Journal of Educational Psychology
2008, Vol. 100, No. 1, 30–47

0022-0663/08/$12.00 DOI: 10.1037/0022-0663.100.1.30

Problem Solving and Computational Skill: Are They Shared or Distinct Aspects of Mathematical Cognition?

Lynn S. Fuchs and Douglas Fuchs
Vanderbilt University

Karla Stuebing and Jack M. Fletcher
University of Houston

Carol L. Hamlett and Warren Lambert
Vanderbilt University

The purpose of this study was to explore patterns of difficulty in 2 domains of mathematical cognition: computation and problem solving. Third graders (n = 924; 47.3% male) were representatively sampled from 89 classrooms; assessed on computation and problem solving; classified as having difficulty with computation, problem solving, both domains, or neither domain; and measured on 9 cognitive dimensions. Difficulty occurred across domains with the same prevalence as difficulty with a single domain; specific difficulty was distributed similarly across domains. Multivariate profile analysis on cognitive dimensions and chi-square tests on demographics showed that specific computational difficulty was associated with strength in language and weaknesses in attentive behavior and processing speed; problem-solving difficulty was associated with deficient language as well as race and poverty. Implications for understanding mathematics competence and for the identification and treatment of mathematics difficulties are discussed.

Keywords: calculations, word problems, cognitive predictors, mathematics

Mathematics, which involves the study of quantities as expressed in numbers or symbols, comprises a variety of related branches. In elementary school, for example, mathematics is conceptualized in strands such as concepts, numeration, measurement, arithmetic, algorithmic computation, and problem solving. In high school, curriculum offerings include algebra, geometry, trigonometry, and calculus. Little is understood, however, about how different aspects of mathematical cognition relate to one another (i.e., which aspects of performance are shared or distinct, or how difficulty in one domain corresponds with difficulty in another). Such understanding would provide theoretical insight into the nature of mathematics competence and practical guidance about the identification and treatment of mathematics difficulties.

The purpose of the present study was to explore the overlap of difficulty with two aspects of primary-grade mathematical cognition and to examine how characteristics differ among subgroups with difficulty in one, the other, both, or neither. The first aspect of performance was computation, including skill with number combinations (e.g., 2 + 5; 8 – 3) and procedural computation (e.g., 25 + 38; 74 – 22). The second aspect of performance was problem solving, including one-step, contextually straightforward word problems (e.g., John had 9 pennies. He spent 3 pennies at the store. How many pennies did he have left?) and multistep, contextually more complex problems (e.g., Fred went to the ballgame with 2 friends. He left his house with $42. While at the game, he bought 5 hotdogs and 3 sodas. The hot dogs cost $10 each, and the sodas cost $5 each. How much did Fred spend?).

The major distinction between computation and problem solving is the addition of linguistic information that requires children to construct a problem model. Whereas a computation problem is already set up for solution, a word problem requires students to use the text to identify missing information, construct the number sentence, and derive the calculation problem for finding the missing information. This transparent difference would seem to alter the nature of the task, but no studies have examined how difficulty in one subdomain corresponds to difficulty in the other and whether students' cognitive characteristics differ as a function of where the mathematics difficulty resides.

By contrast, a related literature does focus on the interplay between math and reading. In these studies, math difficulty typically is defined on a broad measure tapping multiple aspects of performance, then subgroups are formed to determine how performance on mathematics domains differs with or without concurrent reading difficulty. Research has shown that students with difficulty in both math and reading (usually defined in terms of word recognition) experience more pervasive deficits in computation (e.g., Jordan & Hanich, 2000) and problem solving (e.g., Fuchs & Fuchs, 2002; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). This may occur due to a different pattern of

Lynn S. Fuchs, Douglas Fuchs, and Carol L. Hamlett, Department of Special Education, Vanderbilt University; Karla Stuebing and Jack M. Fletcher, Department of Psychology, University of Houston; Warren Lambert, Kennedy Center for Research on Human Development, Vanderbilt University.

This research was supported in part by Grant 1 RO1 HD46154 and Core Grant HD15052 from the National Institute of Child Health and Human Development to Vanderbilt University. Statements do not reflect agency position or policy, and no official endorsement should be inferred.

Correspondence concerning this article should be addressed to Lynn S. Fuchs, Peabody College, Box 228, Vanderbilt University, Nashville, TN 37203. E-mail: lynn.fuchs@vanderbilt.edu

underlying deficits in domain-general abilities associated with comorbidity. Other work (e.g., Jordan, Hanich, & Kaplan, 2003; Landerl, Began, & Butterworth, 2004), however, has shown that students with general math difficulty, with and without reading problems, experience comparable deficits on number combinations. An older body of research suggested that specific math difficulty, usually defined as performance on a broad computational task, is associated with difficulties in nonverbal processing (spatial cognition, working memory) and procedural knowledge (Geary, 1993; Rourke & Finlayson, 1978); concurrent reading and math difficulties reflect more pervasive language and working-memory problems.

This line of work, which speaks to the issue of reading difficulty, is important for generating hypotheses about the nature of mathematics disability as well as its identification and treatment. This literature does not, however, address the issue of whether difficulty within mathematics domains is better conceptualized as shared or distinct. Four large-scale studies speak indirectly to this issue by examining the cognitive characteristics associated with computational and problem-solving skill among representative samples. Studying 353 first through third graders, Swanson and Beebe-Frankenberger (2004) identified working memory as an ability that contributed to strong performance across both areas of mathematical cognition, but some unique cognitive abilities also emerged as important: phonological processing for computation and fluid intelligence as well as short-term memory for simple word problems. In an extension of this work following students' development of calculations and problem-solving skill over 1 year, Swanson (2006) identified predictors of computation (inhibition or controlled attention, vocabulary knowledge, visual-spatial working memory) that differed from problem solving (working-memory's executive system, operationalized as listening span, backward digit span, and digit/sentence span). A latent variable of reading (i.e., phonological processing, timed and untimed reading of real words, timed reading of pseudowords, and comprehension) accounted for skill in both math outcomes.

With a sample of 312 third graders, Fuchs et al. (2006) examined the concurrent cognitive correlates of computation versus simple word problems, this time controlling for the role of arithmetic skill within simple word problems. Teacher ratings of inattentive behavior were identified as a correlate common to both subdomains of math, but the remaining abilities differed: for computation, phonological decoding and processing speed; for word problems, nonverbal problem solving, concept formation, sight word efficiency, and language. The fourth study (Fuchs et al., 2005) used beginning-of-the-year cognitive abilities to predict the development of skill across the year among 272 first graders. Results again suggested some common and some unique patterns of cognitive abilities. The common predictors were working memory and ratings of attention. The unique predictors were phonological processing for computation and nonverbal problem solving for simple word problems.

Across these studies, some findings recur; others are idiosyncratic. But together, the results indicate that some abilities underlying these domains of mathematical cognition are unique. This provides the basis for hypothesizing that the cognitive dimensions underlying difficulty in each of these domains may also be distinct and that difficulty in these two domains may be distinct. This has implications for understanding, identifying, and treating mathematics difficulties. In a related way, such knowledge would help determine whether the distinction in these domains newly introduced into the 2004 reauthorization of the Individuals with Disabilities Education Act is viable. That is, although the reauthorized law, which guides the identification and treatment of students with disabilities throughout schools in the United States, makes a distinction between problem-solving and computational forms of mathematics disability, little prior work is available to assess the validity of this distinction.

In the present study, we extended the literature by addressing these issues directly. We began by conducting preliminary analyses to evaluate the predictors of each math outcome using assessments of math skills and cognitive domains in a large population-based sample of children in third grade. In line with the four earlier studies on representative samples, we hypothesized that different cognitive skills would be associated with each domain, even when we accounted for shared variance in computation and problem solving. Our major analyses, however, focused specifically on determining whether children with extreme deficits in computation or problem solving represent distinct groups. We identified students at the lower end of the distribution on computation, on problem solving, on both, or on neither, and we examined the extent to which students actually experienced difficulty in one domain but not the other. Then, we assessed how the demographic and cognitive profiles associated with these subgroups differed. Using profile analysis, we hypothesized that groups based on computation or problem solving would show different profiles and that the presence of both difficulties would show features of both domains, reflecting a comorbid association. We contrasted profile analysis findings based on a univariate versus a multivariate approach.

We note that the vast majority of prior work examining the cognitive correlates of primary-grade mathematics performance focused on a limited set of cognitive abilities related to a single aspect of math skill, rather than studying how these abilities operate within a multivariate framework to explain different aspects of mathematical cognition. For this reason, the literature provides the basis for deliberate hypotheses about which cognitive abilities may mediate which aspect of third-grade math performance. The literature does not, however, provide the basis for specifying an integrated theory about how these variables operate in coordinated fashion to explain computation versus problem solving. Before describing the method of the present study, we summarize the basis for our hypotheses about which cognitive dimensions might be related to which aspect of mathematical cognition.

In terms of computation, prior work provides the basis for hypothesizing that attentive behavior, working memory, and processing speed may help determine skill. Because computation requires a series of steps, attentive behavior (i.e., low distractibility) may enhance performance. Russell and Ginsburg (1984) provided suggestive evidence on this possibility when they compared math-disabled fourth graders to normal fourth graders and normal third graders. Results indicated that the algorithmic errors of math-disabled students were similar to those of both normal groups, but math-disabled students more closely resembled younger normal counterparts in detecting those errors. More recently, Swanson (2006) showed that inhibitory control predicted the development of computation but not problem-solving skill.

Second, prior work has implicated *working memory* (e.g., Geary, Brown, & Samaranayake, 1991; Hitch & McAuley, 1991; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001), or the capacity to maintain target memory items while processing an additional task (Daneman & Carpenter, 1980). Although the relation between working memory and memory-based retrieval of computation has been repeatedly documented, the nature of that relation is unclear. As described by Geary (1993), working memory involves component skills including, but not limited to, rate of decay (creating difficulties in holding the association between a problem stem and its answer) and attentive behavior (hence the finding that math-disabled children monitor problem solving less well than non-math-disabled children; Butterfield & Ferretti, 1987; Geary, Widaman, Little, & Cormier, 1987). In addition, memory span appears to be related to how quickly numbers can be counted (Geary, 1993).

It is not surprising, therefore, that *processing speed*, or the efficiency with which simple cognitive tasks are executed (R. Case, 1985), represents a promising candidate. Processing speed may dictate how quickly numbers can be counted. With slower processing, the interval for deriving counted answers and for pairing a problem stem with its answer in working memory increases; this creates the possibility that decay sets in before completing the computational sequence. Bull and Johnston (1997) found that processing speed was the best predictor of computational competence among 7-year-olds, subsuming all of the variance accounted for by long- and short-term memory, even with reading performance controlled. More recently, Hecht, Torgesen, Wagner, and Rashotte (2001) provided corroborating data on the importance of processing speed as a correlate of computational skill while controlling for vocabulary knowledge.

As for problem solving, prior work examining which cognitive processes mediate arithmetic word problems has focused heavily on working memory, probably because research (e.g., Hitch & McAuley, 1991; Siegel & Ryan, 1989) shows that children with learning disabilities experience concurrent difficulty with working memory (e.g., Siegel & Ryan, 1989; Swanson, Ashbaker, & Sachse-Lee, 1996) and mathematical problem solving (e.g., L. P. Case, Harris, & Graham, 1992; Swanson, 1993). Also, theoretical frameworks (e.g., Kintsch & Greeno, 1985; Mayer, 1992) posit that word problems involve construction of a problem model, which appears to require working-memory capacity. For example, according to Kintsch and Greeno, when people solve word problems, new sets are formed on-line as the story is processed. When a proposition that triggers a set-building strategy is completed, the appropriate set is formed and the relevant propositions are assigned places in the schema. As new sets are formed, previous sets that had been active in the memory buffer are displaced.

In line with theoretical models implicating working memory, the literature provides support for its importance. For example, Passolunghi and Siegel (2001) found that 9-year-olds, characterized as good or poor problem solvers, differed on working-memory tasks. Other researchers have found corroborating evidence using similar methods (e.g., LeBlanc & Weber-Russell, 1996; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001). At the same time, other studies have raised questions about the robustness of the relation. For example, among typically developing third and fourth graders, Swanson, Cooney, and Brock (1993) found only a weak relation between working memory and problem solution accuracy, and this relation disappeared once reading comprehension was considered. The other leading candidates are attentive behavior, nonverbal problem solving, language ability, reading skill, and concept formation.

In studies involving attention, most work has focused on the inhibition of irrelevant stimuli, with mixed results. Passolunghi and colleagues ran a series of studies suggesting the importance of inhibition. For example, comparing good and poor problem solvers, Passolunghi, Cornoldi, and De Liberto (1999) found comparable storage capacity with inefficiencies of inhibition (i.e., poor problem solvers remembered less relevant but more irrelevant information in math problems). In contrast, Swanson and Beebe-Frankenberger (2004) and Swanson (2006) found no evidence that inhibition contributed to problem-solving skill. Research has, however, rarely studied the role of attention more broadly. An exception is Fuchs et al. (2005), who found that a teacher rating scale of attentive behavior predicted the development of first-grade skill with word problems.

Nonverbal problem solving, or the ability to complete patterns presented visually, has been identified as a unique predictor in the development of problem-solving skill across first grade (Fuchs et al., 2005), a finding corroborated by Agness and McLone (1987). This is not surprising, because word problems, in which the problem narrative poses a question entailing relationships between numbers, appear to require conceptual representations. Language ability is also important to consider given the obvious need to process linguistic information when building a problem representation of an arithmetic word problem. In fact, Jordan, Levine, and Huttenlocher (1995) documented the importance of language ability when they showed that kindergarten and first-grade language-impaired children (receptive vocabulary and grammatic closure $<$ 30th percentile) performed significantly lower than nonimpaired peers on arithmetic word problems. Finally, it is hard to ignore the possibility that reading skill may underlie skill in problem solving. Reading is transparently involved, even when problems are read aloud to children, because reading skill provides continuing access to the written problem narrative after the adult reading has been completed. This potentially reduces the load on working memory and thereby facilitates solution accuracy. Swanson (2006) recently identified reading as a predictor of computational as well as problem-solving skill.

Method

Participants

The data described in this paper were collected as part of a prospective 4-year study assessing the effects of mathematical problem-solving instruction and examining the developmental course and cognitive predictors of mathematical problem solving. The data in the present article were collected with the first-, second-, and third-year cohorts at the first assessment wave, sampling from 1,958 students in 89 third-grade classrooms in 10 Title 1 schools and 3 non-Title 1 schools in a southeastern metropolitan school district.

The sampling process was designed to yield a representative sample. That is, from these 1,958 students, we randomly sampled 990 students for participation, blocking within classroom and within three strata: (a) 25% of students with scores 1 *SD* below the mean of the entire distribution on the Test of Computational Fluency (see *Math Measures*); (b) 50% of students with scores within 1 *SD* of the mean of the entire distribution on the Test of Computational Fluency; and (c) 25% of students with scores 1 *SD* above the mean of the entire distribution on the Test of Computational Fluency. Of these 990 students, we had complete data for the variables reported in the present study on 924 children, who were the basis for the present report. As measured on the two-subtest Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999), IQ averaged 97.68 ($SD = 14.26$). Normal curve equivalent scores on the TerraNova (CTB/McGraw-Hill, 1997), administered the previous spring by the school district, averaged 55.40 ($SD = 16.72$) for the reading composite and 55.75 ($SD = 20.15$) for the mathematics composite. Standard scores on the Woodcock–Johnson III Tests of Achievement (WJ-III; Woodcock, McGrew, & Mather, 2001) Applied Problems averaged 102.25 ($SD = 13.51$), and standard scores on the Woodcock Reading Mastery Tests—Revised Word Identification (Woodcock, 1998) averaged 101.05 ($SD = 10.05$). Of the 924 students, 437 (47.3%) were male, and 499 (54.0%) received subsidized lunch. Ethnicity was distributed as follows: 395 (42.5%) African American, 363 (39.3%) European American, 94 (10.2%) Hispanic, 17 (1.8%) Kurdish, and 55 (6.0%) other. Schools had identified 73 students (7.9%) as having a disability (i.e., learning disability, speech impairment, language impairment, attention-deficit/hyperactivity disorder, health impairment, or emotional behavioral disorder).

Procedure

In this report, we describe only the subset of measures on which we report data. The math measures were administered in large-group arrangement in September of third grade during three sessions each lasting 30 to 60 min. These large-group sessions included three tests of computational skill (Addition Fact Fluency, Subtraction Fact Fluency, and Test of Computational Fluency) and three tests of word problem skill (Simple Word Problems, Algorithmic Word Problems, and Complex Word Problems). Measures of eight (of the nine) cognitive dimensions were administered individually in September and October of third grade during two 45-min sessions: Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, WASI Vocabulary, WJ-III Retrieval Fluency, WJ-III Concept Formation, WASI Matrix Reasoning, Working Memory Test Battery for Children Listening Recall, WJ-III Numbers Reversed, Woodcock Reading Mastery Tests–Revised Word Identification, and WJ-III Visual Matching. Tests were administered by trained examiners, each of whom had demonstrated 100% accuracy during mock administrations. All individual sessions were audiotaped, and 19.7% of tapes, distributed equally across testers, were selected randomly for accuracy checks by an independent scorer. Agreement was between 98.7% and 99.9%. In October, classroom teachers completed the Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior (SWAN) Rating Scale, the ninth cognitive dimension, on each student.

Math Measures

Addition and subtraction fact fluency. The Grade 3 Math Battery (Fuchs, Hamlett, & Powell, 2003) incorporates two math fact retrieval subtests. Addition Fact Fluency comprises 25 addition fact problems with answers from 0 to 12 and with addends from 0 to 9. Problems are presented horizontally on one page. Students have 1 min to write answers. The score is the number of correct answers. Agreement, calculated on 20% of protocols by two independent scorers, was 99.8%. For the representative sample, coefficient alpha was .91.; criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997) Total Math score was .53 for the 844 students for whom we had TerraNova scores. Subtraction Fact Fluency comprises 25 subtraction fact problems with answers from 0 to 12 and with minuends/subtrahends from 0 to 18. Problems are presented horizontally on one page. Students have 1 min to write answers. The score is the number of correct answers. Agreement, calculated on 20% of protocols by two independent scorers, was 98.5%. For the representative sample, coefficient alpha was .92, and criterion validity with the previous spring's TerraNova Total Math score was .51 for the 844 students for whom we had TerraNova scores.

Procedural computation. The Test of Computational Fluency (Fuchs, Hamlett, & Fuchs, 1990) is a one-page test displaying 25 items that sample the typical second-grade computation curriculum, including adding and subtracting number combinations and algorithmic computation. Students have 3 min to complete as many answers as possible. The score is the number of correct responses. Staff entered responses into a computerized scoring program on an item-by-item basis, with 15% of tests reentered by an independent scorer. Data-entry agreement was 99.6%. For the representative sample, coefficient alpha was .94, and criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997) Total Math score was .60 for the 844 students for whom we had TerraNova scores.

Simple arithmetic word problems. Following Jordan and Hanich (2000; adapted from Carpenter & Moser, 1984; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983), Story Problems comprises 14 one-step word problems that express change, combine, compare, and equalize relationships among numbers and require sums or subtrahends of 9 or less for solution. The tester reads each item aloud while students follow along on their own copies of the problems. Students have 30 s to respond to each item before the tester moves to the next one, and students can ask for rereading(s) as needed. The score is the number of correct answers. A second scorer independently rescored 20% of protocols, with agreement of 99.9%. For the representative sample, coefficient alpha was .86, and criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997) Total Math score was .62 for the 844 students for whom we had TerraNova scores.

Algorithmic word problems. Algorithmic Word Problems (Fuchs et al., 2003) comprises 10 word problems, each of which requires one to four steps. The measure samples four problem types, asking students to (a) apply step-up functions, (b) add multiple quantities of items each with different prices, (c) find half, or (d) sum a quantity derived from a pictograph with another addend. The tester reads each item aloud while students follow along on their own copies of the problems. The tester progresses to the next problem when all but one or two students have their pencils down, indicating they are finished. Students can ask for rereading(s) as needed. The maximum score is 44. For the representative sample, Cronbach's alpha was .85, and criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997)

Total Math score was .58 for the 844 students for whom we had TerraNova scores. Interscorer agreement, computed on 20% of protocols by two independent scorers, was .984.

Complex word problems. Complex Word Problems (Fuchs et al., 2003) comprises nine problems representing the same four problem types as the algorithmic word problems within more complex contexts: (a) adding multiple quantities of items with different prices, with information presented in bulleted format and with a selection response format; (b) adding multiple quantities of items with different prices, also asking for money left at the end; (c) a step-up function problem with irrelevant information; (d) a step-up function that requires students to compare the prices of two packaging options; (e) a half problem using the words *share equally* instead of *half*; (f) a pictograph/adding problem asking for money left at the end; (g) a pictograph/adding problem comparing two quantities; (h) a problem with irrelevant information that combines multiple quantities with different prices and pictograph/adding; and (i) a problem with irrelevant information that combines multiple quantities with different prices and a step-up function. The tester reads each item aloud while students follow along on their own copies of the problems. The tester progresses to the next item when all but one or two students have their pencils down, indicating they are finished. Students can ask for rereading(s) as needed. The maximum score is 79. For the representative sample, Cronbach's alpha was .88, and criterion validity with the previous spring's TerraNova (CTB/McGraw-Hill, 1997) Total Math Score was .55 for the 844 students for whom we had TerraNova scores. Interscorer agreement, computed on 20% of protocols by two independent, blind scorers, was .983.

Cognitive Dimensions

Language. Using three measures of language skill, we used a principal components factor analysis to create a weighted composite variable of language. Test of Language Development–Primary Grammatic Closure (Newcomer & Hammill, 1988) measures the ability to recognize, understand, and use English morphological forms. The examiner reads 30 sentences, one at a time. Each sentence has a missing word, and examinees earn 1 point for each sentence correctly completed. As reported by the test developers, reliability is .88 for 8-year-olds; the correlation with the Illinois Test of Psycholinguistic Ability Grammatic Closure is .88 for 8-year-olds. Coefficient alpha on the representative sample was .76. The Woodcock Diagnostic Reading Battery Listening Comprehension (Woodcock, 1997) measures the ability to understand sentences or passages. For 38 items, students supply the word missing from the end of each sentence or passage. The test begins with simple verbal analogies and associations and progresses to comprehension involving the ability to discern implications. Testing is discontinued after six consecutive errors. The score is the number of correct responses. As reported by the test developers, reliability is .80 at ages 5 to 18; the correlation with the Woodcock–Johnson Psycho-Educational Battery—Revised (Woodcock & Johnson, 1989) is .73. Coefficient alpha on the representative sample was .81. WASI Vocabulary (Wechsler, 1999) measures expressive vocabulary, verbal knowledge, and foundation of information with 42 items. The first four items present pictures; the student identifies the object in the picture. For remaining items, the tester says a word that the student defines. Responses are awarded a score 0, 1, or 2 depending on quality. Testing is discontinued after five consecutive scores of 0. The score is the total number of points. As reported by Zhu (1999), split-half reliability is .86 to .87 at ages 6 to 7; the correlation with the Wechsler Intelligence Scale for Children–III Full Scale IQ is .72. Coefficient alpha on the representative sample was .78.

Semantic retrieval fluency. WJ-III Retrieval Fluency (Woodcock et al., 2001) asks examinees to recall related items, within categories, for 1 min per category. Examinees earn credit for each nonduplicated answer. As reported by the test developers, reliability is .78 for 8-year-olds.

Concept formation. WJ-III Concept Formation (Woodcock et al., 2001) asks examinees to identify the rules for concepts when shown illustrations of instances and noninstances of the concept. Examinees earn credit by correctly identifying the rule that governs each concept. Cutoff points determine the ceiling. The score is the number of correct responses. As reported by the test developers, reliability is .93 for 8-year-olds. Coefficient alpha on the representative sample was .82.

Nonverbal problem solving. WASI Matrix Reasoning (Wechsler, 1999) measures nonverbal reasoning with four types of tasks: pattern completion, classification, analogy, and serial reasoning. Examinees look at a matrix from which a section is missing and complete the matrix by saying the number of or pointing to one of five response options. Examinees earn points by identifying the correct missing piece of the matrix. Testing is discontinued after four errors on five consecutive items or after four consecutive errors. The score is the number of correct responses. As reported by the test developer, reliability is .94 for 8-year-olds; the correlation with the Wechsler Intelligence Scale for Children–III Full Scale IQ is .66. Coefficient alpha on the representative sample was .76.

Working memory. With the Working Memory Test Battery for Children Listening Recall (Pickering & Gathercole, 2001), a measure of verbal working memory, the tester says a series of short sentences, only some of which make sense. The student indicates whether each sentence is true or false. After all sentences in a trial (i.e., one to six sentences) are heard and determined to be true or false, the student recalls the final word of each sentence in the order presented. The student earns 1 point for each sequence of final words recalled correctly in the right order, and the score is the total of correct sequences. Testing is discontinued when the student makes three or more errors in any block of items. As reported by Pickering and Gathercole, test–retest reliability is .93. Coefficient alpha on the representative sample was .72. With WJ-III Numbers Reversed (Woodcock et al., 2001), a measure of numerical working memory, the tester says a string of random numbers; the student says the series backwards. Item difficulty increases as more numbers are added to the series. Students earn credit by repeating the numbers correctly in the opposite order. As reported by the test developers, reliability is .86 for 8-year-olds. Coefficient alpha on the representative sample was .84.

Word identification skill. The Woodcock Reading Mastery Tests—Revised Word Identification (Woodcock, 1998) measures real-word reading ability with 100 words arranged in order of difficulty. Students read words aloud. Testing is discontinued after six consecutive errors at the end of a page. The score is the number of correct items. As reported by Woodcock, split-half reliability is .98. Coefficient alpha on the representative sample was .87.

Attentive behavior. The SWAN is an 18-item teacher rating scale (www.adhd.net). Items from the *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.; American Psychiatric Association, 1994) criteria for attention-deficit/hyperactivity disorder are included for inattention (largely distractibility; Items 1–9) and hyperactivity/impulsivity (Items 10–18). Items are rated on a scale of 1 to 7 (1 = *far below*, 2 = *below*, 3 = *slightly below*, 4 = *average*, 5 = *slightly above*, 6 = *above*, 7 = *far above*). In the present study, we report data for the inattentive behavior subscale. Using the nine relevant items, we used a principal components factor analysis to create a weighted composite variable of attentive behavior, or the ability to maintain focus. (Because the principal components factor analysis yielded only one factor, no rotation was necessary.) The SWAN has been shown to correlate well with other dimensional assessments of behavior related to inattention (www.adhd.net). Coefficient alpha on the representative sample was .96.

Processing speed. WJ-III Visual Matching (Woodcock et al., 2001) measures processing speed by asking examinees to locate and circle two identical numbers in rows of six numbers. Examinees have 3 min to complete 60 rows and earn credit by correctly circling the matching numbers in each row. As reported by the test developer, reliability is .91 for 8-year-olds.

Data Analysis and Results

Variable Transformations

In Table 1, we show means, standard deviations, and correlations for the representative sample of 924 students on computation, problem solving, and nine cognitive dimensions (language, semantic retrieval fluency, concept formation, matrix reasoning, verbal working memory, numerical working memory, word identification, attentive behavior, and processing speed). Based on the entire sample of 924 students, we transformed raw scores for each of the three computation and each of the four problem solving measures to z scores ($M = 0.00$, $SD = 1.00$). For dimensions with more than one indicator (computation, problem solving, language, and attention), we used SAS PROC FACTOR to estimate factor scores, using squared multiple correlations as the communalities.

Preliminary Analyses: Screening for Outliers and Nonlinearity and Regressions on the Representative Sample

To identify potential outliers, we plotted the bivariate relations of both problem solving and computation with each cognitive variable. We identified five outlier values whose cases we eliminated from further analyses. Then, to investigate the shape of the relations, we examined the bivariate relations between each math outcome and each cognitive variable. We used linear and quadratic forms of each predictor to investigate the functional relation. Any significant quadratic relations between cognitive dimension and math outcome were marked for inclusion in the full regression analysis. We found significant (although not substantial) nonlinearity between problem solving and computation when predicting computation from problem solving (but not when predicting problem solving from computation). So, in the full regression predicting computation from problem solving, we included the quadratic term for problem solving.

We then completed regression analyses to examine the relation of each math outcome with the nine cognitive dimensions using the entire sample. When the quadratic relation between a cognitive dimension and a math outcome was significant (see above), we retained both the linear and quadratic relations within the full model. (We note that significant nonlinear relations between the cognitive variables and the math outcomes were trivial.) For each math outcome, we also investigated the interaction of the cognitive variables with the math outcome variable we were controlling. For example, when predicting computation, we entered problem solving, all nine cognitive variables, any significant quadratic relations (including the quadratic relation for problem solving), and the product vectors of each cognitive variable with problem solving. We did this to determine whether the relation of the cognitive variables with computation was consistent over the range of problem solving. We found that the block of interaction vectors did not add significantly for either math outcome and deleted the product vectors from the model. Finally, for each math outcome, we ran a pruned model, the variables for which neither the linear nor quadratic relation in the full regression model was significant. In Table 2, we present the results of the full and pruned models predicting computation performance. In Table 3, we present findings for the prediction of problem-solving performance. As shown, after we controlled for problem-solving skill in the prediction of computational skill, significant cognitive predictors were word identification, attentive behavior, processing speed, as well as the quadratic relation for numerical working memory. By contrast, after we controlled for computational skill in the prediction of problem-solving skill, significant cognitive predictors were language, concept formation, matrix reasoning, numerical working memory, as well as the quadratic relation for language and attentive behavior.

Difficulty Status Group Formation

Because the major focus of the present study was to extend understanding about difficulty in computation versus problem solving and because the regressions only informed about the effect of each variable when the other variables are controlled at their means, it was important to examine performance specifically at the lower ranges of performance on the math outcomes. To establish extreme groups impaired on computation, problem solving, or both, we designated math difficulty in the following manner. Any student who scored above the 40th percentile on the problem-solving factor score and above the 40th percentile on the computation factor score was designated as having no difficulty (ND). Any student who scored below the 15th percentile on the computation factor score but above the 40th percentile on the problem-solving factor score was designated as having computational difficulty (CD). Any student who scored below the 15th percentile on the problem-solving factor score but above the 40th percentile on the computation factor score was designated as having problem-solving difficulty (PD). Any student who scored below the 15th percentile on the problem-solving factor score and below the 15th percentile on the computation factor score was designated as having computation and problem-solving difficulty (CPD). This placed 372 students in the buffer zone (i.e., scoring between the 16th and 39th percentiles on either or both math outcome) and

Table 1

Means, Standard Deviations, and Correlations on Representative Sample (n = 924)

	Raw		Transformed		Correlations															
Variable	*M*	*SD*	*M*	*SD*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Basic facts +[a]	12.11	4.99	0.00	1.00	—															
2. Basic facts −[b]	7.00	5.07	0.00	1.00	.59	—														
3. Computation[c]	12.37	6.15	0.00	1.00	.71	.68	—													
4. Simple PS[d]	10.01	3.44	0.00	1.00	.39	.40	.47	—												
5. Algorithmic PS[e]	7.70	5.99	0.00	1.00	.40	.44	.49	.54	—											
6. Complex PS[f]	6.13	6.05	0.00	1.00	.33	.37	.45	.46	.63	—										
7. Real-world PS	9.47	9.88	0.00	1.00	.30	.35	.41	.43	.54	.47	—									
8. Language[g]			0.00	1.00	.23	.26	.33	.53	.46	.43	.47	—								
Grammatic	18.76	6.74	85.59	11.21																
Listening	21.05	4.35	96.48	18.27																
Vocabulary	27.89	6.53	47.24	10.14																
9. Semantic retrieval[h]	496.77	3.64	93.23	14.07	.24	.21	.25	.27	.27	.23	.25	.42	—							
10. Concept formation[i]	15.90	7.19	93.04	13.60	.29	.30	.35	.48	.45	.44	.44	.51	.27	—						
11. Matrix reasoning[j]	15.85	6.41	49.21	11.10	.21	.26	.29	.39	.40	.37	.35	.36	.20	.38	—					
12. Verbal working memory[k]	9.91	3.22	92.37	17.45	.22	.24	.24	.38	.34	.32	.31	.48	.23	.41	.30	—				
13. Numerical working memory[l]	9.39	2.80	95.89	14.05	.23	.28	.24	.32	.33	.26	.25	.27	.20	.29	.32	.36	—			
14. Word ID[m]	57.17	9.87	101.05	10.05	.30	.33	.37	.43	.39	.34	.40	.53	.24	.36	.30	.38	.34	—		
15. Attention[n]	37.02	12.86	0.00	1.00	.38	.39	.47	.50	.50	.44	.47	.45	.24	.41	.36	.33	.33	.51	—	
16. Processing speed[o]	30.74	5.63	98.27	15.48	.43	.37	.47	.32	.32	.30	.32	.24	.32	.30	.29	.23	.25	.26	.41	—

Note. PS = problem solving.
[a] Addition Fact Fluency. [b] Subtraction Fact Fluency. [c] Test of Computational Fluency. [d] Simple Word Problems. [e] Algorithmic Word Problems. [f] Complex Word Problems. [g] A factor score across the Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [h] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency (W score). [i] WJ–III Concept Formation. [j] WASI Matrix Reasoning. [k] Working Memory Test Battery for Children Listening Recall. [l] WJ–III Numbers Reversed. [m] Woodcock Reading Mastery Tests—Revised Word Identification. [n] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [o] WJ–III Visual Matching.

Table 2
Full and Pruned Regression Models Predicting Computation[a] Performance (n = 919)

Predictor	Full			Pruned		
	B	*SE*	*t*	*B*	*SE*	*t*
Intercept	−0.03	0.05	0.62	−0.01	0.05	0.30
Problem solving[b]	0.45	0.04	11.18***	0.44	0.04	11.41***
Language[c]	−0.08	0.04	−2.13*	−0.08	0.03	−2.37*
Semantic retrieval[d]	0.03	0.03	0.99			
Concept formation[e]	−0.01	0.03	−0.33			
Matrix reasoning[f]	−0.03	0.03	−0.88			
Verbal working memory[g]	−0.04	0.03	−1.24			
Numerical working memory[h]	−0.01	0.03	−0.17	−0.02	0.03	−0.62
Word ID[i]	0.11	0.03	3.15*	0.10	0.03	3.00***
Attention[j]	0.12	0.03	3.50***	0.11	0.03	3.39***
Processing speed[k]	0.25	0.03	8.22***	0.26	0.03	8.90***
Problem solving-Q	0.04	0.02	1.87	−0.04	0.02	−1.96
Language-Q	−0.02	0.02	−0.87			
Semantic retrieval-Q	0.01	0.02	0.55			
Matrix reasoning-Q	0.00	0.03	0.07			
Numerical working memory-Q	0.05	0.02	2.77*	0.05	0.02	2.95*
Processing speed-Q	0.03	0.02	1.63	0.03	0.02	1.63

Note. Full, $F(16, 902) = 42.22$, $p < .001$, $R^2 = .43$. Pruned, $F(8, 910) = 83.54$, $p < .001$, $R^2 = .42$. Q indicates the quadratic term.
[a] Addition Fact Fluency, Subtraction Fact Fluency, Test of Computational Fluency. [b] Simple Word Problems, Algorithmic Word Problems, and Complex Word Problems. [c] Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [d] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency (W score). [e] WJ–III Concept Formation. [f] WASI Matrix Reasoning. [g] Working Memory Test Battery for Children Listening Recall. [h] WJ–III Numbers Reversed. [i] Woodcock Reading Mastery Tests—Revised Word Identification. [j] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [k] WJ–III Visual Matching.
* $p < .05$. *** $p < .001$.

resulted in 415 ND, 35 CD, 33 PD, and 64 CPD. All remaining analyses were conducted on this subset of 547 students classified as ND, CD, PD, or CPD (i.e., not in the buffer zone). Because academic performance occurs on a continuum, cutoffs for denoting difficulty or lack thereof are necessarily arbitrary, as is the designation of learning disability in the schools. The 40th percentile is used commonly in the research literature as a cut-point for designating lack of difficulty. Cutoffs for designating difficulty or disability vary more in the literature. We selected the 15th percentile because it is useful for understanding disability as practiced in the schools. In Table 4, we show standard score means and standard deviations for the nationally normed math variables we had available and for the cognitive dimensions on which we had nationally normed data.

Sociodemographic Comparisons

In Table 5, we show frequencies for gender, subsidized lunch, and ethnicity by difficulty status. For each variable, we partitioned the contingency tables to run a series of chi-square tests for determining (a) whether ND differed from students with difficulty (ND vs. CD/PD/CPD), (b) whether CD differed from either variant of problem-solving difficulty (CD vs. PD/CPD), and (c) whether the two variants of problem-solving difficulty differed (PD vs. CPD). To control for multiple comparisons, we adjusted the value of alpha by the number of contrasts on each index (i.e., tested at $p = .05 / 3 = .0167$). There was no significant relation between gender and math difficulty status: $\chi^2(1, N = 547) = 0.02$, *ns*; $\chi^2(1, N = 132) = 0.01$, *ns*; and $\chi^2(1, N = 97) = 0.50$, *ns*, for the three contrasts, respectively. For the proportion of subsidized lunch, however, an interesting set of relations emerged. As might be anticipated, students without math difficulty were significantly less likely to receive subsidized lunch than were students with math difficulty, $\chi^2(1, N = 547) = 6.82$, $p = .009$. More interesting is the fact that students with computational difficulty were significantly less likely to receive subsidized lunch than were students with problem-solving difficulty (PD or CPD), $\chi^2(1, N = 132) = 7.01$, $p = .008$, even though students with PD alone and those with CPD were comparably likely to receive subsidized lunch, $\chi^2(1, N = 97) = 0.46$, $p = .50$. The same pattern emerged for ethnicity. The distribution of ethnicity differed between students with and without difficulty, $\chi^2(5, N = 547) = 33.13$, $p < .001$. The distribution of ethnicity also differed for students with CD versus those with problem-solving difficulty (PD or CPD), $\chi^2(5, N = 132) = 22.89$, $p < .001$, even though students with the two variants of problem-solving difficulty (PD vs. CPD) were similarly distributed across the ethnicity categories, $\chi^2(5, N = 97) = 7.71$, $p = .17$.

Group Comparisons on Cognitive Dimensions

Table 6 presents *z*-score means and standard deviations, by difficulty status, on the computation and problem-solving factor scores and on language (factor score), semantic retrieval fluency, concept formation, matrix reasoning, verbal working memory,

Table 3
Full and Pruned Regression Models Predicting Problem-Solving[a] Performance (n = 919)

Predictor	Full			Pruned		
	B	*SE*	*t*	*B*	*SE*	*t*
Intercept	−0.05	0.04	1.25	−0.07	0.03	2.32*
Computation[b]	0.31	0.03	11.58***	0.29	0.03	11.77***
Language[c]	0.23	0.03	7.29***	0.24	0.03	9.05***
Semantic retrieval[d]	−0.01	0.03	−0.25			
Concept formation[e]	0.16	0.03	5.84***	0.16	0.03	5.95***
Matrix reasoning[f]	0.13	0.03	5.04***	0.14	0.02	5.44***
Verbal working memory[g]	0.03	0.03	1.21			
Numerical working memory[h]	0.05	0.03	1.92	0.05	0.02	2.14*
Word ID[i]	0.02	0.03	0.71			
Attention[j]	0.18	0.03	6.22***	0.17	0.03	6.36***
Processing speed[k]	−0.03	0.03	−1.26			
Language-Q	0.03	0.02	1.90	0.03	0.02	2.00*
Concept formation-Q	0.01	0.02	0.90			
Matrix reasoning-Q	−0.02	0.02	−0.72			
Numerical working memory-Q	0.03	0.02	1.89			
Attention-Q	0.04	0.02	2.32*	0.04	0.02	2.18*

Note. Full, $F(15, 903) = 81.96$, $p < .001$, $R^2 = .58$. Pruned, $F(8, 910) = 152.55$, $p < .001$, $R^2 = .57$. Q indicates the quadratic term.
[a] Simple Word Problems, Algorithmic Word Problems, and Complex Word Problems. [b] Addition Fact Fluency, Subtraction Fact Fluency, Test of Computational Fluency. [c] Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [d] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency (W score). [e] WJ–III Concept Formation. [f] WASI Matrix Reasoning. [g] Working Memory Test Battery for Children Listening Recall. [h] WJ–III Numbers Reversed. [i] Woodcock Reading Mastery Tests—Revised Word Identification. [j] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [k] WJ–III Visual Matching.
* $p < .05$. *** $p < .001$.

numerical working memory, word identification, attentive behavior (factor score), and processing speed.

Preliminary analysis of clustering effects. Before choosing an analytic model, we examined the extent of clustering of children in classes and schools. With clustering, the independence assumption of analysis of variance may be violated, possibly leading to spurious significance levels (Raudenbush & Bryk, 2002). Strong clustering would necessitate a multilevel model rather than a repeated measures approach. Variance components were estimated with SAS PROC MIXED (Littell, Milliken, Stroup, Wolfinger, & Schabenger, 2006). The resulting intraclass correlations showed how much of the total variance in the variables of interest (i.e., the nine cognitive dimensions) was explained by the clustering of children in classroom and school. The effect for school explained 0% of the variance, and the effect for classroom (nested in school) explained less than 5%. Raudenbush and Liu's (2000) ad hoc standards deem 5%, 10%, and 15% as small, medium, and large.

Overall analysis. Because the intraclass correlations in this database were small to nonexistent, we conducted an initial profile analysis using a two-way analysis of variance. The between-subjects factor was math difficulty status (ND vs. CD vs. PD vs. CPD); the within-subjects factor was cognitive dimension (*z* score on language vs. semantic retrieval fluency vs. concept formation vs. matrix reasoning vs. verbal working memory vs. numerical working memory vs. word identification vs. attentive behavior vs. processing speed). The interaction between math difficulty status and cognitive dimension was significant, $F(24, 4337) = 4.67$, $p < .0001$.

To help interpret the interaction between math difficulty status and cognitive dimension, we plotted *z* scores on the nine cognitive dimensions for each of the four difficulty status groups (see Figure 1). As shown, the ND and CD groups performed at a higher level than did the PD and CPD groups. In addition, the difficulty status by cognitive dimension interaction appeared to be evident in the variations in the profile shape across the nine cognitive dimensions as a function of the difficulty status group.

Univariate follow-ups to the interaction. Because the elevation effects in Figure 1 were striking and because many studies have compared univariate differences among math difficulty groups, we initially conducted follow-up tests using Fisher's least significant difference, with math difficulty status as the factor (Seaman, Levin, & Serlin, 1991). Alpha was adjusted for six contrasts per measure, comparing each difficulty status group to all others ($p = .05 / 6 = .008$). Results of these follow-up tests appear under the labels on the horizontal axis in Figure 1. Symbols for the four groups (see key) appear under each cognitive dimension. Groups joined by a horizontal line were not significantly different from one another. The univariate test on each cognitive dimension was significant ($p < .0001$). To help evaluate the magnitude of the univariate differences, we computed effect sizes for each variable (see Table 7) by dividing the difference between group means by the standard deviation pooled across the two groups in the comparison (Hedges & Olkin, 1985). The complicated pattern of differences that emerged is addressed in the Discussion.

Profile analysis of shape effects. The interpretation of a pattern like the one shown in Figure 1 is complicated. The univariate

Table 4
Standard Score Performance by Difficulty Status

Variable	Difficulty status							
	ND (n = 415)		CD (n = 35)		PD (n = 33)		CPD (n = 64)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
TerraNova	58.80	14.97	22.06	14.34	24.83	15.16	17.00	15.60
Applied problems[a]	109.65	12.80	100.31	11.15	93.30	7.72	88.17	9.00
Grammatic[b]	89.43	11.61	86.14	10.30	78.48	8.15	78.83	6.59
Listening[c]	101.92	18.37	104.31	14.11	84.88	11.71	84.36	12.45
Vocabulary[d]	50.88	9.89	49.49	10.20	40.33	6.64	40.75	7.77
Semantic retrieval[e]	96.94	12.94	93.26	12.92	92.36	12.94	86.00	14.07
Concept formation[f]	98.74	12.03	91.80	13.58	84.82	10.59	81.48	10.95
Matrix reasoning[g]	53.35	10.27	49.06	9.94	42.15	9.63	42.45	9.68
Verbal working memory[h]	97.72	14.59	90.23	15.47	83.27	16.07	83.66	15.62
Numerical working memory[i]	99.73	14.13	95.20	10.43	90.30	12.19	89.08	10.19
Word ID[j]	104.93	10.50	100.69	9.45	94.94	6.97	93.08	6.73
Processing speed[k]	104.35	14.83	92.37	12.76	96.42	17.71	87.56	13.85

Note. TerraNova is normal curve equivalents on the TerraNova. All other scores are standard scores. Standard scores are M = 100, SD = 15, except vocabulary and matrix reasoning, for which M = 50 and SD = 10. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.
[a] Woodcock–Johnson III Tests of Achievement (WJ–III) Applied Problems. Applied Problems assesses a variety of math domains with only a limited number of word problems assessed in the third-grade range. [b] Test of Language Development–Primary Grammatic Closure. [c] Woodcock Diagnostic Reading Battery Listening Comprehension. [d] Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [e] WJ–III Retrieval Fluency. [f] WJ–III Concept Formation. [g] WASI Matrix Reasoning. [h] Working Memory Test Battery for Children Listening Recall. [i] WJ–III Numbers Reversed. [j] Woodcock Reading Mastery Tests—Revised Word Identification. [k] WJ–III Visual Matching.

tests do not account for relations among the cognitive dimensions, and they confound level and measure effects. A traditional multivariate analysis of variance determines how a set of measures can be combined into a set of K – 1 univariate composites (discriminant functions), which maximally separates groups (Bernstein, Garbin, & Teng, 1988; Harris, 1975). However, each composite comprises elements that involve differences not only in elevation but also in shape, both of which seem apparent in the profiles shown in Figure 1. By contrast, a multivariate profile analysis accounts for correlations among cognitive dimensions and decomposes the univariate composites into components representing elevation, flatness, and shape. The elevation effects are differences among groups averaged across the dimensions. Flatness effects are differences in dimensions averaged across groups; they indicate whether the profile measures vary or can be represented as a relatively straight line (because we used z scores, we did not anticipate flatness effects). When profiles differ in shape, which is analogous to the Difficulty Status × Dimension interaction already documented, differences among groups vary depending on cognitive dimension, in which case the elevation and flatness components are not interesting.

Thus, we used multivariate profile analysis to conduct four planned contrasts to explore how the profiles of the math difficulty groups differed. In these contrasts, we compared the ND group to the specific computational difficulty group and then in turn compared the specific computational difficulty group to each of the

Table 5
Demographics by Difficulty Status

Demographic variable	Difficulty status							
	ND (n = 415)		CD (n = 35)		PD (n = 33)		CPD (n = 64)	
	n	%	*n*	%	*n*	%	*n*	%
Male	201	48.4	17	48.6	14	42.4	32	50.0
Subsidized lunch[a]	185	23.4	14	45.2	18	66.7	45	73.8
African American	129	31.1	10	28.6	23	70.0	43	67.2
European American	201	48.4	22	62.9	6	18.2	14	21.9
Hispanic	36	8.7	3	8.6	1	3.0	6	9.4
Kurdish	14	3.4	0	0.0	1	3.0	0	0.0
Other	35	8.4	0	0.0	2	6.1	1	1.6

Note. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.
[a] Some schools declined to provide subsidized lunch data, resulting in sample sizes of 363, 31, 27, and 61 for the four difficulty status groups, respectively.

Table 6
Performance by Difficulty Status

Variable	ND (n = 415) *M*	ND *SD*	CD (n = 35) *M*	CD *SD*	PD (n = 33) *M*	PD *SD*	CPD (n = 64) *M*	CPD *SD*
Basic facts +[a]	0.62	0.80	−1.21	0.43	0.40	0.61	−1.44	0.53
Basic facts −[b]	0.62	0.95	−0.95	0.31	0.09	0.74	−1.02	0.45
Computation[c]	0.76	0.79	−1.09	0.20	0.14	0.62	−1.27	0.30
Simple PS[d]	0.76	0.49	0.36	0.53	−1.33	0.54	−1.75	0.59
Algorithmic PS[e]	0.68	0.96	0.00	0.57	−0.98	0.22	−1.04	0.22
Complex PS[f]	0.56	1.10	−0.18	0.55	−0.84	0.22	−0.78	0.22
Language[g]	0.40	0.89	0.32	0.79	−0.85	0.82	−0.81	0.82
Semantic retrieval[h]	0.27	0.93	0.01	0.93	−0.05	0.93	−0.51	1.01
Concept formation[i]	0.42	0.88	−0.09	1.00	−0.60	0.78	−0.85	0.80
Matrix reasoning[j]	0.37	0.92	−0.01	0.90	−0.64	0.87	−0.61	0.87
Verbal working memory[k]	0.31	0.91	−0.16	0.97	−0.59	1.00	−0.57	0.97
Numerical working memory[l]	0.27	1.01	−0.05	0.74	−0.40	0.87	−0.49	0.73
Word ID[m]	0.39	1.04	−0.04	0.94	−0.61	0.69	−0.79	0.67
Attention[n]	0.55	0.90	−0.47	0.94	−0.59	0.72	−1.07	0.53
Processing speed[o]	0.39	0.96	−0.38	0.82	−0.12	1.14	−0.69	0.89

Note. Performance is expressed as z scores in relation to the representative sample of 919. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.
[a] Addition Fact Fluency. [b] Subtraction Fact Fluency. [c] Test of Computational Fluency. [d] Simple Word Problems. [e] Algorithmic Word Problems. [f] Complex Word Problems. [g] A factor score across the Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [h] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency. [i] WJ–III Concept Formation. [j] WASI Matrix Reasoning. [k] Working Memory Test Battery for Children Listening Recall. [l] WJ–III Numbers Reversed. [m] Woodcock Reading Mastery Tests—Revised Word Identification. [n] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [o] WJ–III Visual Matching.

groups involving problem-solving difficulty, thereby reducing the need to compare the ND and problem-solving difficulty groups directly (especially given that the ND group performed at a much higher level than the CPD and PD groups, as shown in Figure 1). We also compared the specific problem-solving difficulty group to the group that manifested both forms of difficulty. To control for Type 1 error (given four planned contrasts), we adjusted the critical value of alpha to .0125 (.05 / 4); however, because the power for each comparison differed, effect sizes (eta-squared) were also considered.

Although separate effects for the elevation, flatness, and shape effects might be explored in a profile analysis approach, the large interaction already documented in the univariate analysis between difficulty status and cognitive dimension mitigated against pursing elevation or flatness effects. Table 8 presents the interaction contrasts exploring the shape effects. As shown, only the contrasts for ND versus CD and for CD versus PD met the critical alpha level. Although the contrasts between PD and CPD and between CD and CPD did not meet the critical alpha level, the effect sizes (both ~0.15) were slightly larger than for the ND versus CD contrast (0.08), the latter having more power. Effect sizes were in the small to medium range.

In interpreting the shape effects, we note that the overall tests for elevation, flatness, and shape are not dependent on the ordering of the dimensions. However, interpretation of individual dimensions may depend on the ordering and the location of pairs of dimensions in the profile. To determine how profile dimensions contribute to the interaction regardless of the ordering of the dimensions, we followed a commonly recommended procedure for the interpretation of profile analysis multivariate analysis of variance: inspection of the canonical structure matrix (Harris, 1975). In following up the shape effect, it is also necessary to remove the effect of elevation (Bernstein et al., 1988; Fletcher et al., 1994). This was done by computing the residuals in a model, which included only the main effects of group and cognitive dimension; the result reduced the elevation of each group to approximately zero. Any variation among group means on the cognitive variables is then due only to the effect of shape.

Figure 2 shows the elevation-adjusted shape profile for each of the four groups along the nine cognitive dimensions. Note that the relation of the groups on the y-axis differs substantially from the relations manifested in Figure 1. This is because the elevation differences that dominated Figure 1 have been removed in Figure 2. In Table 9, we present the canonical structure matrix for each of the four planned contrasts. Within the canonical structure matrix, simple correlations are computed for each variable with the discriminant function representing the multivariate effect for shape, adjusted for elevation (Huberty, 1975). The positive or negative value of the correlations reflects the pattern of mean differences between the contrasted groups. For interpretive purposes, we consider both the magnitude and the direction of these correlations as well as the significance of the univariate test (F

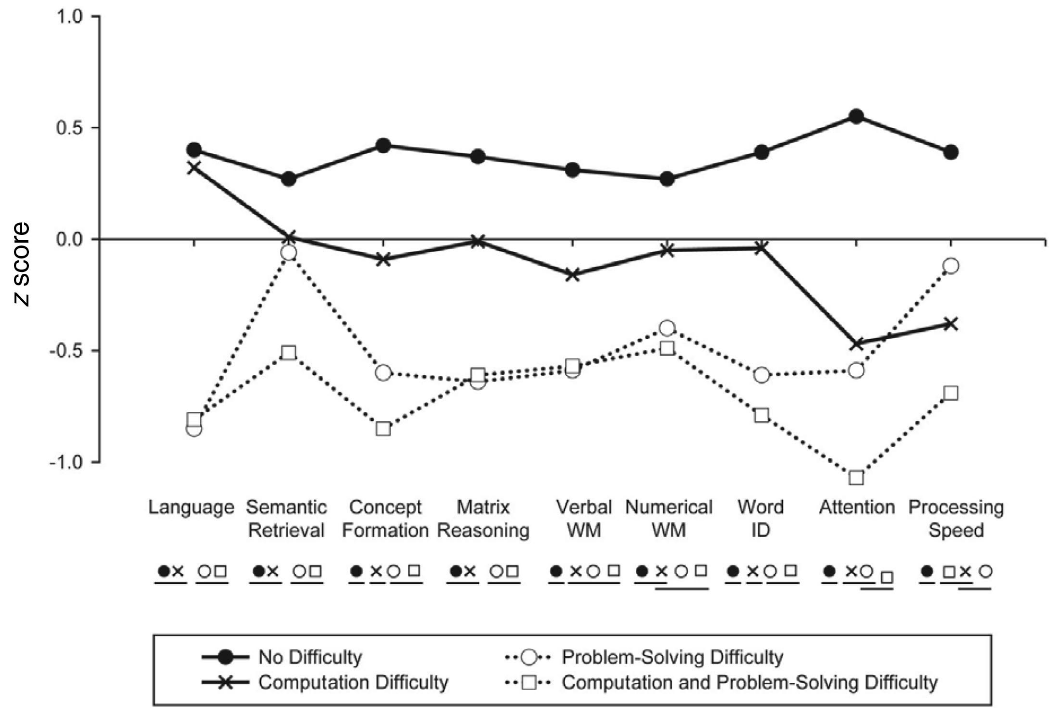

Figure 1. z scores on nine cognitive dimensions by difficulty status. WM = working memory; ID = identification.

value) associated with each cognitive variable for each pair of contrasts (see asterisks in Table 9, where the critical p value has been adjusted to .006 to account for nine univariate dimensions).

As reflected in the canonical structure correlations shown in Table 9, the contrast between the ND and CD groups was accounted for by language (–.44), attentive behavior (.60), and processing speed (.32), which were more heavily weighted than other variables. Accordingly, in Figure 2, the difference in language (positive direction for CD relative to other dimensions), attentive behavior (negative direction for CD relative to other dimensions), and processing speed (negative direction for CD relative to other dimensions) most clearly differentiates the shape of the ND and CD profiles. Among language, attentive behavior, and processing speed, attentive behavior was the most reliable correlate of the shape effect across different methods of interpreting the contributions of the dimensions.

As might be expected, therefore, for the comparison between the PD and CPD groups, Table 9 shows the highest canonical structure coefficients for attentive behavior (.54) and processing speed (.44). In Figure 2, these differences in attentive behavior (negative direction for CPD relative to most other dimensions) and processing speed (positive direction for PD relative to most other dimensions) are striking aspects of the profile and also clearly contrast with the profile for the CPD group, in which attentive behavior was a negative dimension and processing speed is neutral. Again, attentive behavior was the most reliable correlate of the shape effect across different methods of interpreting the contributions of the dimensions.

By contrast, for the comparison between PD and CD, language (–.70) and processing speed (.51) were the variables contributing to the shape effect and reflect the negative direction of language relative to other dimensions for the PD group and the positive direction in language relative to other dimensions for the CD group. In contrast, processing speed was a negative dimension for the CD group but positive for the PD group. Figure 2 shows that semantic retrieval fluency was also positive in the PD group, a likely suppressor effect given the large standardized coefficient for semantic retrieval fluency (.74). Language was the most reliable correlate of the shape effect across different methods of interpreting the contributions of the dimensions. Finally, and in keeping with the contrast between PD and CD, the cognitive dimension accounting for the contrast between the CPD and CD groups was language (–.72). The standardized coefficients showed a similar pattern. Again, this variable moved in opposite directions in the profiles for the two groups (see Figure 2).

Discussion

The purposes of this study were to explore the overlap between difficulty with two aspects of primary-grade mathematical cognition—computation and problem solving—and to examine how demographic and cognitive profiles differ among subgroups with difficulty in one, the other, both, or neither. The goal was to gain insight into whether these domains are shared or distinct aspects of mathematical cognition in extreme groups. This issue is not only theoretically important but also has implications in terms of identifying math disability, as specified in the 2004 reauthorization of the Individuals with Disabilities Education Act, and for designing effective methods for preventing and remediating math difficulty.

Table 7
Effect Sizes (in Absolute Values) for Math Variables and Cognitive Dimensions as a Function of Difficulty Status

	Contrast					
	ND vs.			CD vs.		PD vs.
Variable	CD	PD	CPD	PD	CPD	CPD
Computation[a]	2.80	0.73	3.16	4.83	0.86	5.13
Problem solving[b]	0.89	2.48	2.78	4.13	5.27	0.68
Language[c]	0.09	1.45	1.37	1.44	1.40	0.05
Semantic retrieval[d]	0.28	0.34	0.83	0.06	0.53	0.46
Concept formation[e]	0.57	1.17	1.45	0.60	0.87	0.32
Matrix reasoning[f]	0.39	1.09	1.08	0.70	0.67	0.03
Verbal working memory[g]	0.52	0.98	0.96	0.44	0.42	0.02
Numerical working memory[h]	0.32	0.67	0.78	0.43	0.59	0.12
Word ID[i]	0.42	0.98	1.19	0.70	0.97	0.26
Attention[j]	1.13	1.28	1.64	0.15	0.69	0.80
Processing speed[k]	0.75	0.68	1.46	0.16	0.52	0.58

Note. Performance is expressed as *z* scores in relation to the representative sample of 919. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.
[a] A factor score (Addition Fact Fluency, Subtraction Fact Fluency, Test of Computational Fluency). [b] A factor score (Simple Word Problems, Algorithmic Word Problems, Complex Word Problems). [c] A factor score across the Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [d] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency. [e] WJ–III Concept Formation. [f] WASI Matrix Reasoning. [g] Working Memory Test Battery for Children Listening Recall. [h] WJ–III Numbers Reversed. [i] Woodcock Reading Mastery Tests—Revised Word Identification. [j] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [k] WJ–III Visual Matching.

With respect to overlap, results revealed that difficulty in one domain did not necessarily align with difficulty in the other. This is understandable because correlations between computational and problem-solving skill, as demonstrated elsewhere (e.g., Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004), were only moderate, ranging from .30 to .49. In fact, difficulty occurred in a single math domain as frequently as across math domains. Moreover, specific difficulty was distributed across the two domains with almost identical prevalence.

In a related way, the demographic profiles of the groups also suggest that performance in these two domains of mathematical cognition may be distinct. The demographic profiles of students with specific computational difficulty, in terms of poverty and ethnic background, were more similar to those of students without difficulty than to those of students with problem-solving difficulty or with concurrent difficulty. This suggests that the contextual variables associated with poverty or race exert little effect over the development of computational difficulty. By contrast, students with problem-solving difficulty were significantly poorer and disproportionately more likely to be African American compared to students with specific computational difficulty. This was true regardless of whether problem-solving deficits occurred alone or in combination with computational deficits, with no significant demographic differences between students experiencing specific problem-solving difficulty and those experiencing concurrent problem-solving and computational difficulty. This finding indicates that early or ongoing experience outside of school may account for variance in building a strong foundation for mathematical problem solving.

To identify what kinds of experience outside of school may be key, it is useful to consider analysis of the cognitive dimensions. The regression analyses indicate that a different set of cognitive predictors is associated with the math outcomes, suggesting that these two domains of mathematical cognition may be distinct. However, because the focus of the present study was to extend understanding about difficulty with computation versus problem solving, and because the regressions only inform about the effect of each variable when the other variables are controlled at their means, it is important to examine performance specifically at the lower ranges of performance on the math outcomes.

The univariate profile analyses, which addressed mean differences among difficulty status groups on each cognitive dimension, indicate that language and word identification clearly distinguished problem-solving from computational difficulty. Students with problem-solving difficulty, regardless of whether the problem-solving difficulty occurred alone or in combination with computational difficulty, scored reliably lower than students with neither form of difficulty and lower than those who experienced computational deficits alone. Moreover, students with computational difficulty were statistically indistinguishable from students without difficulty, and students with problem-solving difficulty alone were statistically comparable to students with problem-solving difficulty that occurred in combination with computational difficulty. Two additional variables, concept formation and matrix reasoning, also served to distinguish problem-solving difficulty from computational difficulty (with PD and CPD comparable to each other, and both lower than students with CD and ND, although on these dimensions, students with specific computational difficulty were reliably lower performing than students without difficulty).

It is therefore interesting to consider these abilities in light of the major distinction between mathematical computation and problem solving: the addition of linguistic information that requires individuals to construct a problem model. That is, whereas a computation problem is already set up for solution, a word problem requires students to use text to identify missing information, construct the number sentence, and derive the calculation problem for finding the missing information. This transparent difference would

Table 8
Math Difficulty Status × Cognitive Dimension Interactions for Four Planned Comparison

Contrast	*F*	*df*s	*p*	η^2
ND vs. CD	4.22	8, 441	.0001	.08
PD vs. CPD	1.85	8, 87	.08	.15
CD vs. PD	4.75	8, 59	.0002	.39
CD vs. CPD	2.26	8, 89	.03	.16

Note. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.

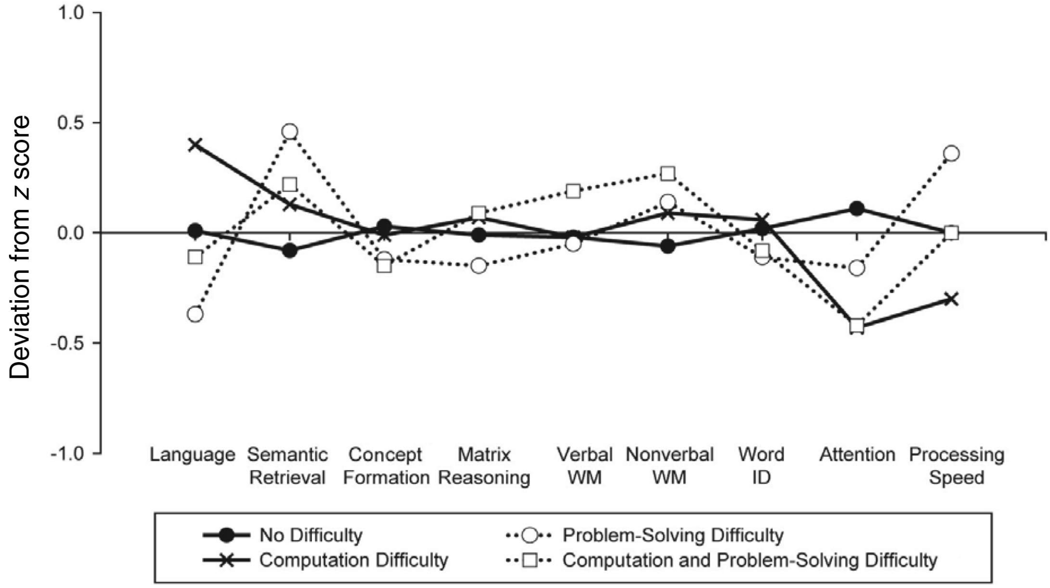

Figure 2. Shape effects by difficulty status. WM = working memory; ID = identification.

seem to alter the nature of the task, and findings corroborate such a hypothesis.

With respect to the contribution of reading skill, although word problems were read aloud to students, with repeated opportunities for rereading whenever students requested, students had the written problem available until they completed it; so, we note that independent, skilled reading may support continuous access to text. In a different way, because the development of word identification skill is facilitated by vocabulary knowledge (cf. Perfetti, 1992), the link between word identification and problem-solving skill suggests that language may play a role in math problem solving. This finding was in fact documented in the univariate analyses. It stands to reason that the ability to make sense of language, as reflected indirectly by word recognition skill and as reflected directly by our language factor score (i.e., grammatic closure, listening comprehension, and vocabulary), should help students cope with narratives in the service of building problem models, and findings corroborate previous work about the role language plays in problem-solving skill (e.g., Fuchs et al., 2006). It also makes sense that nonverbal problem-solving skill, as reflected in concept formation and matrix reasoning, should underlie mathematical problem-solving skill. This corroborates previous work (Fuchs et al., 2005, 2006) and is interpretable because mathematical problem solving requires students not only to build a problem model but also to distinguish relevant from irrelevant

Table 9
Canonical Structure Correlations for Each Cognitive Dimension for the Shape Effect

Variable	ND vs. CD	PD vs. CPD	PD vs. CD	CPD vs. CD
Language[a]	−.44	−.38	−.70*	−.72*
Semantic retrieval[b]	.22	.32	.23	.07
Concept formation[c]	.04	.06	−.08	−.19
Matrix reasoning[d]	−.09	−.30	−.16	.01
Verbal working memory[e]	−.01	−.32	.22	.26
Numerical working memory[f]	−.15	−.19	.04	.26
Word ID[g]	−.05	−.04	−.13	−.20
Attention[h]	.60*	.54*	.21	.01
Processing speed[i]	.32	.44	.51	.38

Note. ND = no difficulty; CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty.
[a] Woodcock Diagnostic Reading Battery Listening Comprehension, Test of Language Development–Primary Grammatic Closure, and Wechsler Abbreviated Scale of Intelligence (WASI) Vocabulary. [b] Woodcock–Johnson III Tests of Achievement (WJ–III) Retrieval Fluency (W score). [c] WJ–III Concept Formation. [d] WASI Matrix Reasoning. [e] Working Memory Test Battery for Children Listening Recall. [f] WJ–III Numbers Reversed. [g] Woodcock Reading Mastery Tests—Revised Word Identification. [h] Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder-Symptoms and Normal-Behavior Rating Scale. [i] WJ–III Visual Matching.
* $p < .006$.

information and to determine how numerical quantities fit into the slots of the problem model (cf. Kintsch & Greeno, 1985).

For the remaining cognitive dimensions, the univariate results were less clear. On verbal and numerical working memory, the performance of the three difficulty groups was indistinguishable. However, on numerical working memory (but not on verbal working memory), students with specific computational difficulty performed comparably to their peers without difficulty.

In terms of working memory, or the capacity to maintain target memory items while processing an additional task (Daneman & Carpenter, 1980), a body of work has established links with computation (Fuchs et al., 2005; Geary et al., 1991; Hitch & McAuley, 1991; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001) and problem solving (e.g., Fuchs et al., 2005; LeBlanc & Weber-Russell, 1996; Passolunghi & Siegel, 2004; Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001). Although other studies have raised questions about the tenability of this association (e.g., Fuchs et al., 2006; Swanson et al., 1993), the univariate results of the present study corroborate a role for working memory, verbal as well as numerical, in both computational and problem-solving difficulty. For computation, students must hold terms and operators in working memory while using various counting strategies to arrive at the correct answer. Over time, repeated associations for the problem stem and its answer, achieved via successful counting, result in representations in long-term memory, which in turn facilitate fluent procedural computation. With respect to word problems, as described by Kintsch and Greeno (1985), when processing a problem narrative, students formulate new sets to construct a problem model. When a proposition that triggers a set-building strategy is completed, the appropriate set is formed and the relevant propositions are assigned their places in the various slots of the set schema. As new sets are formed, previous sets that had been active in the memory buffer are displaced, illustrating the potential importance of working memory. Our univariate analyses lend empirical support for the contribution of working memory to both aspects of mathematical cognition: computation and problem solving. It is important to note, however, that the multivariate profile analyses indicate that working memory is primarily related to elevation (severity), not to shape.

In any case, results of the univariate analyses suggest a pattern of more pervasive cognitive involvement for problem-solving difficulty. Yet, because univariate analyses fail to account for relations among the cognitive dimensions and because in the univariate analyses, elevation and shape are confounded, it is important to consider results of the multivariate profile analyses. At a general level, by explicating the patterns of cognitive abilities associated with computational and problem-solving difficulty, the shape analyses highlight two important notions: (a) that computational difficulty may be distinct from problem-solving difficulty and (b) that the cognitive dimensions associated with performance in a single math domain may also be associated with both computation and problem solving when difficulties occur concurrently. At the same time, the multivariate profile results, which isolate the effects of shape from the effects of elevation, serve to pinpoint more specifically and narrowly which cognitive differences between groups matter. In this way, results demonstrate the need to rely on multivariate, rather than univariate, approaches in the study of mathematics.

Within the multivariate interpretations of the profile analyses, three cognitive dimensions emerged as central to the distinction between computational and problem-solving difficulty. The dominant role of language deficits was substantiated for problem-solving difficulty. Language was the cognitive dimension that served to distinguish the specific problem-solving difficulty group from the specific computational difficulty group and to distinguish the group with concurrent difficulty across problem solving and computation again from the specific computational difficulty group. By contrast, the dominant roles of attentive behavior and processing speed were revealed for computational difficulty, serving to distinguish the specific computational difficulty group from the group without either form of difficulty and to distinguish the group with concurrent difficulty across computation and problem solving from the group with specific problem-solving difficulty.

In the preceding discussion about the univariate analyses, we already considered the role of language in problem solving, but what about the role of attentive behavior and processing speed in computation, which became evident only in the multivariate analyses? Within the univariate analyses, a step-down pattern occurred for attentive behavior, whereby students without difficulty were rated more favorably than the other three groups; students with specific computational difficulty were rated similarly as students with specific problem-solving difficulty but more attentive than students with concurrent difficulty; and students with specific problem-solving difficulty were deemed similarly attentive as students with concurrent difficulty. This ordering, which suggests greater cognitive involvement for problem-solving deficits, is based entirely on elevation differences among groups. By contrast, the multivariate analyses specifically consider the shape (i.e., the profile) by removing the elevation effects. These multivariate profile analyses demonstrate that attentive behavior is implicated in computational but not problem-solving difficulty. Few studies have considered attentive behavior as a predictor of computational skill, but some previous work has suggested its role (e.g., Fuchs et al., 2005, 2006). Moreover, Swanson (2006) recently substantiated the role of inhibitory control, a form of attention, in the development of computational but not problem-solving skill.

At least two explanations seem possible for the role of attentive behavior in computational difficulty. First, attentive behavior may create the opportunity to persevere with the serial execution required for computational math (Luria, 1980) and thereby enhance performance and improve students' responsiveness to instruction. Alternatively, it is possible that teacher ratings of attentive behavior are clouded by students' academic performance and therefore serve as a proxy for achievement rather than indexing attention. Cirino, Ewing-Cobbs, Barnes, Fuchs, and Fletcher (2007) found that although attentive behavior accounts for unique variance in mathematics performance, removing variance due to behavioral ratings of attention does not alter the relations of cognitive measures and mathematical cognition. Present findings, in which attentive behavior was implicated in specific computational difficulty but not specific problem-solving difficulty, reduce the plausibility that teacher ratings simply serve as a proxy for academic achievement and instead provide the basis for hypothesizing that attentive behavior plays a role in computation and for exploring the underlying nature of the relation with alternative measures of attention.

In terms of processing speed, previous work has suggested that processing speed underlies fluency with math facts. For example, Bull and Johnston (1997) found that processing speed subsumed all of the variance in 7-year-olds' arithmetic skill while controlling for word reading ability, item identification, and short-term memory. Processing speed may facilitate counting speed so that as young children gain speed in counting sets to figure sums and differences, problems are successfully paired with their answers in working memory before decay sets in, such that associations in long-term memory are established (e.g., Geary et al., 1991; Lemaire & Siegler, 1995). In addition, Fuchs et al. (2006) demonstrated that processing speed accounted for unique variance in simple arithmetic but not in mathematical problem solving once the relation between processing speed and arithmetic had been accounted for.

In sum, findings lend support for the hypothesis that computation and problem solving may represent distinct domains of mathematical cognition within students at the lower ranges of performance as might be identified with mathematics learning disabilities in the schools. This lends empirical support for the distinction between computational and mathematics problem-solving learning disabilities specified in the 2004 reauthorization of the Individuals with Disabilities Education Act. Results also suggest that poverty and language play critical roles in the development of problem-solving difficulty and that inattentive behavior and poor processing speed may inhibit the development of computational skill. In addition, despite the more substantial math deficits evidenced for students with concurrent difficulty (i.e., an effect size of 0.86 favoring the computational skill of CD over CPD and an effect size of 0.68 favoring the problem-solving skill of PD over CPD), the cognitive deficiencies associated with math performance in a single domain are also apparent when difficulties occur in both domains: for computation, attention and processing speed (as revealed for CD and for CPD); for problem solving, language (as revealed for PD and for CPD). Similarly, poverty or race is associated with problem-solving difficulty, whether it occurs alone or in combination with computational difficulty, and therefore corroborates the relation between language and these sociodemographic variables. Together, these findings suggest that concurrent difficulty with computation and problem solving may not be a unique form of math disability but represents a comorbid association of difficulties in both domains. Additional research should continue to investigate these issues as well as explore the possible role of other cognitive dimensions, including reading comprehension. Matching groups of students on their areas of math strength may also be a productive line of related work. Further work is also needed using larger samples to yield difficulty status groups with greater numbers of students and using more restrictive cutoffs for denoting difficulty that correspond even more closely to the criteria employed to designate children in schools as having mathematics learning disabilities. In addition, related work using different strategies for measuring computational skill, problem-solving skill, and the nine cognitive dimensions is required to corroborate effects. Furthermore and more generally, present study findings indicate that multivariate analytic approaches are required to untangle the role of cognitive abilities underlying the development of mathematical skill.

In the meantime, however, we caution practitioners about the potential need to consider computational skill and problem-solving skill separately in diagnosing and instructing students with learning disabilities. We also note that models of mathematical competence should expand focus on mathematical problem solving and explicitly consider the abilities that underlie the development of this form of mathematical competence. Moreover, findings support the importance of studying instructional procedures that may enhance performance specifically in the area of mathematical problem solving, separate from the issue of how to promote computational skill. Finally, the critical importance of assessing computational and problem-solving skills separately for the presence of math difficulties is apparent. Many mathematics assessments are generic and do not adequately attend to the differentiation of these dimensions of mathematical cognition.

References

Agness, P. J., & McLone, D. G. (1987). Learning disabilities: A specific look at children with spina bifida. *Insights,* 8–9.

American Psychiatric Association. (1994). *Diagnostic and statistical manual of mental disorders* (4th ed.). Washington, DC: Author.

Bernstein, I. H., Garbin, C. P., & Teng, G. K. (1988). *Applied multivariate analyses.* New York: Springer-Verlag.

Bull, R., & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65,* 1–24.

Butterfield, E. C., & Ferretti, R. P. (1987). Toward a theoretical integration of cognitive hypotheses about intellectual differences among children. In J. G. Borkowski & J. D. Day (Eds.), *Cognition in special children: Comparative approaches to retardation, learning disabilities, and giftedness* (pp. 195–233). Norwood, NJ: Ablex.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of additional and subtraction concepts in grades one through three. *Journal of Research in Mathematics Education, 15,* 179–203.

Case, L. P., Harris, K. R., & Graham, S. (1992). Improving the mathematics problem-solving skills for students with learning disabilities. *Journal of Learning Disabilities, 30,* 130–141.

Case, R. (1985). *Intellectual development: Birth to adulthood.* San Diego, CA: Academic Press.

Cirino, P. T., Ewing-Cobbs, L., Barnes, M., Fuchs, L. S., & Fletcher, J. M. (2007). Cognitive arithmetic differences in learning disability groups and the role of behavioral inattention. *Learning Disabilities Research and Practice, 22,* 25–35.

CTB/McGraw-Hill. (1997). *TerraNova technical manual.* Monterey, CA: Author.

Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior, 19,* 450–466.

Fletcher, J. M., Shaywitz, S. E., Shankweiler, D. P., Katz, L., Liberman, I. Y., Stuebing, K. K., et al. (1994). Cognitive profiles of reading disability: Comparisons of discrepancy and low achievement definitions. *Journal of Educational Psychology, 85,* 1–18.

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology, 97,* 493–513.

Fuchs, L. S., & Fuchs, D. (2002). Mathematical problem-solving profiles of students with mathematics disabilities with and without comorbid reading disabilities. *Journal of Learning Disabilities, 35,* 563–573.

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., et al. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology, 98,* 29–43.

Fuchs, L. S., Hamlett, C. L., & Fuchs, D. (1990). *Test of Computational Fluency.* (Available from L. S. Fuchs, 328 Peabody Vanderbilt University, Nashville, TN 37203)

Fuchs, L. S., Hamlett, C. L., & Powell, S. R. (2003). *Grade 3 Math Battery.* (Available from L. S. Fuchs, 328 Peabody, Vanderbilt University, Nashville, TN 37203)

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114,* 345–362.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27,* 787–797.

Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development, 2,* 249–269.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning disabilities. *Journal of Educational Psychology, 93,* 615–626.

Harris, R. J. (1975). *A primer of multivariate statistics.* San Diego, CA: Academic Press.

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computational skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology, 79,* 192–227.

Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis.* Orlando, FL: Academic Press.

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning disabilities. *British Journal of Psychology, 82,* 375–386.

Huberty, C. J. (1975). Discriminant analysis. *Review of Educational Research, 45,* 543–598.

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities, 33,* 567–578.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology, 85,* 103–119.

Jordan, N. C., Levine, S. C., & Huttenlocher, J. (1995). Calculation abilities in young children with different patterns of cognitive functioning. *Journal of Learning Disabilities, 28,* 53–64.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92,* 109–129.

Landerl, K., Began, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition, 93,* 99–125.

LeBlanc, M. D., & Weber-Russell, S. (1996). Text integration and mathematics connections: A computer model of arithmetic work problem-solving. *Cognitive Science, 20,* 357–407.

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124,* 83–97.

Littell, R. C., Milliken, G. A., Stroup, W. W., Wolfinger, R. D., & Schabenger, O. (2006). *SAS system for mixed models* (2nd ed.).Cary, NC: SAS Institute.

Luria, A. R. (1980). *Higher cortical functions in man* (2nd ed.). New York: Basic Books.

Mayer, R. E. (1992). *Thinking, problem solving, cognition* (2nd ed.). New York: Freeman.

Newcomer, P. L., & Hammill, D. D. (1988). *Test of Language Development* (Rev. ed.). Austin, TX: PRO-ED.

Passolunghi, M. C., Cornoldi, C., & De Liberto, S. (1999). Working memory and intrusions of irrelevant information in a group of specific poor problem solvers. *Memory and Cognition, 27,* 779–790.

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with specific arithmetic learning disabilities. *Journal of Experimental Child Psychology, 80,* 44–57.

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology, 88,* 348–367.

Perfetti, C. A. (1992). The representation problem in reading acquisition. In P. B. Gough, L. C. Ehri, & R. Treiman (Eds.), *Reading acquisition* (pp. 145–174). Hillsdale, NJ: Erlbaum.

Pickering, S., & Gathercole, S. (2001). *Working Memory Test Battery for Children.* London: Psychological Corporation.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Newbury Park, CA: Sage.

Raudenbush, S. W., & Liu, X. (2000). Statistical power and optimal design for multisite randomized trials. *Psychological Methods, 5,* 199–213.

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction, 5,* 49–101.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem-solving ability in arithmetic. In *The development of mathematical thinking* (pp. 153–196). San Diego, CA: Academic Press.

Rourke, B. P., & Finlayson, M. A. J. (1978). Neuropsychological significance of variations in patterns of academic skills: Verbal and visual-spatial abilities. *Journal of Abnormal Child Psychology, 6,* 121–133.

Russell, R. L., & Ginsburg, H. P. (1984). Cognitive analysis of children's mathematical difficulties. *Cognition and Instruction, 1,* 217–244.

Seaman, M. A., Levin, J. R., & Serlin, R. C. (1991). New developments in pairwise multiple comparisons: Some powerful and practical problems. *Psychological Bulletin, 110,* 577–586.

Siegel, L. S., & Linder, B. (1984). Short-term memory process in children with reading and arithmetic disabilities. *Developmental Psychology, 20,* 200–207.

Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60,* 973–980.

Swanson, H. L. (1993). An information processing analysis of learning disabled children's problem solving. *American Educational Research Journal, 30,* 861–893.

Swanson, H. L. (2006). Cross-sectional and incremental changes in working memory and mathematical problem solving. *Journal of Educational Psychology, 98,* 265–281.

Swanson, H. L., Ashbaker, M., & Sachse-Lee, C. (1996). Learning disabled readers' working memory as a function of processing demands. *Journal of Experimental Psychology, 61,* 242–275.

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem-solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 96,* 471–491.

Swanson, H. L., Cooney, J. B., & Brock, S. (1993). The influence of working memory and classification ability on children's word problem solution. *Journal of Experimental Child Psychology, 55,* 374–395.

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem-solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology, 79,* 294–321.

Webster, R. E. (1979). Visual and aural short-term memory capacity deficits in mathematics disabled students. *Journal of Educational Research, 72,* 272–283.

Wechsler, D. (1999). *Wechsler Abbreviated Scale of Intelligence.* San Antonio, TX: Psychological Corporation.

Wilson, K. M., & Swanson, H. L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities, 34,* 237–248.

Woodcock, R. W. (1997). *Woodcock Diagnostic Reading Battery.* Itasca, IL: Riverside.

Woodcock, R. W. (1998). *Woodcock Reading Mastery Tests—Revised.* Circle Pines, MN: American Guidance Service.

Woodcock, R. W., & Johnson, M. B. (1989). *Woodcock–Johnson Psycho-Educational Battery—Revised.* Allen, TX: DLM Teaching Resources.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock–Johnson III Tests of Achievement.* Itasca, IL: Riverside.

Zhu, J. (1999). *WASI manual.* San Antonio, TX: Psychological Corporation.

Received December 5, 2006
Revision received August 11, 2007
Accepted August 13, 2007 ■