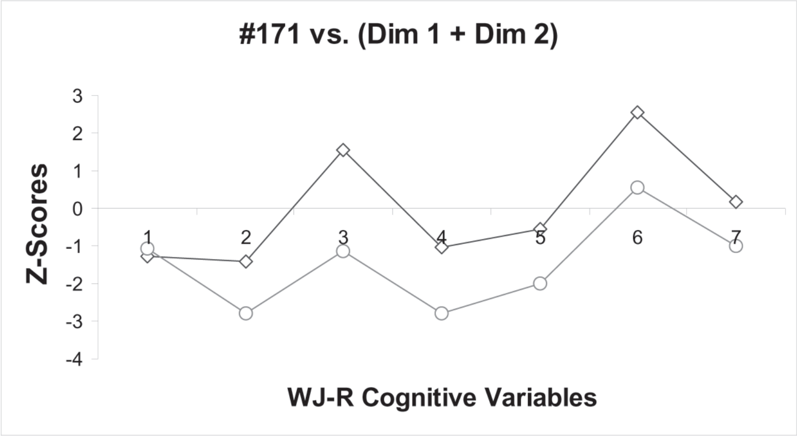


Figure 4
Mirror Image (Circles) of Latent Dimension 2 (Diamonds) and Participant #171 Observed Profile (Circles) vs. Linearly Combined Dimensions (Diamonds)

which means that Participant #171 scored well below average overall. As the PAMS model assumes that an individual's actual profile can be represented as a linear combination of dimension profiles, the profile pattern of Participant #171 was very similar to the Dimension 1 plus Dimension 2 composite as shown in the second graph of Figure 4.

Implications for Relations between Person and Level Parameters

It is important to note that individual differences in the overall profile level (that is an estimate of overall ability or *g*) can be correlated with individual differences in profile patterns. The profile level parameter was significantly correlated with the person parameter that measured the degree of correspondence between person (observed) profile patterns and dimension profile patterns. As for the case of $Cor[\hat{\nu}_{p(1)}, c_p] = .44$, $p < .01$, whenever there is a significant positive correlation between participants' level parameters and their dimension weights for the Dimension 1 profile, it suggests that participants with higher overall profiles show a tendency toward the particular pattern of peaks and valleys that are characteristics of the latent dimension profile. Conversely, whenever there is a significant negative correlation between the level parameter and Dimension 2 weights, $Cor[\hat{\nu}_{p(2)}, c_p] = -.44$, $p < .01$, it suggests that participants with higher overall profiles show a tendency toward a pattern of peaks and valleys that is the mirror image of the latent Dimension 2 profile.

In the Horn-Cattell model of intellectual processing (Woodcock & Mather, 1989), there is a large conceptual distance between the least cognitively complex levels of processing (Short-term Memory and Long-term Retrieval) and the most complex levels of processing (Fluid Reasoning and Comprehension-Knowledge). In contrast there is a shorter conceptual distance among the remaining observed variables. Dimension 1 represents individuals whose profile shapes reflect peaks at the higher levels of a more *g* loaded processing component (Comprehension-Knowledge; see Woodcock & Mather, 1989) relative to a valley at lower levels of a less *g* loaded long term memory observed variable (Long-Term Retrieval). Thus, the significant positive correlation between the level parameter (estimate of Spearman's *g*) and Dimension 1 weights ($r = .44$, $p < .01$) would be somewhat intuitive. However, the profile shape represented by Dimension 2 involves distances between peaks and valleys that are not as conceptually pronounced on the Horn-Cattell continuum of cognitive complexity (Processing Speed and Visual Processing versus Short Term Memory). Therefore, the significant negative correlation between the level parameter (estimate of Spearman's *g*) and Dimension 2 weights ($r = -.44$, $p < .01$) is less interpretable.

Relationship of Person Parameters (or Dimension Weights) with External Variables

The correlations of dimension weights with scores on the four achievement clusters of the WJ-R (Broad Reading, Broad Math, Broad

Written Language, and Broad Knowledge) are shown in Table 3. As expected, individual differences in level parameters displayed the strongest correlations with WJ-R achievement clusters (r = .73 ~ .83, p < .01). That is, higher overall levels of performance on WJ-R cognitive clusters were associated with higher levels of performance on achievement clusters. The next strongest correlation with achievement clusters was associated with individual differences in the first latent core profile: *High Comprehension-Knowledge vs. Low Long-term Retrieval* profile (r = .47 ~ .60, p < .01). The highest correlation was with Broad Knowledge, which consists of brief questions that measure knowledge of facts and concepts in science, social studies, and the humanities. The correlation between person weights on the second latent core profile (*High Speed & Visual Processing vs. Low Short-term Memory*) and achievement clusters was statistically significant, but negative (r = –.33 ~ –.20, p < .01). That is, people with higher levels of achievement tended to have profiles resembling its mirror image (*Low Speed & Visual Processing vs. High Short-term Memory* profile). The highest correlation was with Broad Reading and Broad Written Language clusters, which consisted of Letter-Word Identification, Passage Comprehension, Word Attack, Reading Vocabulary, Spelling Dictation, and Writing Samples subtests.

In addition to examining a linear relationship between dimension weights and achievement, multivariate regression onto the four achievement clusters was conducted, in order to determine the independent contribution of dimension weights and the level parameter on the prediction of achievement clusters. A hierarchical regression was conducted. The dimension weights first and then the level parameter were entered into the regression. The

Table 3
The Correlations of Person Parameters with Scores on the ACHIEVEMENT CLUSTERS of the WJ-R

ACHIEVEMENT CLUSTER	$\nu_{p(1)}$	$\nu_{p(2)}$	c_p
Broad Knowledge	.60**	–.24**	.80**
Broad Reading	.53**	–.33**	.83**
Broad Written Language	.48**	–.32**	.75**
Broad Math Skills	.47**	–.20**	.73**

** $p \leq .01$.

dimension weights explained 32.4% of the achievement scores and then the level parameter explained 39.8% of the achievement scores. Around 72% of the variation in all four achievement cluster scores was explained by both weights and the level parameter. Interactions among independent variables were not statistically significant and disregarded.

Discussion

Why PAMS over Other Competing Approaches?

In the cognitive testing domain, the clinical utility of profile analysis results has not received notable support from researchers because of the substantial portion of *g* factor that is shared by subtests and factors. The test's predictive validity is directly related to the test's *g* loadings (see Jensen, 1998), and the profile patterns which are employed for differential diagnoses for clinical utility have little differential predictive validity after the effects of *g* have been accounted for.

PAMS is designed primarily for those who wish to study individual differences in profile pattern separate from individual differences in profile level. To that end, the PAMS analysis includes detached pattern parameters from the level parameter and the pattern parameters can be used to predict relationships with external variables. In this study, profile patterns were correlated with achievement scores and the results showed significant relationship between the patterns and achievement scores (see Table 3).

The PAMS model of Equation 1 is a linear model that includes latent variables, similar to a factor model. However, in Equation 1 the model is reparameterized in such a way that the parameters can be interpreted in terms of profile patterns and the model can be used to address research questions about profile patterns. The PAMS model includes a profile level parameter with no exact counterpart in exploratory factor models. Further, to identify the solution, PAMS assumes that test parameters sum to zero on each dimension, whereas factor analysis assumes that person parameters (which are factor scores) sum to zero. This PAMS assumption shifts the origin of the PAMS solution to the centroid of the tests, rather than the centroid of the people.

The Q-factor model is a special case of the PAMS model in which the observed profiles have been standardized to have mean 0.0 and variance 1.0. Within the PAMS approach, a Q-factor analysis can be implemented by standardizing the profiles before the analysis. The Q-factor analysis permits the investigator to separate individual differences in profile shape from individual differences in level and scatter. Thus, the PAMS analysis is best suited to those situations where the researcher wishes to focus on individual differences in profile pattern or profile shape separate from profile level. Unlike the more traditional approach to Q-factor analysis, the PAMS approach can be readily applied to samples of any size.

As cluster analysis has been most often applied to the study of intelligence battery profiles, it has been applied in such a way that individual differences in profile level, shape, and scatter are all potentially confounded. In the research reviewed here, the resulting profiles differed primarily in elevation (e.g., Konold et al., 1999; McDermott, 1998). Furthermore, several of the core profiles were flat profiles with no noticeable high or low points in the profile. Such profiles are uninformative as to individual differences in profile pattern and shape. As a description of individual differences, the clusters largely described various general intelligence levels. By including separate parameters for profile level and profile pattern, PAMS assures that the resulting solution will describe whatever individual differences exist in profile level and whatever differences exist in profile pattern.

The cluster approach and the PAMS approach also differ in terms of dichotomous indicators. That is, a person does or does not belong to a cluster; a person can or cannot be represented by the profile pattern associated with that cluster. On the other hand, the PAMS analysis uses a continuous representation of individual differences. In the cluster approach, a researcher may add a continuous measure of profile match to index the degree to which each person's profile matches a pattern identified by the clustering (e.g., Konold, Glutting, & McDermott, 1997; Konold et al., 1999), but a cluster is fundamentally a categorical representation of individual differences. The PAMS model itself includes measures of profile match that could be used by clinicians and researchers. The person parameters in the model, such as those in Table 2, quantify the degree to which a person's observed profile corresponds to one of the dimension profiles. These rescaled correspondence weights can be used by researchers to study the association between ability patterns and achievement test scores (see Table 3).

These rescaled correspondence weights can be useful to clinicians in test interpretation, particularly if patterns are shown to be associated with outcome variables. That is, clinicians can be provided scores (estimates of rescaled correspondence weights) that indicate the degree to which a client's profile matches a latent profile pattern. If the score indicates that the client's pattern closely matches the latent profile pattern and the pattern has a well-established association to an outcome variable, then this association to the outcome may prove useful in interpreting the client's pattern. Research on the association between ability patterns and outcome variables could provide an empirical basis for the clinical interpretation of profile patterns.

However, in intelligence testing, evidence for associations between patterns and outcome variables is lacking (McDermott, 1998; McDermott et al., 1990). As shown in Table 3, the results provided evidence of significant associations between profile patterns and achievement scores. The Dimension 1 profile pattern, *High Comprehension Knowledge vs. Low Long-term Retrieval*, and achievement scores had significant positive associations, and the Dimension 2 profile pattern, *High Speed & Visual Processsing vs. Low Short-term Memory*, and the achievement scores had significant negative associations. These results could provide valuable information to clinicians. Based on these results, clinicians or school psychologists can expect that holding the effect of level parameter constant, students whose cognitive profiles are similar to Dimension 1 will obtain higher achievement scores than students with other profile patterns, or holding the effect of profile patterns constant, students with high level parameters will gain higher achievement scores than students with low profile levels.

The relation observed here between profile patterns and external variables (i.e., achievement scores) may not be generalized to all situations as R. E. Millsap commented (personal communication, 2004): One may find profile patterns emerging in a given situation, yet also find that these patterns contribute little to the prediction of external variables over and above the level information in some cases. Nevertheless, the role given to each (pattern or level information) is different. The level information may usually be utilized to segregate or differentiate a clinical group from a normal group (e.g., LD from non-LD or pathological from normal), while the pattern information may be used to differentiate among clinical groups (e.g., reading disabled vs. computationally disabled or psychotic vs. neurotic).

To examine whether pattern information beyond level information is statistically meaningful, Structural Equation Modeling (SEM) can be used in a confirmatory fashion and then statistical significance of loadings on (dimensional) profile patterns and fit indices need to be examined. If fit indices and statistics of loadings are satisfactory, then pattern information will be valid. Kim, Davison, and Frisby (2001) have been studying this issue.

Nonmetric Distance vs. Singular Value Decomposition MDS Approaches

There is more than one MDS approach based on the PAMS model, just as there are many factor techniques (e.g., maximum likelihood vs. image factor analysis) and many cluster techniques (e.g., average link vs. maximum link cluster analysis). The two major MDS methods are the nonmetric one illustrated in the current study and the singular value decomposition (SVD) described by Gower and Hand (1996) for fitting their rows-regression model. From a purely least squares perspective, the SVD is superior in that, for a given dimensionality *K*, the SVD will provide least squares estimates of all parameters, thereby minimizing the sum of squared discrepancies about the data points, m_{pt}. The MDS solution used here is conditionally least squares in that it provides least squares estimates of the person parameters, ω_{pk} and c_p, conditional on the MDS estimates of scale values, x_{tk}.

While the SVD solution will be least squares optimal, it may not be optimal from an applied perspective. The model based on the SVD solution (e.g., component model) is determinate and draws no distinction between systematic variance in the data and stochastic variance due to such things as the unreliability of the measures. Consider the situation in which the data satisfy the stochastic model of Equation 1, *K* (dimensionality) < *T* (number of tests), and the error variance, σ^2, is not negligible. In the simple MDS of *Step #1* above (p. 605), the proximities will be fit exactly by a solution of *K* dimensions. The SVD, however, will require more than *K* dimensions with the additional components accounting for variation in the data attributable to the error term ε_{pt}. SVD requires that the researcher distinguish between components that represent underlying profile patterns and components accounting for stochastic deviations from the model.

Because the component model is determinate, the SVD or Principal Component (PC) approach poses the same problem in the study of profile patterns as it does in the factor analytic study of trait ability factors: distinguishing components that represent underlying profile patterns from those that represent stochastic variation in the data. In the factor domain, this problem motivated the development of common factor methods. In the study of profile patterns, the problem suggests the simple MDS approach of *Step #1* above, rather than the SVD/PC solution.

Advantages of Bootstrapping in PAMS

Before the bootstrapping technique was applied to estimating standard errors of MDS coordinates, a conventional (but imprecise) rule of thumb has been to interpret as "significant" any MDS coordinates (scale values) equal to or greater than 1.0 in absolute value. However, this approach entirely relies on an arbitrary convention, not on test statistics, and interpretation of the dimensions would be arbitrary and possibly misleading.

For example, in interpreting the Dimension 2 profile in the study, according to the conventional rule, the person would include only variables with absolute scale-values equal to or greater than 1.0 and if so, only the Short-term Memory variable [$\hat{x}_{ST(2)} = -1.43$, $BSE = .37$, $p < .01$] would have been included for interpretation. But actual test statistics based on bootstrap standard error estimates showed that the Dimension 2 coordinate of the Speed of Processing variable [$\hat{x}_{SP(2)}$ = .51, *BSE* = .19, p < .01] and the Dimension 2 coordinate of the Visual Processing variable [$\hat{x}_{VP(2)}$ = .46, *BSE* = .13, p < .01] were both statistically significant although the scale-values were less than 1.0 in absolute value. If bootstrap standard errors had not been available, the marker variables for the *Speed & Visual vs. Short-term Memory* pattern would not have been specified completely. Note that *BSE* refers to the bootstrap standard error estimate. Recognizing the importance of scale-value standard errors, Ramsay (1977) applied the maximum likelihood (ML) estimation procedure to estimating standard errors of MDS scale-values. Although this approach was pioneering, Ramsay's ML approach has been reported as underestimating standard errors by several researchers (Kim, 1999; Weinberg, Carroll, & Cohen, 1984).

Summary and Recommendations

In Profile Analysis via Multidimensional Scaling (PAMS), scale-values are test parameter estimates and can be interpreted in terms of latent profile patterns in a population. The PAMS model assumes that observed profile patterns are represented as linear combinations of the latent profiles, and based on the latent profiles, individuals' observed profiles are specified. In the specification, person (or correspondence) weights quantify the degree of match between the observed profiles and the latent profiles, and the weights determine which pattern of dimension profiles will be dominant in accounting for patterns of the observed profiles. Level parameters are estimated by averaging the subtest scores of people and specify individual differences in overall profile height.

To make it possible to test the statistical significance of parameter estimates in PAMS, the bootstrap approach was utilized to estimate standard errors of scale-values, and in interpreting dimension profiles, only statistically significant scale-values are included and the interpretation is meaningful.

As illustrated above, person parameters can be used to predict external variables. Although we used achievement tests from the same battery for purposes of illustration, data from variables gathered independently from a multi-test battery are appropriate (i.e., self reports of interests/attitudes toward academic subjects, subject area grades, scores on subject-specific tests, etc.). The PAMS procedure is particularly useful for informing researchers which specific *profile patterns* add a significant increment to prediction of external variables *over and above profile level* (or elevation). At this juncture, it remains unclear which person weights on PAMS dimensions are themselves significant (i.e., statistically different from zero). In future research, the bootstrap technique can be applied to estimating standard errors of person weights. Using these error estimates, researchers can then duplicate our procedure by conducting significance tests for person weights (and including only significant weights for interpretation).

References

Aldenderfer, M. S. & Blashfield, R. K. (1984). *Cluster analysis*. Beverly Hills, CA: Sage.

Carroll, J. B. (2000). Commentary on profile analysis. *School Psychology Quarterly, 15*(4), 449–456.

Cattell, R. B. (1967). The three basic factor analysis research design: Their interrelations and derivatives. In D. N. Jackson & S. Messick (Eds.), *Problems in human assessment*. New York: McGraw Hill.

Davison, M. L. (1983). *Multidimensional scaling*. New York: Wiley.

Davison, M. L. (1996, August). *Multidimensional scaling interest and aptitude profiles: Idiogrphic dimensions, nomothetic factors*. Presidential address to Division 5, American Psychological Association, Toronto.

Davison, M. L., Gasser, M., & Ding, S. (1996). Identifying major profile patterns in a population: An exploratory study of WAIS and GATB patterns. *Psychological Assessment, 8*, 26–31.

Davison, M. L. & Kuang, H. (2000). Profile patterns: Research and professional interpretation. *School Psychology Quarterly, 15*(4), 457–464.

Davison, M. L., Kuang, H., & Kim, S-K. (1999). The structure of ability profile patterns: A multidimensional scaling perspective on the structure of intellect. In P. L. Ackerman, P. C. Kyllonen, & R. D. Roberts (Eds.), *Learning and individual differences: Process, trait, and content determinants* (pp. 187-207). Washington, DC: American Psychological Association.

Efron, B. &Tibshirani, R. J. (1993). *An introduction to the bootstrap*. New York: Chapman & Hall.

Gower, J. C. & Hand, D. J. (1996). *Biplot*. London: Chapman & Hall.

Horn, J. L. (1988). Cognitive diversity: A framework for learning. In P. L. Ackerman, R. J. Sternberg, & R. Glaser (Eds.), *Learning and individual differences*. New York: W. H. Freeman.

Horn, J. L., & Cattell, R. B. (1966). Refinement and test of the theory of fluid and crystallized intelligence. *Journal of Educational Psychology, 57*, 250-270.

Jensen, A. (1998). *The g factor: The science of mental ability*. Westport, CT: Prageger.

Kaufman, A. S. (1994). *Intelligent testing with the WISC-III*. New York: John-Wiley & Sons.

Kim, S-K. (1999). *A simulation comparing bootstrap and maximum likelihood estimates of scale-value standard errors in multidimensional scaling*. Unpublished doctoral dissertation, University of Minnesota-Twin Cities, Minneapolis.

Kim, S-K, Davison, M. L., & Frisby, C. L. (2001). *Confirmatory profile analysis via multidimensional scaling (PAMS)*. Manuscript submitted for publication.

Kline, R. B., Guilmette, S., Snyder, J., & Mastellanos, M. (1992). Relative cognitive complexity of the Kaufman Assessment Battery for Children (K-ABC) and the WISC-R. *Journal of Psychoeducational Assessment, 10*(2), 141-152.

Konold, T. R., Glutting, J. J., & McDermott, P. A. (1997). The development and applied utility of a normative aptitude-achievement taxonomy for the Woodcock-Johnson Psycho-educational Battery-Revised. *Journal of Special Education, 31*(2), 212-232.

Konold, T. R., Glutting, J. J., McDermott, P. A., Kush, J. C., & Watkins, M. M. (1999). Structure and diagnostic benefits of a normative subtest taxonomy developed from the WISC-III standardization sample. *Journal of School Psychology, 37*(1), 29-48.

Kruskal, J. B. (1964a). Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika, 29*, 1-27.

Kruskal, J. B. (1964b). Nonmetric multidimensional scaling: A numerical method. *Psychometrika, 29*, 115-129.

Kuang, H. (1998). *The relationship of ability patterns and scholastic achievement in the Woodcock-Johnson cognitive ability tests*. Unpublished doctoral dissertation, University of Minnesota – Twin Cities, Minneapolis.

McDermott, P. (1998). MEG: Megacluster analytic strategy for multistage hierarchical grouping with relocations and replications. *Educational and Psychological Measurement, 58*, 677-686.

McDermott, P. A., Fantuzzo, J. W., & Glutting, J. J. (1990). Just say no to subtest analysis: A critique on Wechsler theory and practice. *Journal of Psychoeducational Assessment, 8*, 290-302.

McDermott, P., Fantuzzo, J., Glutting, J., Watkins, M., & Baggaley, A. (1992). Illusions of meaning in the ipsative assessment of children's ability. *Journal of Special Education, 25*(4), 504-526.

McDermott, P. & Glutting, J. (1997). Informing stylistic learning behavior, disposition, and achievement through ability subtests – or, more illusions of meaning? *School Psychology Review, 26*(2), 163-175.

McGrew, K. S., Werder, J. K., & Woodcock, R. W. (1991). *WJ-R technical manual*. Allen, TX: DLM Teaching Resources.

Moses, J. A. & Pritchard, D. A. (1995). Modal profiles for the Wechsler Adult Intelligence Scale-Revised. *Archives of Clinical Neuropsychology, 11*, 61-68.

Pritchard, D. A., Livingston, R. B., Reynolds, C. R., & Moses, J. A. (2000). Modal profiles for the WISC-III. *School Psychology Quarterly, 15*, 400-418.

Ramsay, J. O. (1977). Maximum likelihood estimation in multidimensional scaling. *Psychometrika, 42*, 241-266.

SAS Institute, Inc. (1990). *SAS/Stat software: Change and enhancement, release 6.07 program manual*. Cary, NC: Author.

Shepard, R. N. (1962a). The analysis of proximities: Multidimensional scaling with an unknown distance function I. *Psychometrika*, *27*, 125-140.

Shepard, R. N. (1962b). The analysis of proximities: Multidimensional scaling with an unknown distance function II. *Psychometrika*, *27*, 219-246.

Skinner, H. & Lei, H. (1980). Modal profile analysis: A computer program for classification research. *Educational and Psychological Measurement, 40*, 769-772.

SPSS, Inc. (1999). *SPSS advanced models 10.0*. Chicago: Author.

Stanton, H. C. & Reynolds, C. R. (2000). Configural frequency analysis as a method of determining Wechsler profile types. *School Psychology Quarterly, 15*, 434-448.

Takane, Y., Young, F. W., & de Leeuw, J. (1977). Nonmetric individual differences multidimensional scaling: An alternating least squares method with optimal scaling features. *Psychometrika, 42*, 7-67.

Watkins, M. W. (2000). Cognitive profile analysis: A shared professional myth. *School Psychology Quarterly, 15*, 465-479.

Weinberg, S. L., Carroll, J. D., & Cohen, H. S. (1984). Confidence regions for INDSCAL using the jackknife and bootstrap technique. *Psychometrika, 49*, 475–491.

Woodcock, R. W. & Johnson, M. B. (1989). *Woodcock-Johnson Psychoeducational Battery - Revised*. Allen, TX: DLM Teaching Resources.

Woodcock, R. W. & Mather, N. (1989). WJ-R Tests of Cognitive Ability — standard and supplemental batteries: Examiner's manual. In R. W. Woodcock & M. B. Johnson (Eds.), *Woodcock-Johnson Psychoeducational Battery — Revised*. Allen, TX: DLM Teaching Resources.

Accepted March, 2004.