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CATTELL-HORN-CARROLL BROAD COGNITIVE ABILITY PROFILES OF LOW MATH ACHIEVERS

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This study extends previous research examining the relations between Cattell-Horn-Carroll cognitive abilities and math achievement. The cognitive profiles of children with normative weaknesses in Math Calculation Skills or Math Reasoning were compared to those of their average-achieving peers. The cognitive profile of the low Math Calculation Skills group ($n = 68$) was similar to that of their average-achieving peers. The low Math Reasoning group ($n = 52$) scored lower than their average-achieving peers on the cognitive abilities as a set and on Fluid Reasoning and Comprehension–Knowledge. When individual profiles were considered, approximately half of the children with normative math weaknesses demonstrated commensurate weaknesses in one or more cognitive abilities, which may inform diagnostic models of learning disabilities.

Current estimates indicate that approximately 5 to 7% of the school-age population has remarkable difficulty in math achievement, a statistic that presents a challenge for a society that demands at least minimal math competency for success in formal schooling, daily living, and employment (Geary & Hoard, 2001; Light & DeFries, 1995). However, the complex array of mathematical domains obfuscates understanding of the general population of poor math learners (Geary, Hoard, & Hamson, 1999). A useful schema for thinking parsimoniously about the array of math domains is to group them into two factors: *math calculation skills* and *math reasoning* (Flanagan, Ortiz, Alfonso, & Mascolo, 2002; *Individuals with Disabilities Education Act*, 1997). Math Calculation Skills refers to the application of mathematical operations (e.g., addition, subtraction) and basic axioms (e.g., commutative property, inverse operations) to solve mathematical problems. Math Reasoning, in contrast, refers to the ability to problem solve using knowledge about math operations and axioms, numerical relationships, and quantitative concepts.

The origins of proficiency in these two math factors are complex. They include ecological variables such as home environment (Mullis, Dossey, Owen, & Phillips, 1991; Walberg, 1984) and math instruction, including quality of textbooks (Carnine, 1991; Mullis et al., 1991; Russell & Ginsburg, 1984). Far less is known about the underlying cognitive abilities that contribute to math calculation skills and math reasoning (Geary, 1994; Rourke & Conway, 1997). The cognitive abilities that have been the focus of most investigations of math skills are information retrieval (Geary, 1990, 1994; Geary, Brown, & Samaranayake, 1991), working memory (Geary, 1994; Hitch & McAuley, 1991; Shafir & Siegel, 1994; Swanson, 1994), and speed of processing (Bull & Johnston, 1997; Geary, 1994).

Assimilating research on the cognitive abilities underlying math performance has been difficult, primarily because researchers have used varying models of cognitive abilities to guide their studies. That is, one researcher may explore the relationship between math and visual–auditory processing whereas another is interested in the connection between math and memory functions. This lack of consistency renders it difficult to draw conclusions about which cognitive abilities are

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the most closely associated with math performance. Furthermore, it has been posited that many important cognitive abilities (e.g., inductive reasoning) have been omitted entirely from the mathematics literature (Floyd, Evans, & McGrew, 2003). Thus, to examine the relative contribution of varying cognitive abilities to mathematics performance and to avoid the exclusion of potentially important explanatory variables, well-validated models must be used to guide our understanding of which cognitive abilities may contribute to or help explain math proficiency. The Cattell–Horn–Carroll (CHC) theory of cognitive abilities provides one such model (Carroll, 1993). The CHC theory is a hierarchical framework of cognitive abilities that consists of three strata describing varying levels of generality: general intelligence or *g* (stratum III), approximately 10 broad cognitive abilities (stratum II), and over 60 narrow cognitive abilities (stratum I).

Several studies have examined the relations between CHC cognitive abilities and math achievement. Both Floyd et al. (2003) and McGrew and Hessler (1995) found that the broad cognitive ability clusters of Comprehension–Knowledge, Fluid Reasoning, and Processing Speed displayed the most consistent relations with measures of math calculation skills and math reasoning. In addition, Floyd et al. found that Short-Term Memory also was a consistent, significant predictor of math achievement. These four factors have been associated with math achievement in several other instances, as has general intelligence (Hale, Fiorello, Kavanaugh, Hoeppner, & Gaitherer, 2001; Keith, 1999; McGrew, Flanagan, Keith, & Vanderwood, 1997; Williams, McCallum, & Reed, 1996).

The aforementioned studies have helped establish a good foundational understanding of the CHC general and broad cognitive abilities that are most predictive of math performance *in the general population of children and adults* (i.e., a normal population). However, research guided by CHC theory has not focused much attention on the patterns of cognitive abilities displayed by individuals with notable *normative* weaknesses in math. Only a recent study by Prevatt and Proctor (2003) found that college students presenting with math difficulty had a relative strength in Visual Processing and a relative weakness in Long-Term Retrieval, yet they displayed no normative strengths or weaknesses.

Using operational measures of the CHC broad cognitive abilities, the purpose of this study is to examine the broad cognitive ability profiles of children who display normative weaknesses in either math calculation skills or math reasoning. Profile analysis was used to compare the group profiles of the exceptionality groups to samples of children with average mathematics achievement. In addition to profile analysis conducted at the group level, the *normative* patterns of performance in the individual profiles of the two exceptionality groups (i.e., math calculation skills and math reasoning) were examined to identify the extent to which normative weaknesses in mathematics were accompanied by commensurate weaknesses in one or more broad cognitive ability areas. By focusing solely on the normative patterns of performance in the cognitive profile, many of the limitations of traditional profile analysis focusing on ipsative or interindividual patterns of performance are overcome (Carroll, 2000; Watkins, 2000).

This line of research is important for a couple of reasons. First, it contributes to the research examining the CHC broad cognitive abilities associated with math achievement. Studying exceptionality groups, such as those with normative weaknesses in mathematics, will likely add to the existing research designed to predict the full range of mathematics ability. Second, it assists in understanding whether there exists a "typical" CHC broad cognitive ability profile(s) for children with mathematics weaknesses. Understanding the group and individual profiles of children with mathematics weaknesses is a particularly important contribution for clinicians and researchers who ascribe to a diagnostic model of learning disabilities that necessitates identifying cognitive weaknesses that have empirically supported relationships with achievement (see Flanagan et al., 2002; Sternberg & Grigorenko, 2002).

Method

Participants

All participants were drawn from the school-age portion of the nationally representative Woodcock–Johnson III (WJ III) standardization sample (Woodcock, McGrew, & Mather, 2001a). Participants in the WJ III standardization sample were selected from the population using a stratified random sampling design that controlled for 10 individual (e.g., race, sex, educational level, occupational status) and community (e.g., community size, socioeconomic status) variables.

Procedure

Three clusters from the WJ III Tests of Achievement (Woodcock, McGrew, & Mather, 2001b) were used to select participants from the WJ III standardization sample: Math Calculation Skills (MCS), Math Reasoning (MR), and Broad Reading. Reliability estimates and validity evidence supporting the use and interpretation of these clusters are presented in McGrew and Woodcock (2001).

Math calculation skills. To select participants who demonstrated math calculation skills in the lowest 16th percentile for their age group, children with age-based standard scores of 85 or below on the MCS cluster were placed in the Low Achievement MCS group (LA MCS). In an effort to focus on normative achievement weaknesses specific to math, this sample was limited to children with standard scores in the average range and above (i.e., standard score $\geq$ 90) on the Broad Reading cluster. A total of 129 children met these two criteria. When cases with complete data for the CHC factor clusters (see Measures section) were considered, a sample of 68 children (31 girls and 37 boys) was selected for analysis. Children ranged in age from 6 to 18 years (M = 13.4, SD = 3.3). Approximately 66% of the sample were White (n = 45), 25% were Black (n = 17), 6% were Asian or Pacific Islander (n = 4), and 3% were American Indian (n = 2). Using father's education level as an index of socioeconomic status (SES), 15% of fathers did not complete high school (n = 10), 31% completed high school (n = 21), and 54% either attended college or obtained a college degree (n = 36). Father's education level was not available for 1 child.

Math reasoning. Consistent with the selection of participants for the LA MCS group, children with age-based standard scores of 85 or below on the MR cluster were placed in the Low Achievement MR group (LA MR). This sample also was limited to children who scored in the average range and above on the Broad Reading cluster. A total of 88 children met these criteria. When cases with complete data for the CHC factor clusters were considered, a sample of 52 children (27 girls and 25 boys) was selected for analysis. Children ranged in age from 6 to 18 years (M = 13.5, SD = 3.5). Approximately 61% of the sample were White (n = 32), 29% were Black (n = 15), 6% were Asian or Pacific Islander (n = 3), and 4% were American Indian (n = 2). Analysis of SES revealed that 33% of fathers did not complete high school (n = 17), 27% graduated from high school (n = 14), and 40% either attended college or obtained a college degree (n = 21).

Average Achievement group. To provide a comparison group that displayed no normative mathematics or reading weaknesses, children with age-based standard scores in the average range (i.e., scores between 90–110) on the MCS, MR, and Broad Reading clusters were first selected. A total of 342 children (164 girls and 178 boys) met these criteria. From this group, a sample of 68 children was randomly selected (via the SPSS 11.0.1 Select Cases subprogram) to compare to the LA MCS group. Children in this selected sample ranged in age from 6 to 18 years of age (M = 12.2, SD = 3.3). Approximately 79% of the sample were White (n = 54), 15% were Black (n = 10), and 6% were American Indian (n = 4). Analysis of SES revealed that 12% of fathers did not complete high school ($n = 8$), 46% graduated from high school ($n = 31$), and 42% either attended college or obtained a college degree ($n = 28$). Father's education level was not available for 1 child. From the total Average Achievement group, another sample of 52 children was randomly selected to compare to the LA MR group. Children in this selected sample ranged in age from 6 to 18 years of age ($M = 12.9$, $SD = 3.4$). Approximately 75% were White ($n = 39$), 19% were Black ($n = 10$), and 6% were American Indian ($n = 3$). Analysis of SES revealed that 14% of fathers did not complete high school ($n = 7$), 44% graduated from high school ($n = 23$), and 42% either attended college or obtained a college degree ($n = 22$).

To ensure adequate selection of groups, preliminary analyses were computed to compare each low math achievement group to its respective Average Achievement group on the selection variables (see Table 1). As expected, the mean score on the MCS cluster of the Low MCS group was significantly below that of the Average Achievement group, $t(67) = 127.49$, $p < .001$, and there was no significant difference between the groups on the Broad Reading cluster, $t(67) = 123.96$, $p < .001$. Similarly, the mean score on the MR cluster of the Low MR group was significantly below that of the Average Achievement group, $t(51) = 143.79$, $p < .001$, and there was no significant difference between the groups on the Broad Reading cluster, $t(51) = 85.88$, $p < .001$.

Measures

Seven clusters from the WJ III Tests of Cognitive Abilities (Woodcock, McGrew, & Mather, 2001c) were used as dependent variables in the profile analyses: Comprehension–Knowledge, Long-Term Retrieval, Visual–Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short-Term Memory. Descriptions of the CHC factor clusters appear in Table 1. Reliability and validity information for the CHC factor clusters is presented in McGrew and Woodcock (2001). For reference, reliability estimates for the CHC factor clusters appear in Table 1. Validity evidence supporting the use and interpretation of these clusters is summarized in Floyd, Shaver, and McGrew (2003).

Table 1
CHC Factor Cluster Means and Standard Deviations for the Low-Achievement Groups and the Average-Achievement Groups

	LA MCS (n = 68)			Average-achievement group: MCS			LA MR (n = 52)			Average-achievement group: MR		
	M	*SD*	Range	*M*	*SD*	Range	*M*	*SD*	Range	*M*	*SD*	Range
Cluster												
MCS	80.16	5.18	58–85	100.90	5.53	90–110	–	–	–	–	–	–
MR	–	–	–	–	–	–	81.15	4.07	65–85	100.33	5.36	90–110
BR	98.34	6.54	90–118	100.31	5.49	90–110	98.31	8.25	90–134	100.90	5.33	90–110
Gc	99.90	10.51	70–122	100.91	10.66	81–133	95.58	10.95	76–125	101.10	8.75	87–128
Glr	100.03	10.34	81–125	102.63	13.36	76–144	99.08	9.55	78–126	103.77	11.07	77–137
Gv	99.06	13.08	70–139	99.43	12.48	64–125	95.63	11.95	75–121	100.94	11.88	64–127
Ga	100.68	13.68	74–142	102.66	13.12	78–144	100.31	14.45	74–136	104.08	12.88	72–150
Gf	96.16	13.57	59–139	99.56	14.11	63–136	90.94	12.72	59–125	100.77	12.14	69–136
Gs	95.94	12.57	65–124	100.19	13.46	77–134	100.25	13.35	65–133	97.04	12.13	57–123

Note. LA MCS = Low-Achievement Math Calculation Skills; LA MR = Low-Achievement Math Reasoning; MCS = Math Calculation Skills; MR = Math Reasoning; BR = Broad Reading; *Gc* = Comprehension–Knowledge; *Glr* = Long–Term Retrieval; *Gv* = Visual–Spatial Thinking; *Ga* = Auditory Processing; *Gf* = Fluid Reasoning; *Gs* = Processing Speed; *Gsm* = Short-Term Memory.

Analyses

Statistical tests grouped under the profile analysis rubric were conducted to compare the cognitive profiles of each LA group to the cognitive profiles of their average-achieving counterparts (Tabachnick & Fidell, 2001). First, the *parallelism* test was used to determine if the patterns of highs and lows on the CHC factor clusters were similar across groups (i.e., if the shape of the profiles were similar). Second, the *flatness* test was used to determine if the combined groups' scores were notably higher or lower on any of the CHC factor clusters (i.e., profile scatter). Third, the *levels* test was used to determine if the LA groups scored significantly lower than the Average Achievement groups on the CHC factor clusters as a set. In addition, planned comparisons were conducted to examine the degree of normative deviation of each CHC factor cluster for each group.

The individual cognitive profiles of children in the LA groups also were examined. To determine the extent to which individual children with normative mathematics weaknesses exhibited commensurate weaknesses in any of the measures of broad cognitive abilities, the percentage of children displaying standard scores of 85 or below on each CHC factor cluster was examined. To provide a contrast to the percentage of normative weaknesses, the percentage of children displaying normative strengths (standard scores of 115 and above) also was examined.

Results

Data screening procedures were conducted before computing each profile analysis, and assumptions regarding multivariate normality, absence of outliers, linearity, and homogeneity of variance–covariance matrices were met for both analyses (Tabachnick & Fidell, 2001). Table 1 presents the CHC factor cluster means and standard deviations for the four groups.

Group-Level Profile Analysis

LA MCS versus Average Achievement group. When the cognitive profiles of the groups were compared, the test for parallelism was nonsignificant, indicating that the LA MCS and the Average Achievement group exhibited similar high and low points in their profiles, $F(6, 129) = .60$, $p = .73$, $\eta^2 = .03$. When averaged over groups, the CHC factor scores deviated significantly from flatness, $F(6, 129) = 3.42$, $p = .004$, $\eta^2 = .14$, indicating variability among cluster scores. The levels test indicated that the groups performed similarly on the CHC factor clusters as a set, $F(1, 134) = 2.98$, $p = .09$, $\eta^2 = .02$. When respective CHC factor clusters were compared between groups, no significant differences were found.

Individual contrasts were conducted within each group to determine whether mean scores for the CHC factor clusters differed significantly from those of the normative population ($M = 100$, $SD = 15$). To control for multiple comparisons and reflect an experimentwise alpha of .05, the alpha rate was set at .007. A series of one-sample z tests were conducted using a criterion z value of ± 2.69. No significant differences were noted in either group.

LA MR versus Average Achievement group. The test for parallelism was significant, indicating that the LA MR and Average Achievement groups exhibited different high and low points in their profiles, $F(6, 97) = 2.62$, $p = .02$, $\eta^2 = .14$. To evaluate deviation from parallelism, confidence limits were computed around the CHC factor cluster means of both groups combined. The alpha rate was set at .0036 for each confidence interval to control for multiple comparisons and to reflect an experimentwise alpha of .05. The LA MR group demonstrated reliably lower performance on the Processing Speed cluster than that of the pooled mean for that cluster.

When averaged over groups, the CHC factor cluster scores deviated significantly from flatness, $F(6, 97) = 4.20$, $p = .001$, $\eta^2 = .21$. The levels test indicated that the LA MR group scored significantly lower than the Average Achievement group on the CHC factor clusters as a set, $F(1, 102) = 8.98$, $p = .003$, $\eta^2 = .08$. When respective CHC factor clusters were compared between groups using an adjusted alpha rate of .007, the LA MR group scored significantly lower than the Average Achievement group on the Fluid Reasoning and Comprehension–Knowledge clusters.

Individual contrasts were conducted within each group to determine whether mean scores for the CHC factor clusters differed significantly from those of the normative population. Using an adjusted alpha rate of .007, a series of one-sample z tests were conducted using a criterion z value of ± 2.69. No significant score deviations were found within the Average Achievement group, but the LA MR group scored significantly lower than the normative population on the Fluid Reasoning cluster.

Individual-Level Normative Profile Analysis

Math calculation skills. Although all children in the LA MCS group displayed normative weaknesses in performing calculations and completing basic operations, only 36 children (53%) displayed at least one CHC factor cluster score of 85 or below, and 17 of these 36 children (25% of the total sample) displayed scores in this range on more than one CHC factor cluster. As evidenced by Figure 1a, children in the LA MCS group most frequently demonstrated cognitive weaknesses commensurate with low achievement on the Fluid Reasoning cluster (14 children or 21%), the Short-Term Memory cluster (21%), and Visual–Spatial Thinking cluster (11 children or 16%).

When normative strengths of children in the LA MCS group are considered, 34 children (50%) displayed at least one CHC factor cluster score of 115 or higher. The most frequent areas of normative strength for the LA MCS group were on the Short-Term Memory cluster (14 children or 21%) and the Auditory Processing cluster (11 children or 16%). For this sample, the percentage of

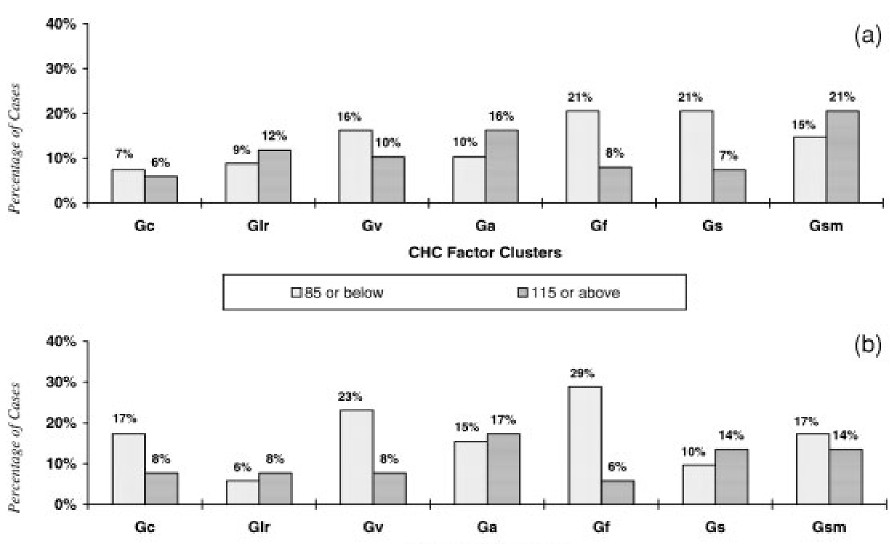

FIGURE 1. Frequencies of normative weaknesses and strengths for the Low Achievement Math Calculation Skills group (a). Frequencies of normative weaknesses and strengths for the Low Achievement Math Reasoning group (b). Gc = Comprehension-Knowledge, Glr = Long-Term Retrieval, Gv = Visual-Spatial Thinking, Ga = Auditory Processing, Gf = Fluid Reasoning, Gs = Processing Speed, Gsm = Short-Term Memory.

children with normative weaknesses for any CHC factor cluster was largely similar to the percentage of children with normative strengths for that CHC factor cluster for five of the seven clusters (the exceptions being the Fluid Reasoning and Processing Speed clusters). In fact, normative strengths were more commonly demonstrated on the Long-Term Retrieval, Auditory Processing, and Short-Term Memory clusters than were normative weaknesses.

Math reasoning. Similar to the number of normative weaknesses demonstrated by the LA MCS group across the CHC factor clusters, 30 children (58%) in the LA MR group demonstrated at least one CHC factor cluster score of 85 or below. Furthermore, 16 of these 30 children (16% of the total sample) displayed scores in this range on more than one CHC factor cluster. As evident in Figure 1b, normative weaknesses for the LA MR group most frequently occurred on the Fluid Reasoning cluster (15 children or 29%), the Visual–Spatial Thinking cluster (12 children or 23%), the Comprehension–Knowledge cluster (9 children or 17%), and the Short-Term Memory cluster (17%).

When normative strengths of children in the LA MR group are considered, 26 children (50%) displayed at least one CHC factor cluster score of 115 or higher. As evident in Figure 1b, the LA MR group most frequently obtained scores in this range on the Auditory Processing cluster (9 children or 17%), the Processing Speed cluster (7 children or 14%), and the Short-Term Memory cluster (14%). Again, the percentage of children with normative weaknesses in any CHC factor cluster was largely similar to the percentage of children with normative strengths in that same CHC factor cluster for four of the seven clusters (with the notable exceptions of Comprehension–Knowledge, Visual–Spatial Thinking, and Fluid Reasoning). In fact, there were *more* normative strengths than there were weaknesses at this level for the Long-Term Retrieval, Auditory Processing, and Processing Speed clusters.

Discussion

The purpose of this study was threefold: to contribute to the research examining the CHC broad cognitive abilities associated with math calculation skills and math reasoning, to shed light on the cognitive ability weaknesses that presumably contribute to math weaknesses, and to assist in answering the question of whether there is a "typical" CHC cognitive profile for children with calculation and math reasoning weaknesses. Previous CHC research had primarily implicated the broad cognitive ability clusters of Comprehension–Knowledge, Fluid Reasoning, Processing Speed, and Short-Term Memory in the explanation of math performance; however, using a different methodology and statistical analysis, we found notably different results.

Math Calculation Skills Profiles

First, we looked at students with achievement weaknesses in math calculation skills or math reasoning and compared their cognitive profiles to those with average math and reading skills. When students who performed poorly on math calculation were compared to the average-achieving group, no difference in the overall level of performance across abilities was indicated. The comparative analyses also showed that none of the specific cognitive abilities "separated" the low-achievement group from the average-achieving group. This lack of significant differences between the LA MCS group and the average-achieving group was surprising, given the results of previous CHC studies. Additionally, none of the CHC factor clusters for this group was significantly below the population mean (M = 100); thus, as a group, they demonstrated no normative cognitive weaknesses.

Although the group-level profile analysis revealed that the mean CHC factor cluster scores for the LA MCS group were similar to an average-achieving group and to the normative population, their individual profiles indicated that approximately half of the children displayed commensurate weaknesses on at least one CHC factor cluster. As evident based on the group-level profile analysis, there was little consistency in the normative weaknesses identified within this group of 36 children. In fact, the normative weaknesses in some participants (e.g., Short-Term Memory) were just as likely to be normative strengths in others. The only exception to this finding was that notably more normative weaknesses than strengths were found in Fluid Reasoning and Processing Speed. Each was a weakness for only 1 in 5 children sampled.

What else, then, might explain these students' poor math calculation skills? Perhaps the most likely explanation is that the poor math calculation skills in this sample are not due primarily to underlying cognitive weaknesses. Rather, they may be attributable to noncognitive influences such as lack of experience, poor motivation, anxiety, poor instruction, or mediocre textbooks. Indeed, in their studies, Geary and colleagues (e.g., Geary, 1990; Geary, Bow-Thomas, & Yao, 1992) found that roughly half of the children who had been identified as having a learning problem in mathematics did not show *any* form of cognitive weakness. Research on noncognitive influences related to mathematical underachievement has indicated, for example, that there is a tremendous amount of variability in the level of difficulty in math curricula around the United States (Travers & Westbury, 1989), thus implying that not all same-grade children are receiving comparable math instruction. Not all educators have embraced the importance of explicit instruction, drill, and practice in the acquisition of basic mathematical skills (Briars & Siegler, 1984). Apart from formal instruction, the home environment plays a role in early numerical skills (e.g., understanding the concept of quantity, counting, and arithmetic), as these skills often develop within the context of parent–child interactions (Saxe, Guberman, & Gearhart, 1987). Finally, lower levels of mathematical ability are associated with higher levels of mathematics anxiety, although math anxiety is not strongly related to general intelligence (Geary, 1994). It is therefore very possible that many children in the LA MCS sample had math calculation weaknesses due to some of the aforementioned noncognitive factors rather than weaknesses in a CHC broad cognitive area.

A second plausible explanation for our findings is related to how the sample was selected. In identifying participants for the LA MCS group, the requirement was an unusually low score on the Math Calculation Skills cluster coupled with at least an average score on the Broad Reading cluster. However, in reality, many students with math difficulty, or even math learning disabilities, also exhibit difficulties in reading and other academic areas. Investigators frequently describe two types of math underachievement associated with learning disabilities (Fleischner & Manheimer, 1997; Silver, Pennett, Black, Fair, & Balise, 1999; Strawser & Miller, 2001). The first dysfunction is a primary math disability, also called nonverbal learning disability, right-hemisphere disability, or dyscalculia. It is described as a primary impairment that specifically affects math (Matte & Bolaski, 1998; Rourke & Conway, 1997). A second dysfunction is described as a math achievement weakness related to verbal learning disabilities (Fleischner & Manheimer, 1997), and it is associated with reading difficulties that consequently impede the ability to complete math word problems. Thus, it is likely that, for many children, reading and math difficulties co-occur because of underlying cognitive weaknesses (Geary, 1994). These cognitive weaknesses, however, may not have appeared in our select sample of children with math-only problems. This might also explain why previous studies (e.g., Floyd et al., 2003; Hale et al., 2001; Keith, 1999; McGrew et al., 1997; McGrew & Hessler, 1995; Williams et al., 1996) found relationships between math calculation and specific cognitive abilities. They not only included scores across the range of mathematical abilities but also most likely included scores from participants with low achievement in both math and reading. Hence, the relationship between math calculation and Processing Speed, Comprehension–Knowledge, Fluid Reasoning, and Short-Term Memory found in previous studies may be tenable only for those with at least average skills in math calculation or for those with "double weaknesses" in both math and reading.

Additional explanations are that (a) the WJ III clusters that were included as dependent variables in this analysis did not adequately measure the cognitive abilities that underlie poor math calculation skills, and (b) low math achievers are such a heterogeneous group that it is impossible to identify a cognitive profile that is generalizable to even a small subset of low math achievers (Strang & Rourke, 1985). Additional research in this area is clearly indicated before we can begin to draw any strong conclusions.

Math Reasoning Profiles

The profile analysis using Math Reasoning as the grouping variable painted a different picture. Overall, the LA MR group scored lower than the Average Achievement group on the aggregate measure of CHC abilities, which may suggest group differences in *g*. As a group, their scores on Fluid Reasoning and Comprehension–Knowledge were lower than those of the Average Achievement group. In addition, their mean score on the Fluid Reasoning cluster was significantly lower than the population mean, indicating a normative weakness. Regarding the individual profiles, similar to the findings of the LA MCS group, more than half of the children in the LA MR group demonstrated at least one cognitive weakness. Most weaknesses were found in the areas of Fluid Reasoning, Visual–Spatial Thinking, Comprehension–Knowledge, and Short-Term Memory. Additionally, exactly half of the LA MR students had normative strengths in one or more CHC areas. The most frequent areas of strength were Auditory Processing, Processing Speed, and Short-Term Memory. The fact that Short-Term Memory emerged as a strength and a weakness for approximately the same number of children, coupled with the fact that it was *not* identified as a group weakness for the sample of LA MR children, suggests that individual differences in Short-Term Memory may not be as involved in poor math reasoning as previously thought. Fluid Reasoning, on the other hand, emerged as a consistent weakness for the group as a whole, in addition to being the most frequent normative weakness (and the least common strength) in the individual profiles. The relationship between math reasoning and Fluid Reasoning resonates with the findings of previous studies examining the statistical relations between math skills and underlying cognitive abilities of reasoning and novel problem-solving skills.

The findings regarding the relationships between Comprehension–Knowledge and math reasoning, and Visual–Spatial Thinking and math reasoning are the most difficult to interpret within the context of the present study. Comprehension–Knowledge emerged as a frequent normative weakness (and an infrequent strength) in the individual profiles, and it was a group weakness for the LA MR group when compared to that of the Average Achievement group. However, there were no group differences when comparing the LA MR score to those of the normative population. Previous studies on the relations between CHC cognitive abilities and math achievement have consistently identified Comprehension–Knowledge as one of the clusters with the strongest relationships with math achievement. Therefore, additional studies using children with normative weaknesses in math reasoning are probably necessary before we can draw any firm conclusions about the role that Comprehension–Knowledge plays within this subpopulation.

Like Comprehension–Knowledge, Visual–Spatial Thinking emerged as a frequent normative weakness and an infrequent normative strength. However, unlike Comprehension–Knowledge, no group differences were found for this cluster. A strong link between math reasoning and Visual–Spatial Thinking has not been found in previous CHC research, although it has been indicated elsewhere (e.g., Geary, 1994; Padget, 1998; Rourke, 1993; Shafir & Siegel, 1994; Strawser & Miller, 2001). In his meta-analysis of studies investigating the correlation between spatial skills and math skills, Friedman (1995) concluded that there is no convincing evidence that spatial skills are strongly related to math ability. However, there seems to be very little consistency in the way that visual–spatial thinking (i.e., spatial skills, visual processing) is defined and measured. This inconsistency in conceptualization and measurement is one likely explanation for the disparate findings regarding the relationship between math skills and visual–spatial thinking, in addition to methodological differences among the various studies.

Although previous CHC research consistently demonstrated a relationship between math reasoning and the underlying cognitive ability of Processing Speed, we were unable to reproduce those findings in the current study. The reasons for these results are not completely understood, but a tentative conclusion is that Processing Speed underlies average or above-average skills in math reasoning, but is unrelated to below-average math reasoning. These results also may have surfaced because students with double weaknesses in mathematics reasoning and reading were omitted from this study. Finally, note that Long-Term Retrieval has not been identified as an important factor for math reasoning in either the present or the previous CHC studies.

Limitations

The results of this research are limited in several ways. First, because the low-achievement groups were empirically derived and excluded children with concomitant reading weaknesses, results of this study may not be directly applicable to the population of children with severe learning difficulties or learning disabilities in mathematics. The results also may not be generalizable to students with learning disabilities who were diagnosed using models or criteria that differ dramatically from the low-achievement model employed in this study (i.e., math achievement score ≤16th percentile, with other achievement scores falling in the average range). Future research should include samples of children who are diagnosed with learning disabilities under competing diagnostic models. Second, several CHC factor clusters may measure some of the same cognitive abilities as the mathematics clusters. For example, the Quantitative Reasoning test, which is included in the MR cluster, probably measures both Quantitative Knowledge and Fluid Reasoning (McGrew & Woodcock, 2001). These relations may have contributed to the finding of frequent concomitant normative weaknesses in Fluid Reasoning when MR was a normative weakness.

Implications for Practice

Together, the results of the group and individual profile analyses suggest that there may not be a unique profile of cognitive abilities for children with normative weaknesses in math calculation skills. In contrast, a common cognitive profile of children with normative weaknesses in math reasoning may show commensurate weaknesses in the areas of Fluid Reasoning and, perhaps, Comprehension–Knowledge. The results of the profile analyses, particularly those performed at the individual level, are problematic for clinicians and researchers who ascribe to a diagnostic model of learning disabilities that necessitates identifying cognitive weaknesses that have empirically supported relationships with underachievement.

In our sample, 1 in 2 children with severe weaknesses in math calculation skills displayed at least one commensurate weakness in a CHC factor cluster area. Similarly, for the math reasoning sample, approximately 1 in 2 children had commensurate cognitive weaknesses. The finding that approximately 50% of our low achievement samples *had no normative cognitive weaknesses* was surprising. Had we been evaluating this sample of 120 children for the presence of a learning disability as defined by a normative weakness in an academic area and a commensurate weakness in at least one specific cognitive ability, only 60 would even be eligible for proceeding in the diagnostic process. For the other 60 children not eligible for a learning disability diagnosis, three options remain: (a) decide that the achievement weakness is not due to a learning disability but rather due to some variable unrelated to the child's cognitive processing, (b) suggest that models requiring both low achievement and at least one concomitant specific cognitive weakness are not valid for diagnosing learning disability in mathematics, or (c) conclude that the tests administered did not accurately capture the child's cognitive abilities, and retest the child using different instruments.

We wish to make a final, positive point regarding an implication of these findings for practice. If, as in the current samples, half of the children with weaknesses in math calculations or math reasoning do not have commensurate cognitive weaknesses, perhaps this bodes well for the probability that interventions will succeed in improving the academic skill. After all, if the cause of the underachievement is not due to a cognitive factor intrinsic to the child, then perhaps the poorly developed math skills are amenable to interventions that modify or enhance instruction or that alter the academic environment. This amenability to intervention is exactly what researchers and practitioners espousing a response to intervention model of diagnosing learning disabilities are targeting (e.g., see National Association of School Psychologists, 2003). Perhaps the children *without* cognitive ability weaknesses are those who will respond to empirically supported interventions, and the sample of those children who fail to respond will present with a more consistent, discernable profile (or profiles) of cognitive abilities.

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