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FACTOR ANALYSES OF THE CALIFORNIA TEST OF MENTAL MATURITY[1]

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THE California Test of Mental Maturity (CTMM) was originally designed and published in 1936 as a group intelligence test. Its purpose was to assess the major mental functions tested in the Stanford-Binet individual intelligence test. The original factor analysis of variables variously used in the CTMM was accomplished in 1938 and, although it was not published, it served to provide guidelines for the organization of the subtests in subsequent batteries. In addition, a short form (CTMM-SF) version of the battery was developed and first published in 1938. Many studies and analyses have been conducted with past editions and revisions of the CTMM and CTMM-SF (e.g. Anderson, 1961a, 1961b; Anderson and Slivinske, in press),[2] but throughout the years they have been maintained in their original purpose.

The 1963 revisions of the CTMM and CTMM-SF are quite similar to past editions and revisions of the instruments in purpose and organization. The tests (including factors and sections) were scaled to the 1960 revision of the Stanford-Binet and, therefore, have adopted the deviation-form of intelligence quotient. Moreover,

[1] The authors are indebted to the California Test Bureau for financial support of this study.

[2] A complete bibliography of research is available free of charge, also, from the California Test Bureau, Del Monte Research Park, Monterey, California; order, "Summary of Investigations Number Three: California Test of Mental Maturity," (1956).

many studies and analyses have been conducted with the new revision of the CTMM and the CTMM-SF to examine properties of the subtests and to strengthen the organization of the subtests into factors and sections. One of the most important sets of studies is the set of factor analytic studies reported herein.

The intent in the construction of the CTMM was to provide composite batteries with subtests of logical reasoning, spatial (or abstract) relations, verbal ability, numerical ability, and memory. The CTMM-SF was designed to contain the logical reasoning factor as well as abbreviated forms of verbal and numerical ability and memory. In addition to the foregoing properties of the subtests, it was recognized that some of the subtests would require more language ability than others; therefore, it was theorized that the subtests should be reorganized in terms of their language and nonlanguage characteristics. Factor analysis provides a means of assessing the integrity of the construction and organization of the subtests in the batteries in terms of the latent structures.

Method

The 1963 revisions include eight levels of the CTMM and CTMM-SF available for testing the age range from five years to adulthood. Six of the levels (viz., 0, 1, 2, 3, 4, and 5) actually span the entire age range of the batteries while the two remaining levels (1h and 2h) are designed to compliment two of the lower level tests. This paper will present the results of the ten factor analyses of the CTMM and CTMM-SF for levels 1, 2, 3, 4, and 5; the analyses of level 0, 1h, and 2h are not available at the time of this report. The grades appropriate for the five levels in these analysis are as follows.

Level	Grades
1	1–3
2	4–6
3	7–9
4	9–12
5	11–College and Adult

The analyses, therefore, cover levels that span all of the school grades except for kindergarten.

Samples. The students selected for testing represent random samples from a larger standardization program plan containing these

and many other studies. The sample sizes for each of the levels was as follows: Level 1 = 285, Level 2 = 416, Level 3 = 273, Level 4 = 312, and Level 5 = 254.

Variables. All of the students in the samples were administered the 1963 revisions of both the CTMM and the CTMM-SF. The order of administration was randomly alternated for subgroups within the samples. The tests were administered by classroom teachers with a minimum of one week between the administration of the two instruments. It was recognized that practice effects might contaminate the data, but it was assumed that the effects on the correlations between the subtests would be uniform and not vitiate the genuine basic structures of the tests.

Analyses. All of the analyses were conducted with the CTMM and CTMM-SF separately at each of the five levels. Means, standard deviations, and K-R #21 reliability coefficients were determined for each of the subtests, and product-moment correlations were computed for each pair of variables. Each of the ten intercorrelation matrices was factored by the centroid method. The resulting factor structures were rotated by the varimax method (Kaiser, 1958) with subsequent rotations and adjustments made by hand (Zimmerman, 1946) according to the criteria of simple structure and positive manifold.

Results

Only the major results of the final rotated factor structures will be presented in the present paper. The complete set of basic data (means, standard deviations, and reliability coefficients, and intercorrelation, unrotated, and rotated factor matrices) can be obtained from the California Test Bureau.[3]

Table 1 presents the major results of the rotated factors at all five levels for the CTMM-SF. All variables with rotated loadings greater than .30 are listed appropriately in the table. It should be noted, however, that throughout the five analyses the communalities for the variables range between .17 and .74 with 63 percent of them above .50 and 86 percent of them above .40.

The results of the Short-Form analyses, as in similar studies

[3] Order, free of charge, "Basic Data for Factor Analytic Studies with the California Test of Mental Maturity, 1963 Revisions," from the California Test Bureau, Del Monte Research Park, Monterey, California.

TABLE 1

*Major Variables, together with Loadings, on the Rotated Factors of the California Short-Form Test of Mental Maturity**

Level	Factor I Logical Reasoning		Factor II Numerical Reasoning		Factor III Verbal/Memory	
1	Analogies	554	Number Problems	699	Delayed Recall	600
	Opposites	530	Numerical Values	662	Verbal Comprehension	598
	Verbal Comprehension	359	Opposites	398	Similarities	347
	Number Problems	346				
2	Similarities	541	Number Problems	609	Delayed Recall	690
	Analogies	469	Numerical Values	569	Verbal Comprehension	680
	Opposites	464	Analogies	428		
	Delayed Recall	330	Verbal Comprehension	400	Number Problems	481
			Delayed Recall	341	Numerical Values	415
					Similarities	372
3	Similarities	568	Numerical Values	683	Verbal Comprehension	742
	Analogies	547	Number Problems	670	Delayed Recall	689
	Opposites	457			Opposites	365
	Delayed Recall	338			Number Problems	330
					Analogies	324
4	Similarities	549	Numerical Values	587	Verbal Comprehension	740
	Analogies	402	Number Problems	520	Delayed Recall	694
	Opposites	345	Analogies	303	Analogies	501
					Number Problems	482
5	Analogies	560	Number Problems	644	Delayed Recall	763
	Similarities	490	Numerical Values	634	Verbal Comprehension	731
	Opposites	446	Similarities	364	Analogies	430
			Analogies	340	Similarities	353

* Decimals omitted from factor loadings.

(e.g., Clark, 1949), reveal a consistency of factors at all age levels under investigation. In general, the first factor is identified as a Logical Reasoning Factor determined, for the most part, by the Opposites, Similarities, and Analogies subtests. Factor II is called a Numerical Reasoning Factor with the main variables being Numerical Values and Number Problems. Factor III is identified as a Verbal/Memory Factor with the Verbal Comprehension and Delayed Recall variables. The loadings on all residual factors were uniformly below .30.

The results of the five factor analyses of the Long-Form CTMM are presented for levels two through five in Table 2a and Level 1

TABLE 2a

*Major Variables, together with Loadings, on Rotated Factors for Levels Two through Five of the California Test of Mental Maturity**

Level	Factor I Logical Reasoning		Factor II Spatial Relations		Factor III Numerical Reasoning		Factor IV Verbal Concepts		Factor V Memory	
2	Analogies	580	Rights and Lefts	402	Number Series	471	Verb. Comp.	735	Immed. Recall	587
	Similarities	490	Man. of Areas	400	Number Problems	461	Inferences	719	Delayed Recall	481
	Man. of Areas	440			Numerical Values	419	Delayed Recall	521	Verb. Comp.	344
	Opposites	438					Number Problems	520	Similarities	330
	Delayed Recall	372					Number Series	400	Numerical Values	309
	Number Problems	363								
	Numerical Values	320								
3	Similarities	525	Man. of Areas	550	Number Series	549	Verb. Comp.	740	Immed. Recall	488
	Opposites	512	Rights and Lefts	539	Numerical Values	405	Inferences	711	Delayed Recall	456
	Analogies	450	Number Problems	390	Number Problems	360	Delayed Recall	584	Analogies	300
			Similarities	364			Number Problems	530		
							Number Series	358		
							Opposites	311		
							Analogies	310		
4	Analogies	600	Man. of Areas	369	Number Series	448	Verb. Comp.	691	Delayed Recall	524
	Similarities	546	Rights and Lefts	328	Number Problems	428	Delayed Recall	537	Immediate Recall	494
	Opposites	480			Numerical Values	385	Number Problems	497	Similarities	327
	Inferences	335					Inferences	494		
							Number Series	397		
							Analogies	300		
5	Similarities	550	Man. of Areas	400	Numerical Values	610	Verb. Comp.	740	Delayed Recall	520
	Analogies	550	Immed. Recall	400	Number Problems	600	Inferences	631	Immed. Recall	350
	Opposites	373	Rights and Lefts	320	Number Series	500	Delayed Recall	530		
	Man. of Areas	328			Analogies	301	Number Problems	410		
							Number Series	380		
							Analogies	310		
							Immed. Recall	309		

* Decimals omitted from factor loadings.

TABLE 2b

*Major Variables, together with Loadings, on Rotated Factors for Level One of the California Test of Mental Maturity**

Factor I Logical Reasoning		Factor II Spatial/Reasoning		Factor III Spatial/Ordering		Factor IV Verbal Reasoning		Factor V Vocabulary	
Analogies	415	Manipulation of Areas	654	Numerical Values	509	Inferences	504	Verbal Comp.	543
Similarities	349	Number Problems	567	Rights and Lefts	479	Immediate Recall	499	Delayed Recall	527
		Opposites	533						
		Analogies	339						
		Inferences	301						

* Decimals omitted from factor loadings.

in Table 2b. The communalities for the variables in all five analyses range from .13 to .79 with 47 percent of them greater than .50 and 77 percent greater than .40. Only those variables with rotated loadings greater than .30 are listed in Tables 2a and 2b.

Similar to the analyses with the CTMM-SF, the results of analyses with the CTMM are very consistent in levels two through five. As indicated in Table 2a, Factor I is identified as a Logical Reasoning Factor with the major variables being Opposites, Similarities, and Analogies; this is the same factor as identified in the Short-Form analyses. Factor II is identified as a Spatial Relations Factor on the basis of the loadings of the Rights and Lefts and Manipulation of Areas subtests. The Numerical Reasoning Factor is the third factor with Number Series, Numerical Values, and Number Problems; this factor was also identified in the Short-Form analyses, but the Long-Form analyses contain the additional variable, Number Series. Factor IV is identified as a Verbal Concepts Factor with Number Problems, Inferences, Verbal Comprehension, and Delayed Recall. The fourth factor seems to be a combination of two verbal subtests, a numerical subtest and a memory subtest, all of which entail a great deal of language ability. Finally, the fifth factor is a Memory Factor as determined by the Immediate Recall and Delayed Recall subtests. All of the loadings on the residual factors are below .30.

The results of the analyses of the Long-Form CTMM at Level 1 do not appear to be completely consistent either with the Short-Form or the other Long-Form analyses. Factor I is identified as a Logical Reasoning Factor with Similarities and Analogies as the dominant variables. Factor II is called a Spatial/Reasoning Factor with the variables Opposites, Manipulation of Areas, and Number Problems. Factor III is a Spatial/Ordering Factor with Rights and Lefts and Numerical Values. The fourth factor is the Verbal Reasoning Factor determined, for the most part, with Immediate Recall and Inferences, while the fifth factor is a Vocabulary Factor with Verbal Comprehension and Delayed Recall. Further interpretation of this analysis is included below.

Discussion

As was stated previously in the Methods section of this paper, it was assumed that the practice effects involved in the administration

of the CTMM and the CTMM-SF, with a minimum of one week elapsed time between the two tests, would be fairly uniform across the correlational domain and hence would not effect the basic structure of the batteries. The consistency of the factor analytic results between and within levels lends a great deal of credence to this assumption. The stability of the correlational structure of the 1963 revisions of the CTMM and the CTMM-SF, as shown in the present study, should prove useful in future research, such as in longitudinal studies of the dynamic import of aptitude and achievement characteristics compared with the stability of intellectual functions.

A comparison of the analyses of the CTMM and the CTMM-SF reveal several interesting characteristics about the subtests. Aside from the analysis of the Level 1 Long-Form, the Spatial Relations Factor is not represented in the Short-Form, but the other four factors in the set of Long-Form analyses can be compared quite reasonably to the three factors generally identified in the set of Short-Form analyses. Both sets of analyses produce the Logical Reasoning Factor with the same three variables generally determining this factor. Also, the Numerical Reasoning Factor is consistent from Short-Form to Long-Form analyses. The Long-Form analyses, however, show the full complexity of the Number Problems as well as the Delayed Recall variables. In the Long-Form analyses, Number Problems appears on the Numerical Reasoning Factor, but the language characteristics resident in this variable force its high loading also on the Verbal Concepts Factor. Similarly, Delayed Recall loads quite high on the Verbal Concepts Factor in both sets of analyses, but it has a high loading, also, on a separate factor with Immediate Recall in the Long-Form analyses. This latter factor was identified as a Memory Factor. Memory factors have not been found widespread in factor analytic studies (e.g., Curtin, 1951, p. 51) but it appears in the present analyses with tenacious consistency. The two memory variables in the CTMM, therefore, might be used to advantage as "marker" variables in other factor analytic studies.

The full meaning of the results of the Level 1 CTMM analysis is not immediately obvious. The Level 1 Short-Form analysis was completely consistent with those at other levels, and since the subtests by name are fairly much the same within each level, one might reasonably have expected the Long-Form analysis to be consistent

with those at other levels. Apparently, however, the introduction of the spatial variables into the structure brings out the perceptual saturation in the variances of the two numerical and the Opposites subtests. This condition is undoubtedly generated by the necessary reliance on pictorial materials for children at this level. Moreover, the Memory Factor fails to appear in this analysis; the results here suggest that, for children of this age, syllogistic-type reasoning ability is closely related to their immediate recall ability, while vocabulary is more closely related to retention of a more durable nature. These interpretations seem reasonable but the results do not assist particularly in defining the structure of the Level 1 CTMM so that it is fully consistent with other levels.

It is noteworthy that only three factors appeared consistently in the Short-Form analyses while five factors were determined in the Long-Form analyses. In both sets of analyses, the number of factors is less than half the number of subtests. Albert (1944a and 1944b) has shown that, in such instances, unique communality solutions are possible (barring a "Heywood" (1931) circumstance) and the stability of communalities under various values of r/n, with r factors and n tests, was studied systematically by Wrigley (1957). Though the circumstance $r = n/2$ appears to be somewhat rare with actual data (e.g., Thurstone, 1947, p. 283), it does attend in the 1963 revisions of the CTMM and CTMM-SF so that the internal structures should prove stable in future analyses.

The factor analyses herein have provided a major basis for the internal organization of the 1963 revisions of the CTMM and the CTMM-SF. In addition to a total score, in general, the Long-Form at *all* levels will provide scales for five factors with the variables grouped as in the results of levels two through five, except that the Verbal Concepts Factor will include only the Inferences and Verbal Comprehension subtests; however, the variables are reorganized for scales in a Language section containing Numerical Reasoning, Verbal Concepts, Inferences, and Delayed Recall, and a Nonlanguage section containing all other subtests. Similarly, the Short-Form tests are scaled with four factors using the results herein with Verbal Comprehension and Delayed Recall on separate factors, but the Language section score is made up of Verbal Comprehension and Delayed Recall together with Number Problems, and the Nonlanguage section score is composed of all other variables. The

organization of the subtests into both factor and section scores which cut across one another allows for a full utilization of the complex characteristics of the Number Problems and Delayed Recall variables.

Summary

The 1963 revisions of the California Test of Mental Maturity and the California Short-Form Test of Mental Maturity were factor analyzed. The analyses included levels one through five, spanning grade levels one through college and adult, and were part of a larger standardization program.

In general, the three factors determined in the Short-Form analyses were the Logical Reasoning Factor, the Numerical Reasoning Factor, and the Verbal/Memory Factor. Five factors determined in the Long-Form analyses included the Logical and Numerical Reasoning Factors, a Spatial Relations Factor, a Memory Factor, and a Verbal Concepts Factor; the Level 1 Long-Form analysis, however, demonstrated the presence of perceptual characteristics resident in the subtests. The results were discussed in terms of the stability of the basic structure as well as the internal organization of the subtests for scaled scores.

REFERENCES

Albert, A. A. "The Matrices of Factor Analyses." *Proceedings of the National Academy of Science,* XXX (1944), 90–95. (a)

Albert, A. A. "The Minimum Rank of a Correlation Matrix." *Proceedings of the National Academy of Science,* XXX (1944), 144–146. (b)

Anderson, H. E., Jr. "The Prediction of Reading and Language from the California Tests." EDUCATIONAL AND PSYCHOLOGICAL MEASUREMENT, XXI (1961), 1035–1036. (a)

Anderson, H. E., Jr. "A Study of Language and Non-language Achievement." EDUCATIONAL AND PSYCHOLOGICAL MEASUREMENT, XXI (1961), 1037–1038. (b)

Anderson, H. E., Jr. and Slivinske, A. J. "A Study of Intelligence and Achievement at the Fourth, Fifth and Sixth Grade Levels." *Journal of Experimental Education,* XXXI (1963), 425–432.

Clark, W. W. "The Differentiation of Mental Abilities at Various Age Levels." *American Psychologist,* IV (1949), 291 (Abstract).

Curtin, J. T. *A Factor Analysis of Verbal and Non-Verbal Test of Intelligence.* Washington, D. C.: The Catholic University Press, 1951.

Heywood, H. B. "On Finite Sequences of Real Numbers." *Proceedings of the Royal Society,* CXXXIV (1931), 486–501.

Kaiser, H. F. "The Varimax Criterion for Analytic Rotation in Factor Analysis." *Psychometrika,* XXIII (1958), 187–200.

Thurstone, L. L. *Multiple-Factor Analysis.* Chicago: University of Chicago Press, 1947.

Wrigley, C. "The Effect upon the Communalities of Changing the Estimate of the Number of Factors." Research Report 13, Contract No. AF 41 (657) –76, University of California, 1957 (dittoed).

Zimmerman, W. S. "A Simple Graphic Method for Orthogonal Rotation of Axes." *Psychometrika,* XI (1946), 51–56.