Journal of Psychoeducational Assessment
1995, *13*, 21-38

THE RELATIONSHIP BETWEEN THE WJ-R *Gf-Gc* COGNITIVE CLUSTERS AND MATHEMATICS ACHIEVEMENT ACROSS THE LIFE-SPAN

Kevin S. McGrew
Saint Cloud State University

Gary L. Hessler
Macomb Intermediate School District

The relationship between the Woodcock-Johnson Psycho-Educational Battery-Revised (WJ-R) cognitive clusters (Long-Term Retrieval, Short-Term Memory, Visual Processing, Auditory Processing, Processing Speed, Comprehension-Knowledge, Fluid Reasoning) and mathematics achievement measures (WJ-R Basic Mathematics Skills and Mathematics Reasoning clusters) was investigated in the WJ-R standardization sample. Multiple regression analysis with the WJ-R cognitive clusters as predictors and the mathematics achievement measures as criteria at 21 different age groups revealed significant relationships among three of the seven cognitive clusters and mathematics achievement at various ages. The degree of relationship between the cognitive clusters and mathematics achievement changed as a function of age. The results have implications for using standardized measures of cognitive abilities to explain problems of mathematics achievement.

Recent reviews of the extant factor analytic research on human intelligence (Carroll, 1993; Horn, 1991; Lohman, 1989) suggest that significant progress has been made in identifying the major factors of intelligence. These reviews do not support a general intelligence model and suggest that our conceptualization of intelligence has changed significantly since the publication of the original Binet and Wechsler scales. These reviews have converged on the multiple intelligence conceptualization referred to as modern *Gf-Gc* theory.

Modern *Gf-Gc* theory has its roots in the fluid/crystallized conception of intelligence (Cattell, 1941; Horn, 1965). Although still referred to as *Gf-Gc* theory, the theory has expanded well beyond the original fluid (*Gf*) and crystallized (*Gc*) intelligence dichotomy. Most contemporary *Gf-Gc* conceptualizations contain the basic abilities identified in the comprehensive research syntheses of Carroll (1993) and Horn (1991). Carroll's work is particularly impressive because he has reviewed and reanalyzed more than 460 different data sets that have been factored analyzed during the past 50 years. Carroll identified eight broad factors of human intelligence—Fluid Intelligence, Crystallized Intelligence, General Memory and Learning, Broad Visual Perception, Broad Auditory Perception, Broad Retrieval Ability, Broad Cognitive Speediness, and Processing Speed.

Correspondence should be addressed to Kevin McGrew, Ph.D., Department of Applied Psychology, St. Cloud State University, 720 Fourth Ave. South, St. Cloud, MN 56301-4498.

Building on the work of Cattell, Horn's (1991) program of *Gf-Gc* research has identified nine broad abilities—Fluid Intelligence, Crystallized Intelligence, Short-Term Acquisition and Retrieval, Visual Intelligence, Auditory Intelligence, Long-Term Storage and Retrieval, Cognitive Processing Speed, Correct Decision Speed, and Quantitative Knowledge. Although using slightly different terms, Carroll and Horn both identify similar broad abilities.

The cognitive section of the Woodcock-Johnson Psycho-Educational Battery—Revised (WJ-R; Woodcock & Johnson, 1989) was designed to measure seven of the *Gf-Gc* factors contained in the Horn-Cattell *Gf-Gc* model of intelligence (Hessler, 1993; McGrew, 1994; McGrew, Werder, & Woodcock, 1991; Woodcock, 1990). It is the use of the *Gf-Gc* theory in the design of the Woodcock-Johnson Tests of Cognitive Ability—Revised (WJTCA-R) that has enabled this battery of tests to provide a comprehensive evaluation of an individual's cognitive strengths and weaknesses (Reschly, 1990; Ysseldyke, 1990).

The WJTCA-R has been judged to be a well-standardized, psychometrically sound measure of cognitive functioning (McGrew, 1994; McGrew et al., 1991; Reschly, 1990; Ysseldyke, 1990). A strength of the WJTCA-R is that it is part of the WJ-R battery that also contains measures of achievement. McGrew (1993) has reported on the relationship between the WJTCA-R *Gf-Gc* clusters and two reading achievement measures (WJ-R Basic Reading Skills and Reading Comprehension clusters) from ages 5 to 80 in the WJ-R standardization sample. His study indicated that the WJTCA-R Long-Term Retrieval, Short-Term Memory, Processing Speed, Auditory Processing, Comprehension-Knowledge, and Fluid Reasoning clusters were related to both the WJ-R Basic Reading Skills and Reading Comprehension clusters, although the strength of the relationships varied as a function of age. A similar analysis by McGrew and Knopik (1993) of the relationship between the WJTCA-R *Gf-Gc* clusters and the two written language measures (WJ-R Basic Writing Skills and Written Expression clusters) has indicated that the WJTCA-R Processing Speed, Auditory Processing, Comprehension-Knowledge, and Fluid Reasoning clusters were related to both the WJ-R Basic Writing Skills and Written Expression clusters, although the strength of the relationships varied with age. A comparable analysis has not been presented in the area of mathematics achievement.

The current study was designed to extend the work of McGrew (1993) and McGrew and Knopik (1993) by investigating the relationship between the WJTCA-R *Gf-Gc* clusters and mathematics achievement across the life-span. The goal was to investigate the validity of using the WJTCA-R *Gf-Gc* cognitive clusters for identifying potential reasons for problems in mathematics achievement. The primary research question was: "Do the seven WJTCA-R clusters demonstrate significant and differential patterns of relationships with different mathematics outcomes across the life-span?"

Given the qualitatively different nature of achievement in reading and writing (e.g., grapho-phonic, orthographic, and linguistic requirements) than in mathematics, it is anticipated that the pattern of relationships between the *Gf-Gc* cognitive clusters and mathematics achievement would be different than the patterns found between the *Gf-Gc* cognitive clusters and reading and writing achievement. Discovering the relationship between the *Gf-Gc* cognitive clusters and mathematics achievement across the life-span may provide important preliminary diagnostic information for the identification and remediation of learning disabilities in mathematics. For example, knowledge of the cognitive processes most closely related to mathematics achievement at particular age levels provides practitioners with valuable information (along with other diagnostic, historical, and observational information) to help establish whether cognitive difficulties underlie learning problems in mathematics for an individual. Furthermore, such knowledge of the relationship between cognitive abilities and mathematics achievement can provide important instructional information so that cognitive deficits can be considered during remedial mathematics activities.

METHOD

Subjects

The subjects were drawn from the WJ-R standardization sample of 6,359 individuals, who ranged in age from 24 months to 95 years. The WJ-R norm sample is a nationally representative sample that has been judged to be technically sound (Kaufman, 1990; Reschly, 1990; Ysseldyke, 1990). The characteristics of the standardization sample are described in detail in the WJ-R Technical Manual (McGrew et al., 1991).

For the purpose of the current investigation, the sample was divided into 21 different age-based samples. The first 15 samples each represented 1 year of age, starting at 5 years and continuing up through age 19 years. Each of the remaining six samples spanned 10 years of adulthood (i.e., 20-29; 30-39; 40-49; 50-59; 60-69; 70-79). Those norming subjects who had complete information on the WJ-R measures described below were included in the investigation (n = 5,386 or 5,398 for the two different mathematics achievement criteria). Sample size and descriptive statistics for the two WJ-R measures of mathematics achievement (described below) for the 21 age groups are presented in Table 1.[1]

Measures

The 35 WJ-R tests can be combined into various clusters, the preferred level of interpretation in the WJ-R (Hessler, 1993; McGrew, 1994). Seven cognitive and two achievement clusters from the WJ-R battery were used in this investigation. Each of these nine clusters is comprised of two individual tests with the exception of Mathematics Reasoning, which is composed of one test. The abilities measured by the seven WJ-R *Gf-Gc* cognitive and two mathematics achievement clusters from the battery are described in Table 2. As reported by McGrew et al. (1991), the correlations between the WJ-R mathematics achievement clusters and other achievement tests (PIAT-R, BASIS, K-ABC, K-TEA, and WRAT-R) ranged from .609 to .869 for independent samples from grades three, three/four, and ten/eleven. This evidence supports the concurrent validity of the WJ-R mathematics clusters.

As indicated in Table 2, the Basic Mathematics cluster is composed of two tests: Calculation and Quantitative Concepts. Calculation is designed to measure the ability to perform mathematical calculations that are fundamental to more complex mathematical reasoning and problem solving. It requires subjects to solve a variety

---

[1]Due to space limitations, descriptive statistics for all seven WJ-R cognitive clusters for all 21 age levels are not reported. This information can be obtained by contacting the first author.

Table 1
Select Descriptive Statistics for 21 Samples Used in Regression Models

Age group (years)	Basic Mathematics Skills			Mathematics Reasoning		
	n	*M*	*SD*	*n*	*M*	*SD*
5	113	413.5	16.6	113	437.6	12.0
6	301	435.2	16.3	301	449.8	13.3
7	229	455.8	14.7	230	465.4	17.3
8	324	475.0	14.2	325	483.7	16.6
9	305	484.6	12.9	306	490.8	16.0
10	311	499.6	13.7	312	501.6	14.2
11	237	507.5	14.5	237	504.1	14.8
12	169	518.2	12.8	169	513.1	13.5
13	265	526.8	12.5	266	518.4	15.2
14	209	529.3	14.9	209	520.6	17.4
15	168	532.7	17.9	168	523.7	22.2
16	265	538.7	17.7	265	531.1	20.2
17	257	540.2	18.9	257	534.5	22.1
18	248	540.9	18.6	248	533.9	21.8
19	231	543.2	21.5	231	537.5	24.2
20-29	703	541.8	18.7	702	540.2	20.6
30-39	343	537.1	16.9	343	539.3	18.3
40-49	232	531.3	18.9	234	536.1	19.1
50-59	174	524.4	20.5	174	531.6	21.4
60-69	163	520.0	19.9	163	527.2	21.9
70-79	148	513.7	28.2	148	522.0	26.8

of problems by using addition, subtraction, multiplication, and division, as well as more advanced problems that require geometric, trigonometric, logarithmic, and calculus operations. Whole numbers, percents, negative numbers, decimals, and fractions are involved in some of the calculations, as is the use of single, double, and triple digits. Subjects use paper and pencil, and the test is untimed (Hessler, 1993). Quantitative Concepts is designed to measure the breadth and depth of an individual's knowledge of mathematical concepts, symbols, and vocabulary. Subjects are required to point to or state answers to questions on number identification, sequences, shapes, symbols, mathematical terms, and formulae. Although subjects are tested relative to their understanding of mathematical concepts, symbols, and vocabulary, no application is required (Hessler, 1993).

The Mathematics Reasoning cluster is composed of the Applied Problems test. Applied Problems is designed to measure skill in analyzing and solving practical problems in mathematics. The test requires subjects to comprehend the nature of a problem that is read to them, recognize relevant information, identify and perform necessary calculations, and state the answer. The test includes items on number concepts, money, measurement, time, fractions, averaging, probability, interest, algebra, and geometry. Items require addition, subtraction, multiplication, and/or division, separately and in combination; story problems involve one-, two-, or three-step calculations and sometimes include distracting information (Hessler, 1993).

The cluster scores used in this investigation were the *W* scores, a special transformation of the Rasch ability scale (Woodcock, 1978; Woodcock & Dahl, 1971). The *W* scale is an equal-interval scale centered on a value of 500 that is the approximate average performance of a beginning fifth-grade student.

Table 2
Seven WJ-R Gf-Gc Cognitive (Predictors) and Two Mathematics Achievement (Criteria) Cluster Descriptions

Cluster name	Symbol	Cluster description
Fluid Reasoning	*Gf*	A combination of the Analysis-Synthesis and Concept Formation tests that measures the ability to reason, form concepts, and problem solve, often with unfamiliar information or procedures
Comprehension-Knowledge	*Gc*	A combination of the Picture Vocabulary (receptive) and Oral Vocabulary (expressive) tests that measures a person's breadth and depth of vocabulary knowledge
Visual Processing	*Gv*	A combination of the Visual Closure and Picture Recognition tests that measures the ability to analyze and synthesize nonlinguistic visual stimuli
Auditory Processing	*Ga*	A combination of the Incomplete Words and Sound Blending tests that measures the ability to analyze and synthesize auditory linguistic stimuli. Phonological processing
Processing Speed	*Gs*	A combination of the Visual Matching and Cross Out tests that measures the ability to rapidly perform automatic cognitive tasks, especially when under pressure to maintain focused concentration
Short-Term Memory	*Gsm*	A combination of the Memory for Sentences and Memory for Words tests that measures the ability to temporarily store verbal information and then use it within a few seconds
Long-Term Retrieval	*Glr*	A combination of the Memory for Names and Visual-Auditory Learning tests that measures the ability to store information and retrieve it later through association. Paired-associate learning for whole words
Basic Mathematics Skills		A combination of the Calculation and Quantitative Concepts tests that together measure paper-and-pencil mathematics calculation skills and knowledge of mathematics concepts, vocabulary, and symbols
Mathematics Reasoning		Consists of the Applied Problems test that measures the ability to perform mathematics problem solving, reasoning, and applications tasks

Procedure

Gf-Gc/mathematics achievement regression analyses. To determine the relationship between the seven WJ-R cognitive clusters and two types of mathematics achievement (viz., basic skills and reasoning) two regression models were estimated in each of the 21 samples. In each model the predictor variables were the seven WJ-R cognitive clusters. In one of the models the criterion measure was the WJ-R Basic Mathematics Skills cluster, while in the other the criterion was the WJ-R Mathematics Reasoning cluster. The standardized regression coefficients were used as the primary unit of analysis. The standardized regression coefficients indicate the portion of standard deviation units that the criterion measure changes as a function of one standard deviation change in a predictor. For example, if a standardized regression coefficient of .25 is reported between the WJ-R Processing Speed cluster and the WJ-R Basic Mathematics Skills cluster, this would mean that for every standard deviation change observed in Processing Speed scores, this is associated with an average .25 standard deviation change in Basic Mathematics Skills scores.

Due to different patterns of missing data, the sample sizes at each age level differed for each of the regression models. Across the 21 age groups, a total of 5,401 subjects had complete data for the prediction of Basic Mathematics Skills, while 3,371 had complete data for the prediction of Mathematics Reasoning.

Age-related analyses. To investigate the possibility of age-related changes in the ability of the WJ-R cognitive clusters to predict the different mathematics criteria, the standardized regression coefficients for each cognitive predictor for each of the mathematics criteria were plotted by age. For example, in the prediction of Basic Mathematics Skills, there were 21 different Auditory Processing regression coefficients that corresponded to the 21 different age groups. Using the mean age for each age group as the X axis, each of these 21 regression coefficients was plotted on a graph. Visual inspection of the graph allowed for a rough determination of any age-related changes in the predictive relationship between Auditory Processing and Basic Mathematics Skills. This was repeated for each WJ-R cognitive cluster for each of the two mathematics criteria measures.

Inspection of the graphs revealed apparent age-related changes that were examined for systematic trends through the use of the distance weighted least squares (DWLS) smoothing function (Wilkinson, 1990). The DWLS smoothing function produces a true, locally weighted curve running through points on the smoothed line, each of which is based on a weighted quadratic multiple regression on all the points (Wilkinson, 1990). Given the exploratory nature of this study, the DWLS function was selected over linear or low-order polynomial smoothing models that presuppose the shape of the function and curve. The resulting smoothed curves then were plotted together with the original regression coefficients to display visually the age-related changes in the relationship between a WJ-R cognitive cluster and each mathematics criterion. Because the original standardized regression coefficients contain sampling error, the smoothed values that utilize information from all the data points are probably better estimates of the population parameters (Zachary & Gorsuch, 1985).

RESULTS

Select statistics for the two regression models estimated at each of the 21 age levels are presented in Table 3.[2]

Gf-Gc/mathematics achievement regressions. Across both mathematics criteria, and across age groups, all regression models produced overall F-statistics that were significant beyond the .001 level. The multiple Rs ranged from .69 to .88 for the prediction of Basic Mathematics Skills and from .66 to .82 for Mathematics Reasoning. (See Table 3.) The magnitude of these multiple Rs indicates that the combined WJ-R cognitive clusters are highly related to both areas of mathematics achievement. For example, most of the multiple Rs are in the .70 to .80 range, values that indicate that the combined WJ-R cognitive clusters account for approximately 50% to 70% of mathematics achievement variance.

Inspection of the number of significant coefficients for the WJ-R cognitive clusters across age levels (Table 3) indicated that the Processing Speed, Comprehension-

[2]Due to space limitations, all statistics for all regression models are not reported. This information can be obtained by contacting the first author.

Table 3
Standardized Regression Coefficients and Multiple Rs for Regression Models with the WJ-R Basic Mathematics Skills and Mathematics Reasoning Measures as Criteria and WJ-R Gf-Gc Clusters as Predictors for 21 Age Groups

	Glr		*Gsm*		*Gs*		*Ga*		*Gv*		*Gc*		*Gf*		R	
Age group	BMS	MR	BMS	MR	BMS	MR	BMS	MR	BMS	MR	BMS	MR	BMS	MR	BMS	MR
5	.02	.08	.10	.22*	.37*	.28*	.08	.01	.08	.19*	.18	.12	.15	.06	.70	.66
6	.11*	.06	.04	.11*	.41*	.31*	.09	−.03	.02	.11*	.09	.20*	.18*	.19*	.71	.68
7	.04	.08	−.02	.12*	.30*	.22*	.18*	.09	.00	.02	.18*	.15*	.28*	.33*	.76	.80
8	−.04	.00	.08	.06	.27*	.22*	.05	.09	−.01	.09*	.36*	.28*	.24*	.25*	.77	.76
9	.10*	.08	.07	.14*	.28*	.22*	.05	.08	−.07	−.01	.33*	.22*	.18*	.27*	.74	.74
10	.04	.06	.12*	.15*	.39*	.23*	−.04	.00	−.10*	.03	.37*	.28*	.16*	.22*	.73	.69
11	.01	.03	.02	.05	.32*	.18*	.07	.06	−.16*	−.09	.44*	.41*	.21*	.28*	.77	.75
12	.01	.05	.12*	.12	.20*	.13*	−.04	−.02	.01	.01	.45*	.42*	.24*	.24*	.76	.72
13	.06	−.08	.05	.07	.20*	.18*	.06	.03	−.09	−.03	.36*	.40*	.28*	.34*	.69	.70
14	.08	−.01	.00	.03	.29*	.12*	−.08	−.04	−.04	.06	.40*	.41*	.32*	.35*	.79	.74
15	.13*	.06	.06	−.06	.26*	.29*	−.13*	−.07	−.07	−.03	.42*	.43*	.31*	.30*	.82	.78
16	.07	.05	−.02	.02	.23*	.16*	.03	.02	.00	−.03	.33*	.36*	.37*	.33*	.79	.74
17	.12*	.04	.00	−.02	.25*	.19*	.03	.01	−.12*	−.06	.35*	.46*	.30*	.29*	.79	.78
18	.17*	.06	.11*	.10	.24*	.14*	−.04	.05	−.07	.02	.29*	.25*	.25*	.33*	.74	.72
19	.08	.04	.09	.05	.14*	.18*	−.02	.02	−.06	−.03	.38*	.38*	.36*	.35*	.80	.80
20-29	.16*	.03	.11*	.16*	.14*	.18*	−.16*	−.11*	−.13*	−.06	.32*	.38*	.36*	.30*	.69	.71
30-39	.03	.04	.06	.02	.18*	.11*	−.08	.00	.00	−.04	.54*	.50*	.19*	.19*	.76	.70
40-49	.10	.09	.12*	.14*	.14*	.13*	−.16*	−.16*	−.10*	−.08	.51*	.55*	.24*	.19*	.76	.76
50-59	.05	.11	−.01	−.10	.18*	.01	.07	.20*	−.04	−.08	.51*	.38*	.18*	.27*	.83	.71
60-69	.05	.02	.01	.14	.10	.08	−.10	−.12	.00	.02	.66*	.49*	.17*	.19*	.81	.72
70-79	.04	.05	.02	−.07	.18*	.03	−.02	−.03	.01	.00	.46*	.56*	.32*	.34*	.88	.82
# significant coefficients	6	0	5	7	20	18	4	3	5	3	19	20	20	20		

Note.—BMS = Basic Mathematics Skills; MR = Mathematics Reasoning; *Glr* = Long-Term Retrieval; *Gsm* = Short-Term Memory; *Gs* = Processing Speed; *Ga* = Auditory Processing; *Gv* = Visual Processing; *Gc* = Comprehension-Knowledge; *Gf* = Fluid Reasoning; R = multiple R.
*Indicates coefficients that were significant ($p < .05$) within each age group model.

Knowledge, and Fluid Reasoning clusters displayed the most consistent relationship with mathematics achievement across the life span. Based on a count of the number of significant regression coefficients, the Fluid Reasoning (20), Processing Speed (20), and Comprehension-Knowledge (19) clusters demonstrated the most consistent relationship with Basic Mathematics Skills. Long-Term Retrieval and Short-Term Memory also were related to Basic Mathematics Skills, although less consistently (6 and 5 significant coefficients, respectively). Auditory Processing and Visual Processing were found to have little relationship with Basic Mathematics Skills. Similar to Basic Mathematics Skills, the top three cognitive clusters related to Mathematics Reasoning were Comprehension-Knowledge (20), Fluid Reasoning (20), and Processing Speed (18). Two cognitive clusters (Short-Term Memory and Visual Processing) also were related to Mathematics Reasoning (7 and 3 of their coefficients were significant, respectively), although the relationships were primarily at the early age levels. Finally, Long-Term Retrieval and Auditory Processing clusters were found to have little relationship with Mathematics Reasoning.

Age-related analyses. The results of the age-related analyses are summarized by the 14 graphs presented in Figures 1 through 4. For each WJ-R cognitive cluster, the plotting of the raw (connected by dotted line) and smoothed (connected by solid line) coefficients for each of the two mathematics criteria are presented side by side. Each graph's label lists the mathematics achievement cluster criterion and the respective *Gf-Gc* cognitive cluster abbreviations (*Glr* = Long-Term Retrieval; *Gsm* = Short-Term Memory; *Gs* = Processing Speed; *Ga* = Auditory Processing; *Gv* = Visual Processing; *Gc* = Comprehension-Knowledge; *Gf* = Fluid Reasoning).

Also included in each figure are two parallel dashed lines that correspond to standardized regression coefficients of .10 and .30. These lines are guides for interpreting the significance of the smoothed values in each figure. Based on a review of the significance tests for all standardized regression coefficients in this study, as well as two related studies in reading and written language that used the same methods (McGrew, 1993; McGrew & Knopik, 1993), it was determined that most coefficients at or above .10 were statistically significant ($p < .05$). Although there were exceptions to this rule, .10 also was judged to be a good practical significance criterion. Although some standardized coefficients in the .08s to .09s were significant, any change of less than one-tenth of a standard deviation in a mathematics criterion as a function of a full standard deviation change in a predictor is probably not practically meaningful. Thus, the .10 line was established as the minimal level of statistical and practical significance. Furthermore, it was decided that regression coefficients could be classified as either moderately or strongly related to the mathematics criteria. Moderate significance was defined arbitrarily as being from .10 to .29. Standardized coefficients at or above .30 (approximately one-third of a standard deviation) were considered to be strong.

A review of the smoothed lines in Figures 1 through 4 reveals a number of different relationships between the WJ-R cognitive clusters and the two mathematics criteria across the 21 age levels. The Long-Term Retrieval cluster (top half of Figure 1) appears unrelated to Basic Mathematics Skills and Mathematics Reasoning in a meaningful sense throughout all ages. Except for the age range of 15 to 20 in Basic Mathematics Skills, the fitted curves consistently fall below the criteria for meaningful

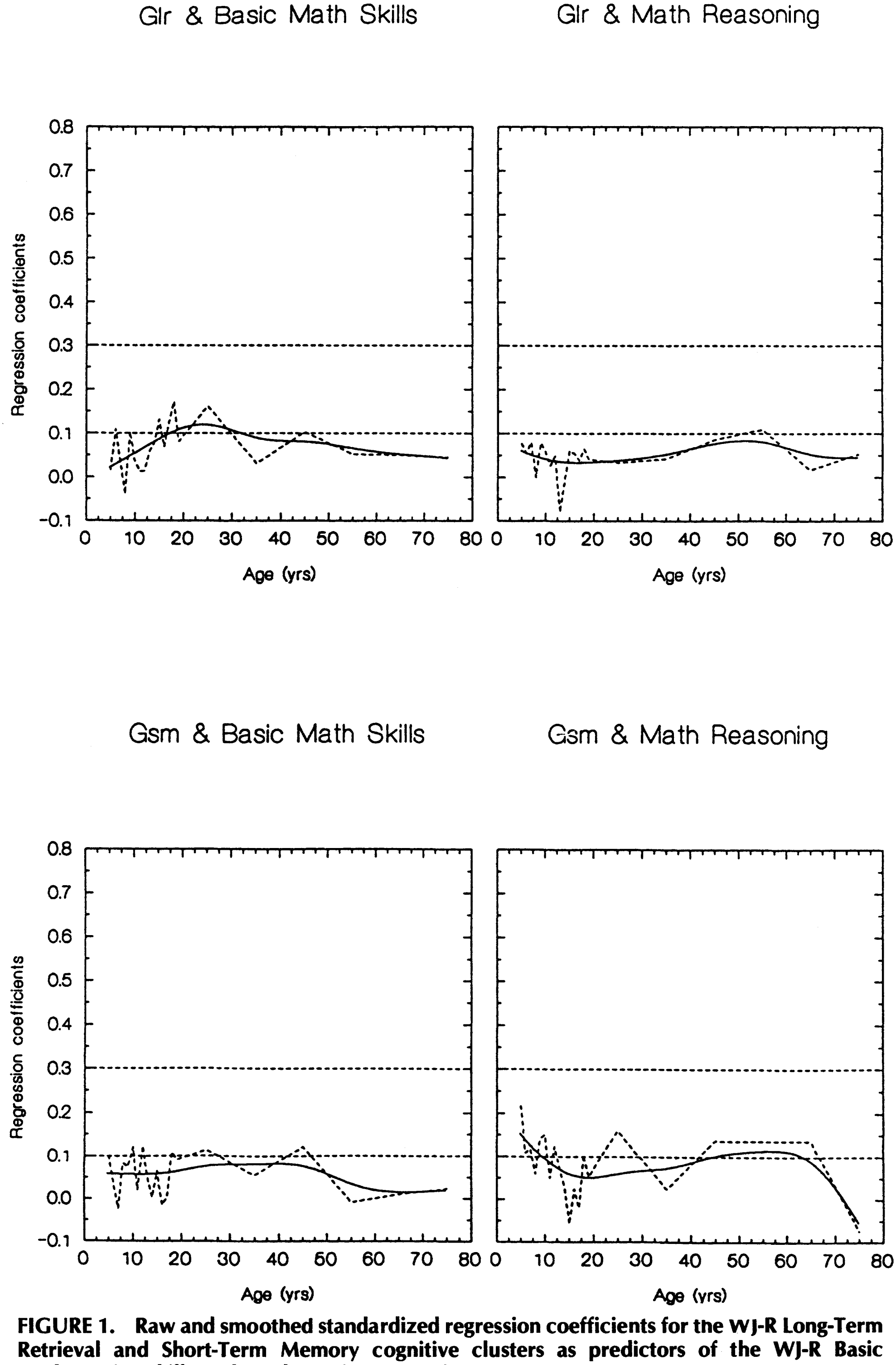

FIGURE 1. Raw and smoothed standardized regression coefficients for the WJ-R Long-Term Retrieval and Short-Term Memory cognitive clusters as predictors of the WJ-R Basic Mathematics Skills and Mathematics Reasoning across 21 age groups.

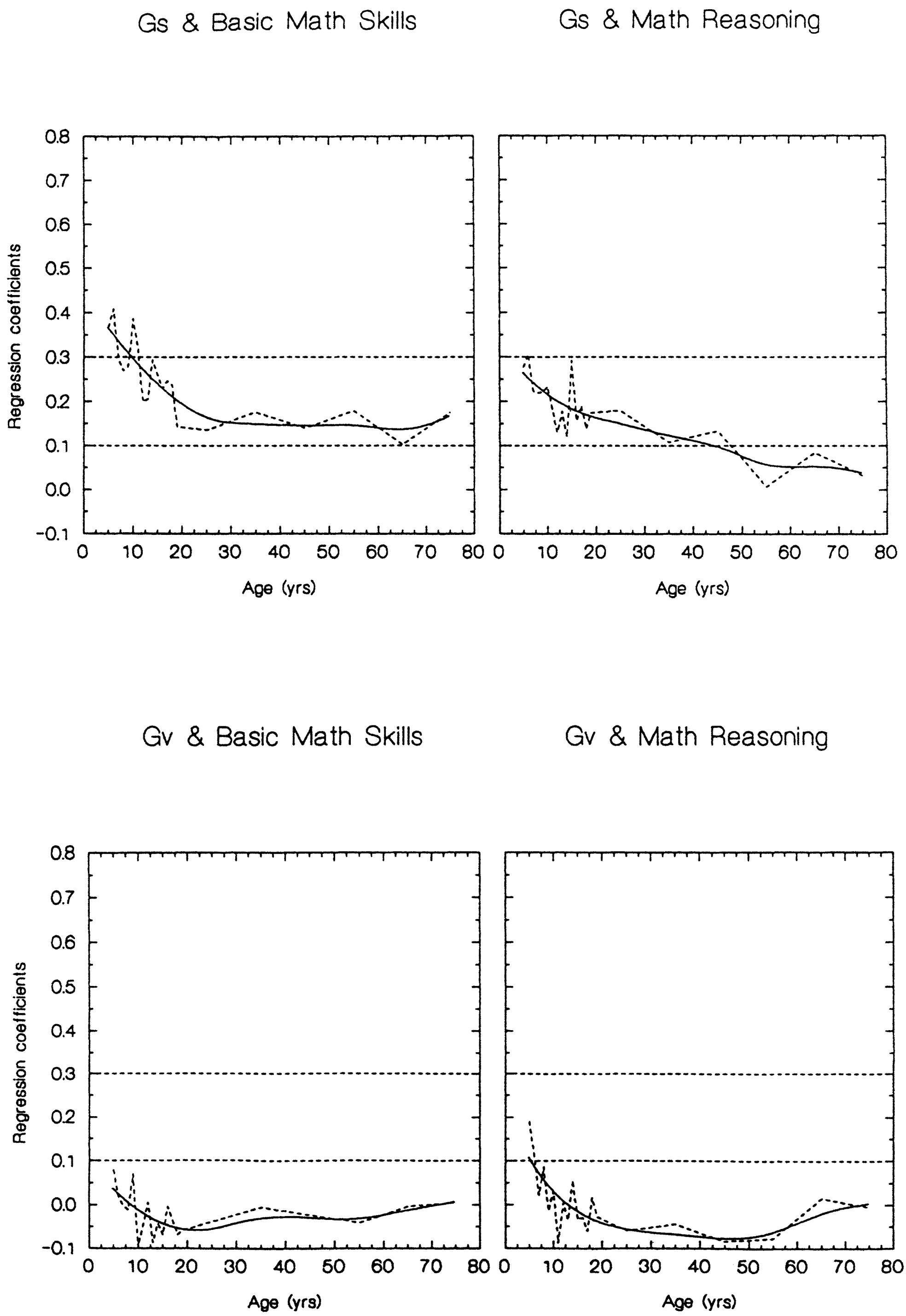

FIGURE 2. Raw and smoothed standardized regression coefficients for the WJ-R Processing Speed and Visual Processing cognitive clusters as predictors of the WJ-R Basic Mathematics Skills and Mathematics Reasoning across 21 age groups.

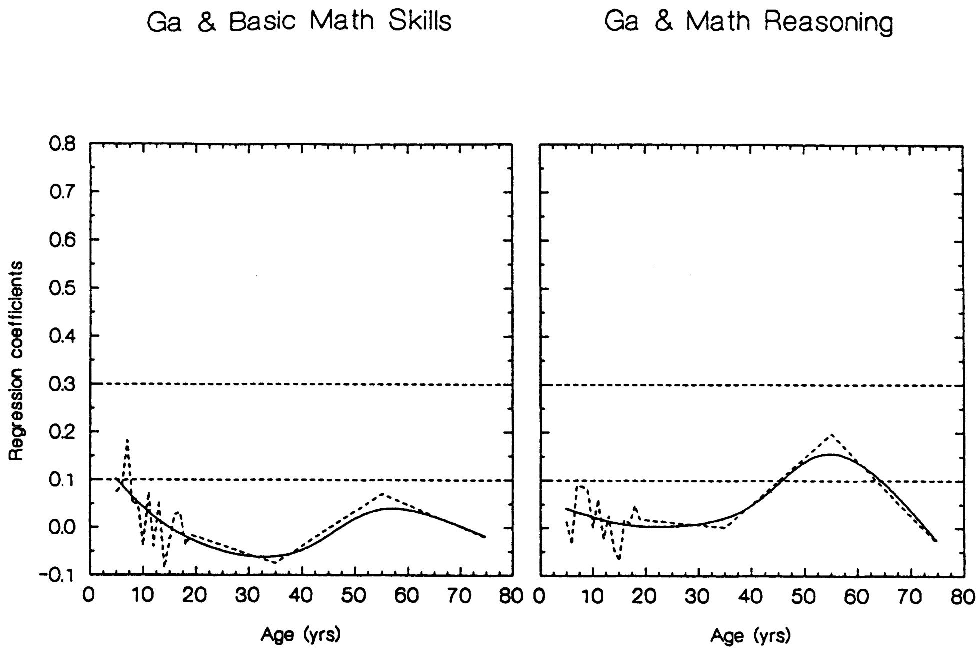

FIGURE 3. Raw and smoothed standardized regression coefficients for the WJ-R Auditory Processing cognitive cluster as predictor of the WJ-R Basic Mathematics Skills and Mathematics Reasoning across 21 age groups.

significance. This finding is basically consistent with the previous results of the analyses by McGrew (1993) and McGrew and Knopik (1993), who found that no substantial relationship exists between Long-Term Retrieval and reading and written language achievement across the life span.

Similar to the curves for Long-Term Retrieval, the Short-Term Memory cluster (bottom half of Figure 1) demonstrates the lack of a significant relationship with Basic Mathematics Skills throughout the life span. However, in the case of Mathematics Reasoning a moderate association is suggested from 5 to 10 years of age and perhaps in the 40s and 50s, although no significant relationships exist at the other age levels. Due to their sporadic pattern, it is possible that the moderate relationship between Short-Term Memory and Mathematics Reasoning in mid-life may be due to chance. This finding of a relatively low correlation between Short-Term Memory and mathematics achievement is consistent with a low relationship with written language achievement (McGrew & Knopik, 1993), but is different from Short-Term Memory's association with reading achievement, which is moderately strong across much of the life span (McGrew, 1993).

Performance on the Processing Speed cluster (top half of Figure 2) appears to be related significantly to Basic Mathematics Skills throughout the life span. Processing Speed's relationship with Basic Mathematics Skills is strongest from 5 to 11 years of age, when it systematically decreased until approximately 20 years of age. The curve then plateaus and remains in the moderate range. Processing Speed also demonstrates a moderate relationship with Mathematics Reasoning, beginning with a high point at age 5, but it then systematically declines until no meaningful

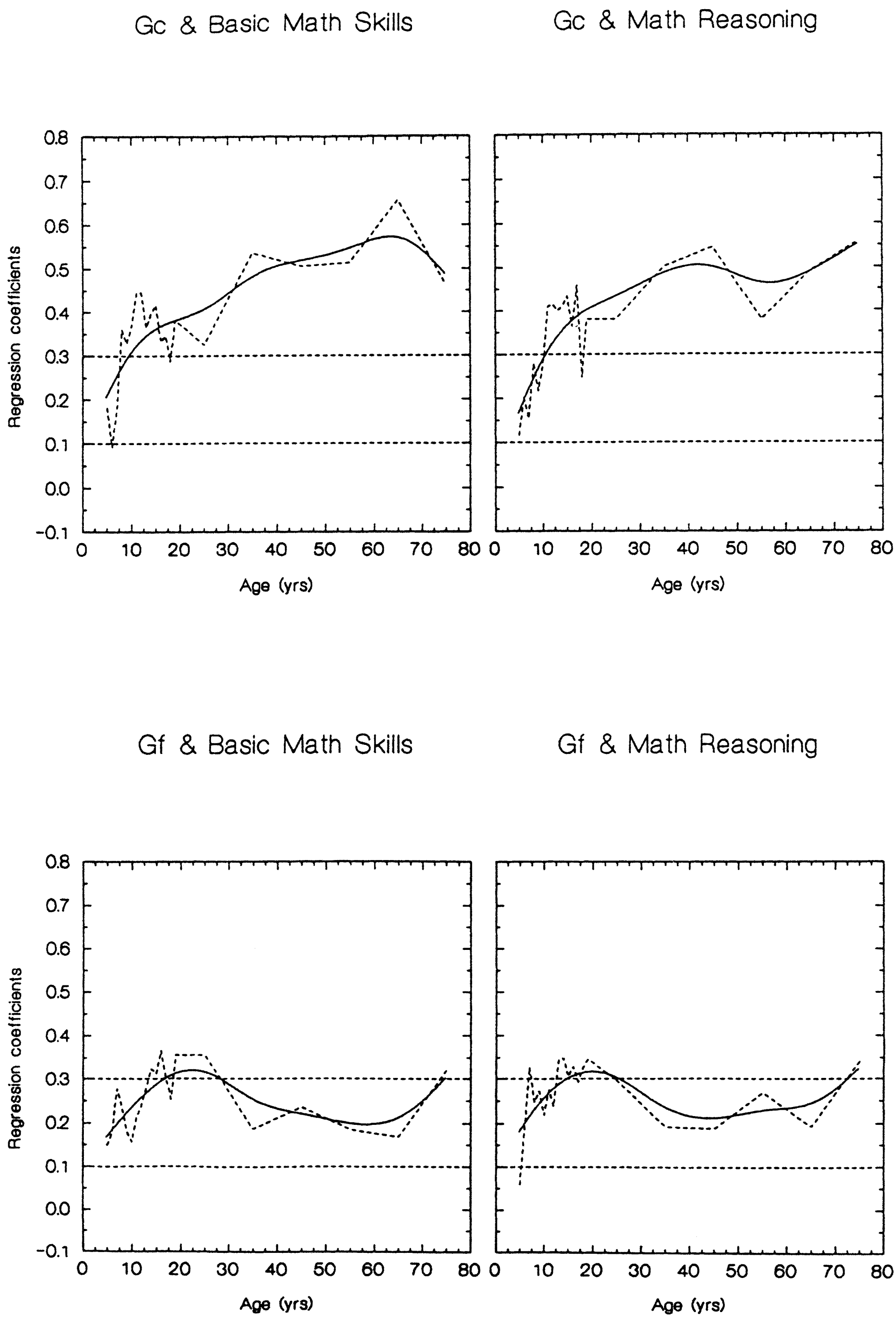

FIGURE 4. Raw and smoothed standardized regression coefficients for the WJ-R Comprehension-Knowledge and Fluid Reasoning cognitive clusters as predictors of the WJ-R Basic Mathematics Skills and Mathematics Reasoning across 21 age groups.

association exists beyond approximately age 40. While significantly related to both areas of mathematics achievement, Processing Speed's relationship, therefore, appears to decrease with age, although it remains more substantial with Basic Mathematics Skills than with Mathematics Reasoning in later life. Although the developmental patterns and magnitudes are different, both achievement in mathematics and written language appear to be related substantially to Processing Speed across much of the life span (McGrew & Knopik, 1993). In contrast, Processing Speed and achievement in reading appear to be related most strongly up to age 10, after which time the relationship declines substantially and becomes nonsignificant (McGrew, 1993).

Perusal of the bottom half of Figure 2 indicates that the relationship between Visual Processing and both areas of mathematics achievement is negligible. This suggests that performance on this measure of visual processing of basically nonlinguistic stimuli has little diagnostic significance for mathematics achievement, although Visual Processing appears to have a moderate degree of relationship to Mathematics Reasoning at 5 to 8 years of age (Table 3). This finding is consistent with previous analyses (McGrew, 1993; McGrew & Knopik, 1993), which also suggest low statistical and practical relationships between Visual Processing and achievement in reading and written language. Similarly, the relationship between Auditory Processing and both Basic Mathematics Skills and Mathematics Reasoning resulted in few significant regression coefficients, and the fitted curves consistently fell below the level of statistical and meaningful significance (Figure 3). The lone exception is the relationship between Auditory Processing and Mathematics Reasoning in middle age, where the fitted curve ascends into the moderate range, but then rapidly drops below the .10 minimal significance level at approximately age 60. This finding of a low relationship between Auditory Processing and mathematics achievement is in contrast to the relatively robust relationships between Auditory Processing and achievement in reading (McGrew, 1993) and written language (McGrew & Knopik, 1993) and, thereby, suggests that the auditory processing (e.g., phonological awareness) requirements in mathematics as measured by the WJ-R are not the same as those in more linguistically based areas of achievement.

In contrast to all the other smoothed curves, those for Comprehension-Knowledge (top of Figure 4) consistently increase from the youngest to the oldest ages, with a leveling off and possible slight decline that starts with age 60 in the case of Basic Mathematics Skills. The magnitude of the regression coefficients reflected in Table 3 and Figure 4 (most in the strong classification—at or above .30) suggests that comprehension-knowledge abilities are those that are associated most closely with both types of mathematics achievement. The association between Comprehension-Knowledge and mathematics achievement as measured by the WJ-R probably is strengthened by the oral language requirements of the Quantitative Concepts test, which comprises half of the Basic Mathematics Skills cluster and of the Applied Problems test that makes up the Mathematics Reasoning cluster. Nevertheless, the strong relationship between Comprehension-Knowledge and mathematics achievement is consistent with the similarly strong associations between Comprehension-Knowledge and achievement in reading (McGrew, 1993) and written language (McGrew & Knopik, 1993).

The results for the Fluid Reasoning cluster (bottom of Figure 4) suggest that the abilities measured by this cluster generally are correlated with mathematics achievement to a moderate degree across all ages. Fluid Reasoning's relationship to both mathematics criteria demonstrates a systematically increasing trend from the relatively low points at age 5. The development of the relationship peaks at a strong level around age 20 and then slowly declines to a low point (still in the moderate range) at approximately age 60. This is followed by an apparent increasing trend in late adulthood. Inspection of previous similar analyses by McGrew (1993) and McGrew and Knopik (1993) indicates that Fluid Reasoning's relationship with mathematics achievement is consistently stronger across the life span than its relationship with achievement in reading or written language.

DISCUSSION

Similar to the conclusion reached in McGrew's (1993) reading achievement study and in McGrew and Knopik's (1993) written language achievement study, the current results support the validity of using the WJTCA-R cognitive clusters in psychoeducational assessment. The finding that the combined WJTCA-R clusters accounted for approximately 50% to 70% of mathematics achievement variance across most of the life-span provides support for the concurrent validity of the WJTCA-R. These results, together with those reported by McGrew (1994), suggest that the WJTCA-R may be a valuable instrument for assessing those cognitive abilities associated with reading, writing, and mathematics achievement as assessed by the WJ-R Achievement Battery.

Performance on three of the seven WJTCA-R cognitive clusters (Processing Speed, Comprehension-Knowledge, and Fluid Reasoning) was related consistently and significantly to performance on measures of mathematics calculation and basic knowledge (WJ-R Basic Mathematics Skills) and mathematics problem solving and applications (WJ-R Mathematics Reasoning). This suggests that mathematics achievement is related to the abilities of fluid or novel reasoning (Fluid Reasoning), verbal and vocabulary knowledge (Comprehension-Knowledge), speed or automaticity of cognitive processing (Processing Speed), and other skills that have been shown to be related to learning mathematics skills.

Among the seven *Gf-Gc* measures included in this study, measures of verbal and vocabulary knowledge and abstract and fluid reasoning and problem solving were related most consistently to both aspects of mathematics achievement across the life span. These findings are consistent with cognitive-based mathematics research. Research has suggested a significant relationship between a person's mathematics skills and breadth of declarative knowledge (in this case verbal or vocabulary knowledge) (Gagné, 1985; Glover, Ronning, & Bruning, 1990; Mayer, 1986; Smith & Rivera, 1991). In addition, the ability to reason and problem solve abstractly and to categorize and integrate information is important for mathematics achievement (Ginsberg, 1989; Glover et al., 1990; Mayer, 1986; Resnick & Ford, 1981; Smith & Rivera, 1991).

The strength of association between the Processing Speed and measures of mathematics achievement varied more as a function of age. The relationship between the automaticity or processing speed and both forms of mathematics achievement decreased with age, although this decline was more dramatic in the case of Mathematics Reasoning than with Basic Mathematics Skills. Processing Speed appears to be a strong correlate of Basic Mathematics Skills from 5 to 10 years of age and a moderate correlate throughout the life span. The relationship between Processing Speed and Mathematics Reasoning appears to be moderate in magnitude until approximately 40 years of age, when the relationship becomes nonsignificant. The relatively strong relationship between Processing Speed and mathematics achievement is no surprise because the ability to perform quickly and automatically basic mathematics procedures allows an individual to devote more conscious attention and working memory to other aspects of mathematics (Cawley, 1985; Hasselbring, Goin, & Bransford, 1988; Kirby & Becker, 1988; Hessler, 1993; Mather, 1991; Pellegrino & Goldman, 1987; Resnick & Ford, 1981; Zentall, 1990).

Long-Term Retrieval, Auditory Processing, and Visual Processing were not consistent correlates with mathematics achievement. The finding of a lack of relationship between Long-Term Retrieval and Visual Processing and achievement in mathematics is consistent with their relatively poor association with achievement in reading (McGrew, 1993) and written language (McGrew & Knopik, 1993). In contrast to its relatively strong relationship to achievement in reading and written language (McGrew, 1993; McGrew & Knopik, 1993), Auditory Processing was not correlated with mathematics achievement in a consistent way. This difference in relationships probably is related to the stronger linguistic basis of achievement in reading and written language than in mathematics.

Short-term memory, as measured operationally by the WJTCA-R Short-Term Memory cluster, also was not associated consistently with mathematics achievement, although a significant relationship may exist during the elementary school years in Mathematics Reasoning. This appears to contradict other research that has suggested a significant relationship between short-term or working memory and mathematics achievement (Hessler, 1993; Mather, 1991). One possible explanation for this finding is that the WJTCA-R Short-Term Memory cluster measures something different than the working memory cited in prior research studies. Factor analytic studies indicate that one of the two tests contained in the WJTCA-R Short-Term Memory cluster (i.e., Memory for Sentences) is a factorially complex measure of both short-term memory and comprehension-knowledge (McGrew, 1994). The fact that the tests that constitute the Short-Term Memory cluster both measure auditory short-term memory of contextual (Memory for Sentences) and non-contextual (Memory for Words) verbal information may be related to the lack of Short-Term Memory's consistent relationship with mathematics achievement. It is possible that this relationship would have differed if tests that used other types of short-term memory stimuli (e.g., digits, letters, visual symbols) had been used. Additional studies are needed to clarify the relationship between the individual WJTCA-R tests and clusters and mathematics achievement.

The results of this study have practical implications for psychoeducational assessment. Assessment professionals should consider using the WJTCA-R Comprehension-Knowledge, Processing Speed, and Fluid Reasoning clusters when they are evaluating individuals with mathematics achievement problems. The Fluid Reasoning and Comprehension-Knowledge clusters appear important to use throughout the life-span when persons with problems in either mathematics calculation or reasoning are being assessed. The Processing Speed cluster also should be used for problems in both areas of mathematics. However, its value appears strongest during the school years, although it also may be useful for assessing individuals with problems in basic mathematics skills throughout the life span and for assessing individuals with problems in mathematics reasoning until approximately 40 years of age.

For example, practitioners who use the WJ-R and who wish to determine whether an individual's learning problems in mathematics are related to central processing disorders should examine the individual's performance on the Comprehension-Knowledge, Fluid Reasoning, and Processing Speed clusters (along with other relevant test, observational, and historical information). By examining an individual's pattern of cognitive and academic strengths and weaknesses according to the age-related developmental patterns outlined in this paper, practitioners can help to establish the degree to which relevant cognitive abilities are associated with academic deficits. Hessler (1993) has described various means for analyzing performance on the WJ-R according to age or grade norms and/or mean performance levels.

The information presented in this manuscript also can be of value for instructional programming. Cognitive deficits that pertain to achievement in mathematics can be considered during remedial mathematics activities, thus strengthening instruction. For example, instruction to individuals who experience difficulties with Processing Speed might benefit from direct instruction in mathematics that considers the individual's difficulty with fluency and automaticity in learning mathematics facts and procedures. Garnett (1992) provides excellent recommendations for individuals with these difficulties. Furthermore, individuals with deficits in Fluid Reasoning probably would benefit from instruction in mathematics that includes consideration of their difficulties to reason and problem solve abstractly and to categorize and integrate mathematics information. The individuals probably would profit from cognitive oriented instruction (Fleichner & Garnett, 1983). Some implications for instruction would involve additional time for active construction of mathematical relationships, attention to the learner's prior knowledge, emphasis on highlighting the associations between related mathematical concepts and procedures, and the like (Fleishner & Garnett, 1983). Similarly, individuals who are experiencing a deficit in Comprehension-Knowledge, along with difficulties in mathematics, also will benefit from mathematics instruction that considers the pertinent cognitive problems. In this case, the nature of the individual's informed understanding of mathematical concepts will need to be considered during instruction, and the necessary vocabulary, prior knowledge, and verbal reasoning strategies will need to be taught during mathematics instruction. Ginsburg (1989) provides numerous examples and recommendations for such instruction. Mather (1991) also presents much information related to direct instruction in mathematics that considers an individual's cognitive deficits.

Several study limitations suggest caution in the interpretation of the results and provide direction for future investigations. First, the current study was cross-sectional in nature. A longitudinal investigation of the relationships between the WJTCA-R cognitive clusters and mathematics achievement would be the next logical step to examine predictive validity further. Second, there exists the possibility that certain *Gf-Gc* abilities measured by the WJTCA-R cognitive clusters also may have indirect paths of influence that are mediated through other *Gf-Gc* abilities. The exploratory method used in this study specified that all seven WJTCA-R clusters be included in a full regression model. In such a model each cluster has a single direct path or influence on the respective mathematics achievement criteria. A study that specifies a complex causal model, estimating both direct and indirect influences, would need to be conducted to capture the total direct and indirect influence on each *Gf-Gc* ability. Third, the failure to find a significant relationship between a specific cognitive cluster and mathematics achievement does not mean that the specific ability measured by that cluster is completely unrelated to the achievement criterion. The findings only indicate that when considered together with other measures of *Gf-Gc* abilities, the abilities measured by the other WJTCA-R clusters accounted for most of the variance in the mathematics achievement criteria. Fourth, the focus of this study was on the age-based relationship between the WJ-R's cognitive clusters and mathematics achievement clusters. The association of gender and race was not considered in this relationship, but it certainly is a possible issue to consider in future research. Fifth, this study employs the WJ-R standardization sample, and, therefore, it does not represent an independent validity study. The use of an independent sample to cross-validate these results would represent important future research. Finally, the current study only indicates that the WJTCA-R includes measures of *Gf-Gc* abilities that are related differentially to mathematics achievement. Moving from this finding to the design and evaluation of *Gf-Gc* based academic intervention studies is the next critical step that needs to be pursued through additional research.

REFERENCES

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies.* New York: Cambridge University Press.

Cattell, R. B. (1941). Some theoretical issues in adult intelligence testing. *Psychological Bulletin, 38,* 592. (Abstract)

Cawley, J. F. (1985). Thinking. In J. F. Cawley (Ed.), *Cognitive strategies and mathematics for the learning disabled* (pp. 131-161). Rockville, MD: Aspen Systems.

Fleishner, J. E., & Garnett, K. (1983). Arithmetic difficulties among learning-disabled children: Background and current directions. *Learning Disabilities, 2,* 111-125.

Gagné, E. D. (1985). *The cognitive psychology of school learning.* Boston: Little, Brown.

Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. *Learning Disabilities Research and Practice, 7,* 210-216.

Ginsberg, H. P. (1989). *Children's arithmetic: How they learn it and how you teach it* (2nd ed.). Austin, TX: Pro-ed.

Glover, J. A., Ronning, R. R., & Bruning, R. H. (1990). *Cognitive psychology for teachers.* New York: Macmillan.

Hasselbring, T. S., Goin, L. I., & Bransford, J. D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20,* 1-7.

Hessler, G. L. (1993). *Use and interpretation of the Woodcock-Johnson Psycho-Educational Battery—Revised.* Chicago: Riverside.

Horn, J. L. (1965). *Fluid and crystallized intelligence: A factor analytic and development study of structure among primary mental abilities.* Unpublished doctoral dissertation, University of Illinois.

Horn, J. L. (1991). Measurement of intellectual capabilities: A review of theory. In K. S. McGrew, J. K. Werder, & R. W. Woodcock, *WJ-R technical manual.* Chicago: Riverside.

Kaufman, A. (1990). *Assessing adolescent and adult intelligence.* Needham, MA: Allyn & Bacon.

Kirby, J. R., & Becker, L. D. (1988). Cognitive components of learning problems in arithmetic. *Remedial and Special Education, 9,* 7-16.

Lohman, D. F. (1989). Human intelligence. An introduction to advances in theory and research. *Review of Educational Research, 59,* 333-373.

Mather, N. (1991). *An instructional guide to the* Woodcock-Johnson Psycho-Educational Battery-Revised. Brandon, VT: Clinical Psychology.

Mayer, R. E. (1986). Mathematics. In R. F. Dillon & R. J. Sternberg (Eds.), *Cognition and instruction* (pp. 127-154). New York: Academic Press.

McGrew, K. (1993). The relationship between the Woodcock-Johnson Psycho-Educational Battery—Revised *Gf-Gc* cognitive clusters and reading achievement across the life-span. *Journal of Psychoeducational Assessment,* WJ-R Monograph.

McGrew, K. (1994). *Clinical interpretation of the Woodcock-Johnson Tests of Cognitive Ability-Revised.* Boston: Allyn & Bacon.

McGrew, K., & Knopik, S. N. (1993). The relationship between the WJ-R *Gf-Gc* cognitive clusters and writing achievement across the life-span. *School Psychology Review, 22,* 687-695.

McGrew, K., Werder, J., & Woodcock, R. (1991). *WJ-R technical manual.* Chicago: Riverside.

Pellegrino, J. W., & Goldman, S. R. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities, 20,* 23-32, 57.

Reschly, D. (1990). Found: Our intelligences: What do they mean? *Journal of Psychoeducational Assessment, 8,* 259-267.

Resnick, B. & Ford, W. W. (1981). *The psychology of mathematics for instruction.* Hillsdale, NJ: Erlbaum.

Smith, D. O., & Rivera, D. P. (1991). Mathematics. In B.Y.L. Wong (Ed.), *Learning about learning disabilities* (pp. 345-374). New York: Academic Press.

Wilkinson, L. (1990). *SYGRAPH: The system for graphics.* Evanston, IL: SYSTAT.

Woodcock, R. (1978). *Development and standardization of the Woodcock-Johnson Psycho-Educational Battery.* Chicago: Riverside. Riverside.

Woodcock, R. (1990). Theoretical foundations of the WJ-R measures of cognitive ability. *Journal of Psychoeducational Assessment, 8,* 231-258.

Woodcock, R., & Dahl, M. (1971). *A common scale for the measurement of person ability and test item difficulty* (AGS Paper No. 10). Circle Pines, MN: American Guidance Service.

Woodcock, R., & Johnson, M. (1989). *Woodcock-Johnson Psycho-Educational Battery—Revised.* Chicago: Riverside.

Ysseldyke, J. (1990). Goodness of fit of the Woodcock-Johnson Psycho-Educational Battery-Revised to the Horn-Cattell *Gf-Gc* theory. *Journal of Psychoeducational Assessment, 8,* 268-275.

Zachary, R., & Gorsuch, R. (1985). Continuous norming: Implications for the WAIS-R. *Journal of Clinical Psychology, 41,* 86-94.

Zentall, S. S. (1990). Fact-retrieval automatization and math problem solving by learning disabled, attention-disordered, and normal adolescents. *Journal of Educational Psychology, 82,* 856-865.