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A FACTORIAL STUDY OF NUMBER ABILITY

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In order to investigate certain hypotheses concerning the nature of number ability, and, secondarily, the nature of perceptual speed, a battery of thirty-four tests was given to 223 Chicago high school seniors and the data were factored by the centroid method. Seven primary factors were identifiable upon rotation. Several deductions are made relative to the interpretation of the factors and relative to the consistency of the data with the hypotheses which were to be tested.

I. Introduction

Since the development of multiple factor methods by L. L. Thurstone, at least nine multiple-factor studies of cognitive abilities have been carried out in his laboratory. The general results of these studies have been in very high agreement. The same factors have occurred repeatedly in different test batteries with subjects from eighth-grade level to college freshmen. One of the clearest of the primaries and one that has been repeatedly obtained is number ability, characteristically defined by tests of addition and multiplication. These two tests are referred to as "simple" tests in that they have a large projection on only one or only a very few common factors.

It is not very satisfying to define an ability in terms of the content of the tests which measure that ability. This procedure should be regarded as a makeshift until investigations may be carried out which give insight into the psychological nature of the fundamental process characterizing the ability. For example we may ask: Is the ability defined by addition and multiplication specific to numerical manipulations or does it represent some more fundamental unity that transcends numerical operations? Is it inherent in the organism or a product of training? With the idea of gaining some insight into these questions we shall review some of the facts gained from case

* I wish to express my great appreciation of the aid of Professor L. L. Thurstone whose generosity made this study possible. Grateful acknowledgment is made of the aid of Mr. Ledyard Tucker in the use of the I.B.M. machines for obtaining the intercorrelations and the centroid factor loadings, and to both him and Mr. Harold Bechtoldt for aid in the testing of subjects.

histories of arithmetical prodigies and from the functional analysis of cases of acalculia or arithmetic disability.

Prior to the isolation of number ability by multiple factorial methods, there have been indications that this ability represents a functional unity in the mind. Scripture* in his article on arithmetical prodigies gave a brief account of more than fifteen individuals, dating from about 100 A.D., who gave evidence of phenomenal reckoning powers. Mitchell† also summarized what was known about mathematical prodigies and gives a brief account of his own case. Other psychologists have contributed to the knowledge of reckoning ability by psychological studies of individual cases.‡ Intensive experimental investigations of individual cases are lacking, most of the literature being biographical in nature. However, certain things which are of interest here seem to be clear. Mitchell states, "Skill in mental calculations is . . . . independent of general education; the mathematical prodigy may be illiterate or even densely stupid, or he may be an all-around prodigy and veritable genius."§ With the possible exception of memory for numbers, case histories of rapid mental calculators indicate that these individuals do not necessarily show any unusual ability in any other line of endeavor. Their most common characteristic is an unusual interest in and liking for numbers for their own sake. This interest is accompanied by a great deal of practice in their manipulation.

These data on mathematical prodigies seem to indicate that educational forces are of slight importance to this ability. A large proportion of the cases cited had little or no formal education and gave evidence of their unusual ability at a very early age. It might be argued that their interest and liking for numbers and numerical manipulation caused them to practice a great deal, and that this is essentially the counterpart of modern education. This may be true, but it seems more reasonable that an inherently superior native ability gave impetus to the interest, rather than vice versa. It is very unlikely that an average, or even below average, child could, by reason of interest, develop an ability to do numerical calculations which would transcend the ability of all his contemporaries.

* E. W. Scripture. Arithmetical prodigies. *American J. Psychol.*, 1891, 4, 1-59.

† Frank D. Mitchell. Mathematical prodigies. *Amer. J. Psychol.*, 1907, 18, 61-143.

‡ A. Binet. *Psychologie des Grands Calculateurs et Joueurs d'échecs*, Paris: Hachette, 1894, pp. 364 ff.

E. H. Lindley and A. L. Bryan. An arithmetical prodigy. *Psychol. Review*, 1900, 7, 135.

§ F. D. Mitchell, *op, cit., p.* 131.

Studies of individuals with marked arithmetical disability are also very suggestive. Two cases have been reported in detail by Werner and Straus.* Case I revealed a specific deficiency in the finger schema and a general inability to grasp relations in space. Case II revealed a specific disturbance in the comprehension of visual patterns and in general a deficiency in grasping simultaneously the multiple aspects of a visual unity. All of these disturbances were found to differentiate a group of subjects selected for poor arithmetic ability from a group of subjects selected for good arithmetic ability. Guttmann, summarizing the evidence for congenital arithmetic disability arising from structural or functional anomalies of the brain, states, "There is little doubt that focal brain lesions may produce acalculia . . . . "†

The evidence from such cases of deficiency in number ability tends to indicate that the deficiency is more fundamental than mere lack of ability to manipulate numbers. This would be the case if the psychological process which we call number ability were inherent in the organism. Numbers, as a cultural invention, would be a much later development than a mental ability evolved by the slow processes of evolution.

It is reasonable, therefore, that tests like addition and multiplication really measure some more fundamental process than mere number manipulations *per se*. Hence it should be possible to devise tests of a non-numerical character which would measure this fundamental process. To do this one must make an hypothesis as to the nature of this process characterizing number ability.

If it is correct to assume that two senior students in high school know equally well that, say, eight and six are fourteen, and similarly for other possible number combinations, then the difference between their scores on a test of addition represents a difference in the rapidity with which the individuals can recall and manipulate these well-established associations. One might call this an agility in manipulating a symbolic system according to a specified set of rules, with two restrictions, that the symbolic system be familiar and that the rules be highly practiced. In a loose sense it might be descriptively referred to as mental agility, though this term would have more extensive implications than are desired here.

* Henry Werner and Alfred Straus. Problems and methods of functional analysis in mentally deficient children. *J. Abn. & Soc. Psychol*, 1939, 34, 37-62.

† E. Guttmann, Congenital arithmetic disability and acalculia, *British J. med. Psychol*, 1937, 16, 18.

If number ability were basically such a process as this—the quick recollection and manipulation of well-established associations—then this fundamental process would transcend numerical operations. Inasmuch as "well-established associations" are products of educational and environmental forces, such forces would obviously play a role in the establishment of the ability. In view of the fact, however, that the learning curve has a horizontal asymptote, it is possible to set up tests of a non-numerical character in which the associations to be established are so simple that the asymptote is approached very early. These new tests would then measure primarily the rapidity with which these "well-established associations" could be manipulated.

From this hypothesis as to the nature of number ability certain deductions may be made which are susceptible to experimental verification. Briefly these may be stated as follows:

1) Tests involving new rules for manipulation of a symbolic system are better tests of number ability after practice than they are before.

2) Of several tests involving manipulation of a symbolic system, the one in which the symbolism is more familiar provides the better measure of number ability.

Another hypothesis as to the basic process involved in number ability may also be tested. It has been suggested by H. D. Landahl that the operation characterizing the tests defining the number factor is in the nature of a serial response, in that each response to a pair of numbers leads to the next response. This may be submitted to experimental verification by varying the length of the addition or multiplication problems. If the serial response hypothesis of number ability is correct, the addition of two digits would not be a measure of number ability, but the addition of four digits would.

Inasmuch as the tests of Identical Forms and Verbal Enumeration have not been very satisfactory as measures of perceptual speed, several cancellation tests were incorporated in the battery to see if they might not give a better definition of this factor. This clarification was of particular interest because of the fact that in a very simple number problem such as the addition of two digits, perception of the numbers and knowledge of the answers seem almost simultaneous. With good tests of the perceptual speed factor it is possible to determine what relation, if any, exists between it and number ability.

Tests were designed to check the validity of the hypotheses as to the nature of number ability and to determine what relation number ability bears to perceptual speed. These tests were incorporated with a variety of other tests covering the domain of known mental abilities, and the entire battery was analyzed by Thurstone's centroid method.

II. *The Experiment*

THE SUBJECTS

In the spring semester of 1939, the Chicago high schools offered the members of their senior classes a course in self-appraisal. Of their own volition the students in these classes took personality and aptitude tests with the intention of gaining some evaluation of their capabilities. Because the subjects for this experiment were taken from six such classes in six south-side high schools, it may be assumed that their motivation was fairly good and fairly constant. The six schools and the number of complete records obtained from each are as follows.

Englewood High School	87
Fenger High School	34
Hirsch High School	28
Hyde Park High School	21
Morgan Park High School	23
Parker High School	30
Total	223

Each class was visited for approximately an hour on each of five consecutive days, and from three to five tests were given to the subjects at each session.

TABLE 1

Tests for the Primaries

Number	Test	Primary
19	Identical Forms	P
20	Verbal Enumeration	P
21	Addition	N
22	Multiplication	N
23	Completion	V
24	Same-Opposite	V
25	Cards	S
26	Figures	S
27	Initials	M
28	Word-Number	M
29	Letter Grouping	I
30	Marks	I
31	Number Patterns	I
32	Arithmetic	D
33	Number Series	D
34	Mechanical Movements	D

THE TESTS

The tests used in this study may be divided into two groups: 1) the tests for the primaries and 2) the experimental tests.

The tests for the primaries, listed in Table 1, consisted of the sixteen tests in the American Council on Education *Tests for Primary Mental Abilities*, experimental edition, 1938. These tests were given by Miss Ruth Wright, of the Bureau of Child Study, Board of Education of Chicago, as part of the regular testing program of the high schools.

The experimental section of the battery consisted of eighteen tests selected and designed to answer specific problems. Each test is described below together with a statement of the purpose for which the test was designed.

Two-Digit Addition (1).—This test and the following two were designed to investigate the hypothesis that the number process was essentially in the nature of a serial response. If the hypothesis is correct, the saturations of tests (1), (2), and (3) on the number factor should progressively increase. The fore-exercise of this test, the whole of which consisted of 238 problems, was as follows:

Look at the following addition of problems.

8	9	3	8	5	3	7	14	18
6	7	5	6	2	2	2	4	1
—	—	—	—	—	—	—	—	—
14	16	9	14	7	5	7	18	19
		X				X		

Because they are wrong two of the answers are underlined with an X.
In the following problems cross out every answer that is wrong.

7	5	8	9	12	17	3	8	7	12	8
6	4	3	6	2	2	9	6	5	3	9
—	—	—	—	—	—	—	—	—	—	—
13	8	11	15	14	19	13	14	11	14	17

Three-Digit Addition (2).—This test consisted of 170 problems. The fore-exercise follows.

Look at the following addition problems.

8	5	4	9	7	6	4	8	5	9	4	2
7	3	8	3	2	5	1	6	9	3	6	6
6	1	4	2	6	2	7	3	2	1	7	4
—	—	—	—	—	—	—	—	—	—	—	—
21	9	17	14	15	14	13	17	16	13	16	12
		X			X	X				X	

Because they are wrong four of the answers are underlined with an X.
In the following problems cross out every answer that is wrong.

7	8	4	3	9	5	3	1	2	4	3	8
6	3	7	2	5	8	3	2	8	6	9	4
3	7	8	9	4	2	4	9	5	3	7	2
16	18	17	14	18	15	14	12	15	14	19	14

Four-Digit Addition (3).—This test consisted of 153 problems. The fore-exercise follows.

Look at the following addition problems.

5	6	7	8	4	2	9	2	3	5	7	7
6	3	1	5	1	3	8	9	5	6	3	8
6	3	7	8	1	3	2	6	5	3	8	5
4	7	2	1	8	3	9	2	8	5	3	7
21	18	17	22	15	11	27	19	21	18	21	27
	X			X		X			X		

Because they are wrong four of the answers are underlined with an X.
In the following problems cross out every answer that is wrong.

5	6	7	3	8	3	9	8	2	3	1	3
7	3	8	6	9	1	3	6	2	8	4	3
3	6	8	3	6	9	3	2	7	9	1	3
4	2	6	8	3	6	4	9	3	2	6	9
19	17	19	20	25	19	19	24	14	22	12	18

AB (4).—The *AB* and the *ABC* tests were designed to contrast with the Forms test to check the hypothesis that of several tests involving manipulation of a symbolic system, the one in which the symbolism is most familiar provides the best measure of number ability. The familiar alphabetical character of the elements in the two former tests contrasts markedly with the unfamiliar character of the elements used in the Forms test. Also the greater simplicity of the *AB* test compared to the *ABC* test would throw some light on the extent to which the speed element characterizes number ability. The fore-exercise of the *AB* test follows:

Only three letters, *A*, *B*, and *C*, are used in this test. They are combined in various ways, but there are only two rules. The rules are:

(1) A combination of any two different letters is equal to the third letter.

Examples: $A\,B = C$, $A\,C = B$, $B\,C = A$,
$B\,A = C$, $C\,A = B$, $C\,B = A$.

(2) If a letter is combined with itself, the combination is equal to that letter.

Examples: $A\,A = A$, $B\,B = B$, $C\,C = C$.

Below are some problems for you to practice on. If the answer to a problem is *A*, put a check (√) in the first column, if *B* put a check (√) in the second column, if *C* put a check (√) in the third column. The first three problems have been worked for you. Work them yourself to see that they are right and then go on with the others.

	A	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*
A B			√	*A A*				*C B*			
C A		√		*B A*				*A B*			
B B		√		*C A*				*A C*			
A C				*B C*				*B A*			

This test consisted of 210 problems.

ABC (5).—This test consisted of 60 problems. The fore-exercise follows.

Only three letters, *A*, *B*, and *C*, are used in this test. They are combined in various ways, but there are only two rules. The rules are:

(1) A combination of any two different letters is equal to the third letter.

Examples: $A\,B = C$, $A\,C = B$, $B\,C = A$,
$B\,A = C$, $C\,A = B$, $C\,B = A$.

(2) If a letter is followed by itself, the combination is equal to that letter.

Examples: $A\,A = A$, $B\,B = B$, $C\,C = C$.

The example below has been worked out according to these rules.

$$A\,B\,A = B$$

Here is the way you solve the problem. Combine the first two letters. The combination $A\,B = C$. Then combine this *C* with the next letter, which is *A*. The combination $C\,A = B$.
Work the following examples and write the answers in the blanks.

$C\,B\,B =$ ——— $C\,A\,C =$ ———

You should have written *C* in the first blank and *A* in the second blank.
Here is a longer example which has been worked out.

$$C\ A\ B\ C = A.$$

The first two letters, *C A*, equal *B*. This letter combined with the next gives *B B*, which is equal to *B*. This letter, combined with the next, gives *B C* = *A*. Find the answer to the following example and write it in the blank.

$$B\ C\ A\ C = \text{——}$$

You should have written *B*.

Here are some more problems for you to practice on. Write the answers in the blanks.

B C C A C = — *B A B B A* = — *A C B C A* = —

Forms (6).—This test was of exactly the same nature as the *ABC* test except that three non-meaningful geometrical designs replaced the letters *A*, *B*, and *C*.

Alphabet I (7).—This and the following two tests were designed to check the hypothesis that practice with a set of rules would improve a test as a measure of number ability. If the hypothesis is correct, the three tests, Alphabet I, Alphabet II, and Alphabet III should show progressively increasing validity as measures of number ability. The fore-exercise of Alphabet I follows.

In this test the letters of the alphabet are combined in various ways according to three rules.

(1) In a combination of two letters like *MP*, the letters are in the same order as they occur in the alphabet and there are several letters between them, *N* and *O*. Two letters like *RT* are also in alphabetical order and have one letter between them, *S*. Similarly one could write other combinations of two letters in alphabetical order in which there are three or more letters between them.

A combination of two letters in alphabetical order which has letters between them is equal to the letter in the alphabet which follows the second letter in the combination. For example, the two letters in the combination *MP* have letters between them, so the combination is equal to *Q* because *Q* follows *P* in the alphabet. Similarly, *RT* = *U* because the letters *RT* have a letter between them and *U* follows *T* in the alphabet.

Other examples: *CE* = *F*, *HK* = *L*, *DV* = *W*.

(2) In a combination of two letters like *PM* the

letters are in backward order and have several letters between them, for example, *N* and *O*. Two letters like *TR* are also in backward order and have one letter between them, *S*. Similarly, one could write other combinations of two letters in backward order with three or more letters between them. A combination of two letters in backward order which has letters between them is equal to the letter in the alphabet which precedes the second letter of the combination. For example, the two letters *PM* have letters between them, and so the combination is equal to *L* because *L* precedes *M* in the alphabet. Similarly, $TR = Q$ because the letters *TR* have a letter between them and *Q* precedes *R* in the alphabet.

Other examples:

$$HC = B, \quad VN = M, \quad JH = G.$$

(3) A combination of two letters which have no letters between them is equal to the second letter of the pair.

Examples: $OP = P, \quad GH = H, \quad RS = S,$
$PO = O, \quad HG = G, \quad SR = R.$

Here are some problems for you to practice on. The first few have been answered for you. Check them to see that they are right and then go on with the others. Write the answer in the blank space.

$DF = G$	$JL = M$	$RS =$	$FH =$	$EC =$
$GJ = K$	$TS = S$	$LM =$	$BI =$	$YF =$
$RI = H$	$WY =$	$UT =$	$HJ =$	$OL =$

This test consisted of 192 problems.

Alphabet II (8).—Consisting of a second page of problems, this test was similar to Alphabet I. The subjects upon finishing Alphabet I had a rest period of several minutes and then took Alphabet II. The test consisted of 192 problems.

Alphabet III (9).—This test consisted of 192 problems similar to those in Alphabet I and Alphabet II. There was a brief period of rest between the tests Alphabet II and Alphabet III.

Digit Cancellation (10).—In conjunction with the next two tests, digit cancellation was designed to clarify the definition of a perceptual speed factor and thereby indicate what relation this factor would have to the number factor. The fore-exercise of this test was as follows:

Look at the rows of numbers below. A parenthesis has been put around each number 5.

3	7	2	(5)	9	0
7	(5)	2	8	6	3
1	0	8	6	2	7
4	6	9	(5)	(5)	0

Put a parenthesis around each number 5 in the following rows.

4	1	9	5	2	3
8	6	7	1	5	2
5	9	0	4	7	2
3	6	5	8	5	2

You are given several longer rows of numbers below. Put a parenthesis around each number 5.

8 7 6 0 3 5 2 1 0 5 8 4 7 9 3

Scattered X's (rowed) (11).—This test consists of ten pages of letters in rows through which *x*'s were scattered. The fore-exercise is below.

Look at the rows of letters below. A parenthesis has been put around each letter "*x*."

z	b	s	h	s	g
(x)	t	j	r	(x)	y
l	m	v	(x)	(x)	o
e	(x)	d	f	k	a

Put a parenthesis around each letter "*x*" in the following rows.

m	b	t	j	g	r
a	e	x	d	v	x
c	x	l	o	f	h
j	x	e	x	x	k

You are given several longer rows of letters below. Put a parenthesis around each letter "*x*."

v u t o c m z c f x m l z q k

Scattered X's (pied) (12).—This test consists of seven pages of pied letters with seven *x*'s on each page. The subjects were instructed to find as many *x*'s on a page as they were able and then to turn to the next page.

Identical Numbers (13).—This test, in conjunction with Highest Numbers and Digit Cancellation, was selected for the purpose of determining whether tests involving numbers and numerical concepts necessarily had a projection on number if they did not involve manipulation of the numbers. The test, the fore-exercise of which follows, consists of 45 columns with 29 three-digit numbers in each.

> The number at the top of the first column of figures is 634. A mark has been made under each 634 in the column. In the second column, a mark has been made under the 876, because 876 is the number at the top of that column. In the third column, the two 795's have been marked, because 795 is the number at the top of the third column.
> The number at the top of each of the other columns is repeated one or more times in that column. Find those numbers as quickly as possible and put a mark under each of them. Go right ahead.

634	876	795	423	279	374
693	643	583	837	363	282
850	328	795	115	643	663
634	932	189	423	279	539
513	879	342	528	375	314
398	375	795	969	470	375
696	470	896	274	887	576
634	697	247	423	699	374
574	876	319	627	291	850
628	294	468	423	983	677
634	982	543	962	585	846

Highest Number (14).—This test consists of 80 columns with 40 three-digit numbers in each. The fore-exercise is as follows:

> You are given five columns of numbers below. Find the highest number in each column and put a parenthesis around it. The first two columns have been done correctly. You mark the other three. Go right ahead. Do not wait for any signal.

1	2	3	4	5
298	189	229	189	149
142	237	340	376	580
389	(746)	187	234	689
527	642	246	427	327
462	248	546	167	486
127	543	827	349	682
482	329	628	256	595
(536)	735	821	453	337
227	670	342	113	428
162	167	119	297	321

Size Comparison (15).—It was thought desirable to set up a test of a "quantitative" nature but not involving numerical concepts to see if number ability had a characteristic of quantitative thinking. The test consists of 69 items. The following is the fore-exercise.

Look at the following pairs of words.

sardine —(s)hark
(G)ermany —Cuba
(P)lanet —palace

A parenthesis has been put around the first letter of the word in each pair which means the larger of the two things. A shark is larger than a sardine; Germany is larger than Cuba; a planet is larger than a palace.
In the following pairs of words put a parenthesis around the first letter of that word in each pair which means the larger of the two things.

Texas —Maine	clock —watch	grape —lemon
thumb —foot	tree —forest	wrench —nut
inch —mile	rock —pebble	cup —barrel

Substitution I (16).—This test, in conjunction with Substitution II and Substitution III, was designed to see if an increasing familiarity with the translation of an arbitrary symbolism would have any

TABLE 2

Time Limit and Scoring Formulae used on the Experimental Tests

Number	Test	Time Limit in Minutes		Scoring Formula
		Fore-exercise*	Test	
1	Two Digit Addition	1	4	R-W
2	Three Digit Addition	2	4	R-W
3	Four Digit Addition	2	4	R-W
4	AB	3	3	2R-W
5	ABC	8	5	2R-W
6	Forms	13	5	2R-W
7	Alphabet I	10	10	R
8	Alphabet II		5	R
9	Alphabet III		5	R
10	Digit Cancellation	2	4	R
11	Scattered X's (rowed)	1	4	R
12	Scattered X's (pied)	2	4	R
13	Identical Numbers	2	3	R
14	Highest Number	2	3	R
15	Size Comparison	2	1.7	R-W
16	Substitution I	10	10	R
17	Substitution II		5	R
18	Substitution III		5	R

* The time limit for the fore-exercise was not rigidly adhered to except in the case of the substitution tests.

significance in relation to number ability. This test and the two following tests each consist of 90 words in code which are to be translated. The same code is used throughout and is given at the top of each test sheet.

Substitution II (17).—This test consists of a second page of code words similar to Substitution I and of the same length. The code is repeated on this test blank.

Substitution III (18).—This test is a third page of code words which are similar to those of the two preceding tests. The same code is also repeated on this test blank.

The time limit and scoring formula used for each of the eighteen tests comprising the experimental section of the battery are given in the following table.

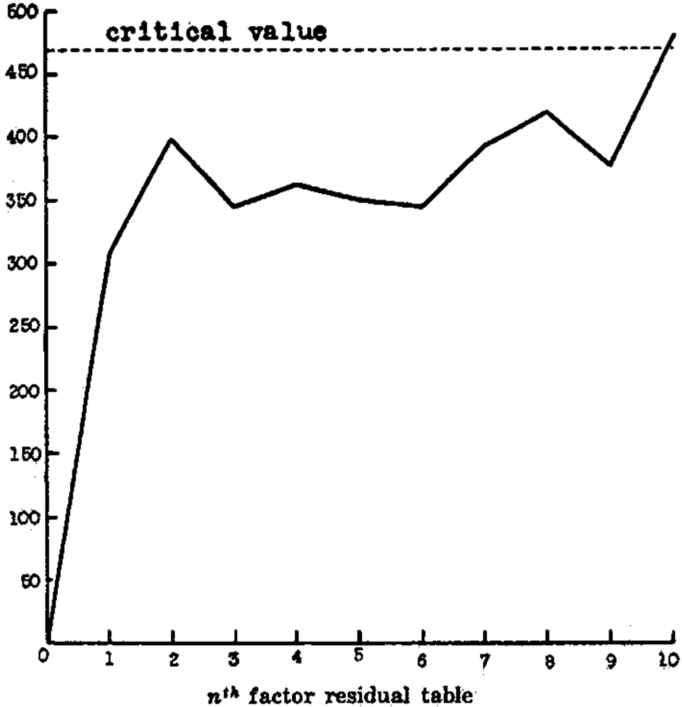

FIG. 1.—Number of negative signs in n^{th} factor residual table after sign change.

THE FACTOR ANALYSIS

In order to obtain the product moment correlation coefficients between tests by means of the punched card technique and I. B. M. machines, it was necessary that all test scores be expressed in no more than two digits. For this reason the scores of certain of the

TABLE 3
Correlations between the Tests

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1																																		
2	849																																	
3	788	886																																
4	488	502	515																															
5	407	455	470	567																														
6	301	385	396	515	569																													
7	363	449	476	588	601	584																												
8	383	472	489	588	591	516	921																											
9	399	498	507	644	595	582	906	925																										
10	452	446	445	432	292	283	358	329	322																									
11	268	274	235	316	178	218	184	142	160	509																								
12	133	122	159	129	081	075	178	213	175	297	497																							
13	540	495	503	444	307	370	348	325	384	508	370	214																						
14	462	441	451	448	378	368	364	409	411	428	307	153	466																					
15	345	394	391	484	447	302	441	453	442	279	222	123	312	359																				
16	281	288	326	372	346	415	424	405	419	255	254	188	350	333	282																			
17	196	195	219	318	313	425	303	294	308	208	203	104	348	338	253	331																		
18	257	238	245	305	332	392	321	310	306	282	235	096	348	344	256	335	918																	
19	265	239	212	344	217	301	285	255	287	335	463	285	295	344	291	304	256	246																
20	354	347	310	404	316	336	318	303	304	368	298	176	455	281	536	192	165	179	300															
21	519	657	646	396	371	355	266	288	301	390	277	140	388	338	275	306	213	252	187	343														
22	738	703	689	408	358	284	345	355	387	384	192	141	405	356	213	285	167	251	221	328	556													
23	157	266	245	321	338	246	485	444	458	118	010	092	126	187	532	098	082	006	162	366	121	149												
24	259	334	323	440	458	277	484	527	511	181	001	−012	242	263	578	140	090	084	161	468	192	206	709											
25	175	253	225	345	351	307	442	418	407	218	226	128	184	259	266	259	149	163	476	220	210	187	236	222										
26	212	256	229	318	343	306	417	372	354	176	268	172	058	284	351	248	156	147	516	238	184	175	311	249	784									
27	155	223	178	170	132	171	266	274	256	110	077	085	167	124	167	200	097	147	119	126	150	208	163	190	240	145								
28	113	115	135	207	107	185	146	161	148	080	032	008	128	086	122	294	207	236	084	082	166	130	008	053	043	025	360							
29	226	327	292	391	305	317	422	433	423	327	311	208	259	268	178	267	171	166	309	215	296	230	218	295	264	283	192	067						
30	178	260	231	352	394	410	503	480	473	169	115	098	197	241	310	331	260	246	268	287	241	138	268	356	405	389	064	014	328					
31	256	306	325	246	152	260	225	226	241	168	242	200	194	330	151	188	121	134	347	123	354	272	029	052	335	305	105	021	301	344				
32	165	303	327	270	312	188	410	390	267	076	008	036	059	164	286	199	095	085	098	149	237	194	333	354	392	339	357	166	133	349	281			
33	263	361	358	320	329	218	480	472	467	166	089	151	143	224	355	174	070	058	167	203	274	251	383	384	402	342	124	[illegible]	234	338	232	336		
34	164	209	210	261	258	165	345	304	319	183	183	215	243	199	326	278	220	187	270	222	165	151	295	227	377	382	068	070	224	252	237	315	312	

tests were linearly translated to two digits, a process which does not affect the correlation coefficient. The correlation matrix is presented in Table 3.

The correlations were factored on the I.B.M. machines by Thurstone's centroid method. Application of a criterion developed for the purpose of detecting significant common factor variance in a matrix of intercorrelations indicated that ten factors were significant and the eleventh was not.* This criterion is based on the number

TABLE 4
Centroid Factorial Matrix

	I	II	III	IV	V	VI	VII	VIII	IX	X	h^2
1	64	–39	24	33	18	06	06	21	12	–12	84
2	72	–27	24	33	28	10	16	11	07	–09	90
3	71	–27	20	32	25	08	17	10	11	–04	84
4	72	03	–06	18	–07	–11	–06	06	–07	12	60
5	66	19	–13	17	05	06	–04	14	–13	15	58
6	62	03	–28	07	08	–09	–12	10	–24	14	58
7	77	37	–24	12	–05	–14	25	10	13	–05	92
8	77	36	–24	17	–06	–14	24	11	12	–12	93
9	78	34	–24	18	–07	–15	23	14	10	–08	93
10	54	–32	13	05	–23	–21	09	–04	04	12	54
11	42	–35	16	–26	–29	–32	04	–13	–11	03	61
12	29	–17	14	–24	–23	–20	23	–18	–04	–10	38
13	57	–35	02	19	–20	–07	02	–04	06	18	57
14	58	–19	03	03	–08	–09	–03	14	06	17	44
15	61	15	15	12	–26	23	–23	–05	04	08	62
16	59	–23	–52	–28	–03	25	–06	–05	13	–09	84
17	49	–31	–58	–28	–14	32	–15	10	09	06	92
18	51	–36	–57	–24	–10	30	–15	08	12	04	92
19	50	–10	13	–32	–13	–25	–21	12	–07	–09	53
20	53	–03	20	14	–26	08	–16	–12	–16	11	49
21	58	–30	14	12	28	08	08	–11	–19	09	61
22	57	–33	17	26	26	04	07	08	–03	–14	63
23	46	47	22	16	–26	25	–10	–10	–06	–22	71
24	54	43	13	29	–25	26	–13	–08	–12	–10	76
25	55	31	17	–43	20	–18	–19	16	14	07	77
26	54	29	23	–45	12	–13	–25	21	07	–13	79
27	32	05	–09	06	17	–16	–15	–35	19	–19	39
28	23	–06	–25	08	17	–06	–21	–37	17	–05	37
29	50	05	–02	–08	–04	–19	15	–08	–19	–10	37
30	53	25	–08	–20	06	06	12	07	–22	14	48
31	41	–08	13	–27	19	–12	15	07	–17	06	37
32	44	36	12	–10	28	22	11	–20	19	14	58
33	52	34	20	–11	18	16	24	–15	12	11	60
34	44	11	12	–26	–10	09	07	–06	11	03	33

* A paper presenting the logic upon which the criterion is based and the critical values for a given number of tests in the battery is in preparation.

of negative entries in the residual matrices after sign change. The critical value in the case of 34 variables is 468. When this value is attained no more significant common factor variance remains. The number of negative signs in the tenth factor residual table after sign change was 478. The number of negative signs in each residual table after sign change is presented in Figure 1.

The loadings on the ten centroid factors are presented in Table 4. This table was given to a disinterested person, who rearranged the table by rows and gave the variables a new set of numbers. The code remained in the possession of this person throughout the time the

TABLE 5
Rotated Factorial Matrix*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
1	74	02	05	–03	–01	–09	–06	–02	03	06
2	72	04	–03	02	–02	06	03	04	–02	02
3	66	01	–04	03	00	07	00	05	–01	07
4	10	09	09	06	09	–06	14	24	–04	30
5	06	11	04	–01	–14	04	23	27	01	25
6	02	–03	10	09	–05	–07	37	26	04	25
7	–02	–03	01	03	01	06	–03	64	02	03
8	02	00	00	04	00	–01	–04	66	02	–01
9	03	–01	00	01	00	–02	–02	66	01	04
10	18	–01	04	–01	46	–05	–05	04	–02	24
11	03	–01	19	–01	65	–03	06	–03	00	09
12	01	03	03	–03	55	07	–02	08	03	–13
13	22	04	–06	02	31	–09	–04	00	06	35
14	17	–04	16	–05	16	00	–01	04	06	29
15	–02	49	11	01	03	06	–08	–04	10	31
16	–01	00	02	15	02	04	00	03	69	–03
17	–08	00	03	00	–04	–03	01	–05	78	13
18	–01	–03	02	04	–03	–05	–01	–06	76	13
19	03	06	50	–03	28	–06	06	01	06	04
20	05	41	03	01	22	–02	10	–05	–04	30
21	45	04	–07	12	12	17	32	–10	–02	10
22	64	02	00	06	00	–05	12	00	00	–02
23	–06	66	03	–02	–04	03	–06	18	–02	–03
24	–05	64	–04	00	–09	–02	03	21	–04	12
25	–08	–08	65	04	–02	36	01	02	–04	06
26	–01	07	71	–02	–02	23	00	02	02	–08
27	03	01	08	53	05	–01	–05	04	–06	–04
28	–03	–03	–03	55	–02	–03	00	–06	05	08
29	04	04	05	04	28	02	22	30	–04	–08
30	–10	06	09	–11	03	29	30	24	08	04
31	17	–14	20	–09	20	25	26	04	–02	–06
32	–03	11	02	17	–11	57	–02	–02	–01	03
33	02	13	01	04	04	56	00	08	–04	–02
34	–06	15	15	–05	19	30	–09	00	15	00

* All entries have been multiplied by 100 to eliminate the decimal.

rotations were being made. The centroid matrix was rotated by means of extended vectors and the oblique rotational method. In rotating "blindly" in this manner the only criterion determining rotations was to go strictly to configuration. Seventeen rotations yielded a highly satisfactory simple structure in which no more rotations were evident. On June first the code was obtained and opened for the first time. No further rotations were made. The structure obtained by rotating "blindly" yielded the same psychologically meaningful factors as those obtained in numerous other studies in which the rotations were made with a knowledge of the names of the test vectors.

The factorial matrix obtained after rotation is presented in Table

TABLE 6

Direction Cosines of the Reference Vectors*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
I	21	16	17	09	16	12	11	20	13	15
II	–52	23	11	01	–33	25	00	44	–30	–05
III	35	33	28	–23	24	28	–15	–40	–46	–01
IV	36	15	–48	16	–29	–47	–04	24	–40	29
V	43	–45	11	36	–49	35	35	–15	–20	–20
VI	12	57	–34	–26	–41	32	–06	–39	62	–09
VII	16	–28	–58	–38	33	34	–03	46	00	–39
VIII	29	–26	39	–72	–44	–25	–08	24	14	02
IX	01	–25	14	22	–11	17	–90	–15	18	09
X	–34	–26	–11	–10	06	46	13	–26	–20	82

* All entries have been multiplied by 100 to eliminate the decimal.

TABLE 7

Cosines of the Angles Between the Reference Vectors*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
N	99									
V	–08	101								
S	–03	–11	100							
M	–12	–08	–02	101						
P	–17	01	–10	00	100					
D	–16	–07	–04	–04	07	102				
I	02	–01	–10	04	–02	01	100			
A	–12	–20	–19	–16	00	–31	11	99		
B	–02	15	–08	–22	–12	03	–21	–17	99	
C	–27	–04	04	08	–01	02	–02	–25	–24	99

* All entries have been multiplied by 100 to eliminate the decimal.

5, and the transformation matrix (Λ) leading from the centroid matrix (F_c) to the final rotated matrix (V) by the equation

$$V = F_c \Lambda$$

is given in Table 6.

The correlations between the reference vectors, obtained by the matrix multiplication ($\Lambda' \Lambda$), are presented in Table 7.

TABLE 8

Direction Cosines of the Primary Vectors*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
I	70	47	52	46	39	52	17	78	56	63
II	–39	37	16	02	–46	37	–12	40	–33	–12
III	28	36	20	–30	20	22	–22	–25	–58	–12
IV	34	21	–46	12	–28	–26	–19	16	–31	32
V	29	–33	06	32	–44	35	29	–07	–11	–17
VI	07	41	–30	–14	–38	25	02	–16	34	01
VII	15	–22	–46	–27	26	36	–13	26	–06	–20
VIII	15	–22	32	–61	–33	–15	–03	13	07	14
IX	10	–17	06	27	–09	20	–87	08	10	10
X	–13	–25	–17	–21	00	29	13	–10	–05	61

* All entries have been multiplied by 100 to eliminate the decimal.

The correlations between the primary vectors, obtained by the equation

$$R_{pq} = H H',$$

are shown in Table 9.

TABLE 9

Correlations Between the Primary Vectors*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
N	100									
V	24	100								
S	21	21	100							
M	28	19	17	100						
P	24	05	16	10	100					
D	33	24	20	22	03	100				
I	00	–02	10	00	06	–08	100			
A	44	35	34	36	14	46	–04	100		
B	27	03	22	32	21	15	21	37	100	
C	44	18	17	18	15	23	05	46	39	100

* All entries have been multiplied by 100 to eliminate the decimal.

The matrix H' is given by the equation

$$H' = (\Lambda' \Lambda)^{-1} D ,$$

where D is a diagonal matrix the entries in which are the normalizing constants of the corresponding columns of Λ. The columns of this matrix (H') contain the direction cosines of the primary vectors (see Table 8).

TABLE 10

Correlations of the Tests with the Primaries*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
1	91	23	23	22	22	21	–04	38	27	45
2	94	30	20	30	20	40	02	48	26	45
3	91	26	18	30	21	40	00	48	28	48
4	49	33	34	30	25	27	16	62	34	62
5	41	34	29	28	01	35	23	62	34	56
6	34	16	32	29	11	20	41	53	42	53
7	42	31	33	37	14	49	–06	95	38	46
8	44	34	32	38	13	44	–07	96	37	44
9	46	33	33	36	14	43	–04	96	38	48
10	47	14	21	15	59	13	–01	33	26	47
11	25	07	32	09	74	03	14	16	24	23
12	16	09	16	07	59	12	01	18	14	01
13	52	17	12	20	46	11	01	35	36	60
14	47	13	32	15	32	21	05	39	35	55
15	33	64	31	22	15	30	–03	43	28	53
16	28	08	25	45	22	22	17	41	91	34
17	18	01	21	26	16	08	21	29	96	44
18	24	–01	20	31	17	08	19	30	96	46
19	25	17	63	13	42	10	16	28	28	23
20	35	52	22	15	32	16	13	30	18	47
21	66	19	12	29	28	34	34	27	27	38
22	78	20	18	27	20	21	12	33	24	33
23	19	80	24	17	00	29	–09	47	04	17
24	26	80	19	21	–02	28	00	54	11	35
25	21	17	77	22	09	53	06	43	19	23
26	22	29	84	17	11	42	07	40	20	12
27	21	15	19	60	11	15	–06	26	14	08
28	12	04	05	59	04	06	02	15	24	16
29	28	21	27	22	36	23	22	46	21	17
30	19	23	31	10	11	47	38	50	31	29
31	32	01	33	05	29	35	29	25	18	13
32	25	31	18	34	–07	72	–05	39	12	20
33	33	36	21	24	08	74	–03	48	11	21
34	20	28	31	13	26	42	–04	33	26	19

* All entries have been multiplied by 100 to eliminate the decimal.

Applying the transformation (H') to the centroid matrix in the manner

$$R_{fp} = F_c H',$$

the correlations of the tests with the primary vectors are obtained (see Table 10). These correlations are the validity coefficients of the tests as measures of the primaries.

III. *Interpretation and Discussion*

The most interesting part of a factor analysis is the psychologi-

TABLE 11

Simplified Rotated Factorial Matrix*

	N	*V*	*S*	*M*	*P*	*D*	*I*	*A*	*B*	*C*
1	74									
2	72									
3	66									
22	64									
21	45						32			
23		66								
24		64						21		31
15		49								30
20		41			22					
26			71			23				
25			65			36				
19			50		28					
28				55						
27				53						
11			(19)		65					
12					55					
10					46					24
32						57				
33						56				
34					(19)	30				
6							37	26		25
30						29	30	24		
31			20		20	25	26			
8								66		
9								66		
7								64		
29					28		22	30		
5							23	27		25
17									78	
18									76	
16									69	
13	22				31					35
4								24		30
14										29

* All entries have been multiplied by 100 to eliminate the decimal.

cal interpretation of the factors obtained. To facilitate the interpretation we have reproduced the rotated matrix (Table 11), excluding all projections under .20 and rearranging the rows to make the structure more apparent.

The number factor.—Considering first the number factor, *N*, it may be seen that Two-Digit Addition has the highest projection, with Three-Digit Addition next, followed by Four-Digit Addition. The order of these three tests in their projection on number indicates that the simpler tests are better tests of this factor. This is borne out by

(1) Two-Digit Addition	.74	(22) Multiplication	.64
(2) Three-Digit Addition	.72	(21) Addition	.45
(3) Four-Digit Addition	.66	(13) Identical Numbers	.22

the relation of Multiplication and Addition, the former being higher than the latter. The multiplication problems are the product of two digits by one digit, whereas the addition problems are the sum of six two-digit numbers. These results are strong evidence against the hypothesis that number ability is in the nature of a serial response process. The only other test having a projection greater than .20 is Identical Numbers, with a projection of .22. These results on the number factor indicate that it has a speed characteristic.

The rotated factorial matrix does not yield any information on the various hypotheses that the battery was designed to investigate because the primaries are oblique. However, information on these points may be obtained from the correlation matrix (Table 4) and from the validity coefficients of the tests (Table 10) as measures of the primary. These data are assembled in the following four tables.

The three alphabet tests were designed to check the hypothesis

TABLE 12

Correlations of Alphabet Tests with Number Tests

	Two-Digit Addition	Three-Digit Addition	Four-Digit Addition	Multiplication	Addition
(7) Alphabet I	.363	.449	.476	.345	.266
(8) Alphabet II	.383	.472	.489	.355	.283
(9) Alphabet III	.399	.493	.507	.337	.301

that practice with a set of rules improves a test as a measure of number ability. If this is correct, the correlations of tests Alphabet I, Alphabet II, and Alphabet III with the number tests should progressively increase. This block of correlation coefficients is presented above in Table 12.

TABLE 13

Validity Coefficients of Alphabet Tests with the Number Primary

(7) Alphabet I	.42
(8) Alphabet II	.44
(9) Alphabet III	.46

The correlations all progressively increase with the one exception of the correlation between Alphabet III and Multiplication. Similarly, the validity coefficients of the three alphabet tests as measures of number ability show a slight increase as the subjects become more practiced with the set of rules. It is not to be expected that the amount of practice obtained by the subjects while taking these alphabet tests will yield a familiarity with the system in any way comparable with their familiarity with the number system.

The AB, ABC, and Forms tests were designed to throw light on the hypothesis that the more familiar or well-established the symbolism the better the test is as a measure of number. If this hypothesis is correct, the correlations of the AB and ABC tests with those for number ability should be distinctly higher than the correlation of the Forms test with the number tests. Table 14 shows that the predictions of the hypothesis are borne out without exception. The validity coefficients of these three tests as measures of number ability also support this conclusion.

TABLE 14

Correlations of AB, ABC, and Forms Tests with Addition and Multiplication Tests

	Two-Digit Addition	Three-Digit Addition	Four-Digit Addition	Multiplication	Addition
(4) AB	.488	.502	.515	.408	.396
(5) ABC	.407	.455	.470	.358	.371
(6) Forms	.301	.385	.396	.284	.355

TABLE 15

Validity Coefficients of AB, ABC, and Forms Tests with the Number Primary

(4) AB	.49
(5) ABC	.41
(6) Forms	.34

The consistent difference between the AB and the ABC test in the foregoing data also substantiate the conclusion that the simpler or less involved the manipulations called for in the test, the better is the test as a measure of number ability.

It is almost impossible to prove conclusively the original hypothesis that number ability represents the rapidity with which well-established associations may be recalled and manipulated. The only conclusion which may safely be drawn is that the hypothesis is not disproved. All the data bearing on deductions from the hypotheses are in agreement with it.

The verbal factor.—The second factor, V, has the following four tests high on it:

(23)	Completion	.66
(24)	Same-Opposite	.64
(15)	Size Comparison	.49
(20)	Verbal Enumeration	.41

There are no other tests with projections higher than .20. This factor is obviously the verbal factor. Completion and Same-Opposite are the two tests designed to isolate the verbal factor. Verbal Enumeration has been found previously to have a projection on the verbal factor. The fact that the new test, Size Comparison, has a projection on this factor is in agreement with the interpretation of the factor as verbal ability.

The space factor.—The third factor, *S*, is defined by the following tests:

(26)	Figures	.71
(25)	Cards	.65
(19)	Identical Forms	.50
(31)	Number Patterns	.20

This factor is readily seen to be the space factor, inasmuch as Figures and Cards are the two tests for this primary. Identical Forms has also been found previously to have a projection on the space factor. It is interesting to observe that none of the three tests are pure, the first two having projections on deduction and the third having a projection on perceptual speed. This indicates that we do not yet have clear insight into the psychological nature of this factor, or perhaps that the freedom given the subjects in adopting a work method results in some subjects using deductive ability to do the tasks in the Figures and Cards tests.

Memory.—The fourth factor, *M*, is obviously the Memory factor with no other tests high than the two included for the purpose of isolating this ability. Although this factor is called memory ability, it

(28)	Word-Number	.55
(27)	Initials	.53

should be more strictly defined as rote learning, as both Word-Number and Initials are tests of rote learning with immediate recall. The score is more representative of a point on the ascending learning curve than of a point on the descending forgetting curve.

Perceptual Speed.—The fifth factor, *P*, we have identified as perceptual speed. The tests with highest projections on this factor are the three designed for the purpose of clarifying the definition of perceptual speed.

(11) Scattered *x*'s (rowed)	.65	(19) Identical Forms	.28
(12) Scattered *x*'s (pied)	.55	(29) Letter Grouping	.28
(10) Digit Cancellation	.46	(20) Verbal Enumeration	.22
(13) Identical Numbers	.31	(31) Number Patterns	.20

Certain suggestions are contained in these results which would be worth further investigation. There were considerably more cancellations per unit time in the Digit Cancellation test than in the Scattered *X*'s (rowed) test, so the indications are that the more scanning and the less cancelling called for by a test the better it is as a test of perceptual speed. The Scattered *X*'s (pied) test would probably be a better test of perceptual speed if administered in another manner. The subjects were instructed that there were only seven *x*'s on a page and that they might go on to the next page whenever they wanted to. It would probably improve the test if they were required to get *all* the *x*'s on each page before going on to the next, or perhaps, even better, if they were given a time limit of about twenty seconds on each page. An inspection of the fifth column of Table 11 indicates that all the tests with projections of .20 or greater involve the rapid scanning of the test content.

The Deductive Factor.—The sixth factor, *D*, may be identified as the deductive factor. The three tests included in the battery for the identification of this factor (Arithmetic, Number Series, Mechanical Movements) have their highest projections on it.

(32) Arithmetic	.57	(30) Marks	.29
(33) Number Series	.56	(31) Number Patterns	.25
(25) Cards	.36	(26) Figures	.23
(34) Mechanical Movements	.30		

Two of the space tests have significant projections on this factor for a possible reason already indicated. Two other tests, Marks and Number Patterns, have projections of .29 and .25 respectively on this factor. This is in agreement with the interpretation of this factor as deduction. It is interesting to note that with the definition of number ability by new tests with a greater speed characteristic than those in former studies, Arithmetic and Number Series have lower projections on the number primary.*

* Dr. H. O. Gulliksen has pointed out that these deductive tests seem to have a serial response character.

The inductive factor.—Those tests which have their highest projection on the seventh factor, *I*, are:

(6) Forms	.37	(31) Number Patterns	.28
(30) Marks	.30	(5) ABC	.23
(21) Addition	.32	(29) Letter Grouping	.22

Addition, which is primarily a number test, has a projection of .32 on this factor. Letter grouping and *ABC* also have projections greater than .20. Because of the small amount of variance accounted for by this factor, it is both difficult and risky to interpret. Inasmuch as Marks, Number Patterns, and Letter Grouping were included in the battery to identify the inductive plane, this factor might very tentatively be called induction.

Other factors.—The eighth factor, *A*, is primarily identified by the triplet of alphabet tests, a number of other tests having projections between .20 and .30. A factor defined by a triplet of tests so similar in nature as the three alphabet tests cannot be interpreted with any certainty. For that reason no attempt will be made here to indicate the psychological significance of this factor. Similarly, the ninth factor, *B*, is identified by the three substitution tests, with no other test having a projection on it as high as .20. This factor is another defined primarily by a triplet, and no attempt will be made to identify it.

The last factor, *C*, has very low variance and may be considered to be a residual factor.

Discussion

The reasons why such tests as *AB*, *ABC*, Forms, and the three alphabet tests have no projection on the number factor is that most of their variance is explained by other common factors, in this case primarily factors *A* and *C*, which have no "pure" tests to identify them. The result is that by rotating strictly to configuration, the number plane is passed through these tests, causing the configuration to be oblique. Hence, as may be seen from Table 9, the number primary correlated .44 with each of these two primaries, *A* and *C*. It is these correlations which have absorbed the potential projections of these tests on the number factor. It is assumed in this argument that the tests for the number factor do not call upon any non-number factor present in the alphabet tests. An alternative assumption is possible. It may be assumed that the alphabet tests are "pure" tests involving no number ability, but that the number tests are complex in that they involve not only the number factor but also the common factor defined by the alphabet tests. However, it seems more reasonable that the simple number tests involve only one of the two factors and the more complex alphabet tests involve both of them.

Indications that word tests have significance to the number factor are apparent in several studies already made. One of the tests described by Thurstone in his monograph, *Primary Mental Abilities*,* is Free Writing, the score on which is the total word count of a theme the subject is asked to write. This test had a projection of .295 on the number factor. In Thurstone's recent monograph, *Factorial Studies of Intelligence*,† the four highest tests on the word factor are Prefixes, First Letters, First and Last Letters, and Suffixes.

In the test Prefixes, the subjects were instructed to write all the words they could that began with "con." In First Letters the subjects were instructed to write all the words they could that began with the letter "s." In First and Last Letters the subjects were instructed to write all the words they could that began with "t" and ended with "e." In Suffixes they were instruction to write all the words they could that ended with "tion."

None of these tests had a projection on the number factor because in rotating strictly to configuration the number plane was passed through these tests. But, as in the case of the present study, the result was oblique primaries and the correlation between the axes absorbed the potential projections of these tests on number. The correlation between the number and word primaries in this battery was .33.

In another study, as yet unpublished, the three tests for the word factor were First Letters, Four-letter Words, and Suffixes. The number factor and word factor were correlated .47 in this study.

By refinement of these word tests it should be possible to devise word tests which do not involve number ability and, as opposed to these, others in which the content is verbal but which do not involve the word factor. It is not certain whether the manipulatory character or the associative character of these word tests is the basis of their relation to number. A type of word test for the investigation of this problem may be suggested here. The subject could be given a list of words which were the first members of pairs of words commonly associated, such as cup-saucer, man-woman, knife-fork. He could then be required merely to write the first letter of the associated word. Such a test would involve a minimum of manipulatory processes. On the other hand, a test like Free Writing, described above, probably has more of a manipulatory than an associative character.

* L. L. Thurstone. Primary mental abilities, *Psychometric Monographs*, 1938, No. 1. Chicago: Univ. Chicago Press, pp. 121.

† L. L. Thurstone. Factorial Studies of Intelligence, *Psychometric Monographs*, 1941, No. 2. Chicago: Univ. Chicago Press, pp. 94.

With regard to the interpretation of the factors in general, it should be pointed out that the identification of one of the factors as perceptual speed does not imply that it is *the* speed ability. Certain aspects of the classical problem of speed versus power may be reinterpreted in the light of the results of multiple factor analysis. There does not appear to be a single speed ability, but a number of them. If a test is composed of items of equal difficulty and, though not necessarily, of items of a low level of difficulty, then the scores on that test represent the amount done in a given time and measure a speed ability. In this sense at least three of the factors obtained in this study, number, space, and perceptual speed, are speed abilities.

VI

Summary and Conclusions

A battery of thirty-four tests was given to 223 high school seniors in the city of Chicago. The battery included sixteen tests for the identification of seven of the primary mental abilities and eighteen experimental tests for the investigation of particular hypotheses. The study was undertaken primarily to investigate the hypothesis that number ability represents the agility with which an individual can manipulate a symbolic system according to a specified set of rules, with two restrictions: that the symbolic system be familiar, and that the set of rules be highly practiced. Certain deductions were made from this hypothesis and non-numerical tests were designed to check these deductions. A secondary problem was to obtain a clearer identification of the perceptual speed factor in order that its relation to number ability might be determined.

The ten factors of the centroid matrix were rotated to simple structure without knowledge of the identity of the tests. Seven of the ten factors were given a psychological interpretation, two were triplets and non-interpretable, and one was a residual factor. The seven primaries that were given psychological interpretation were number, verbal, space, memory, perceptual speed, deduction, and induction.

Conclusions.—1) The number factor is most clearly identified by very simple number tests.

2) The perceptual speed factor is most clearly identified by cancellation tests, and the more scanning that is involved and the less cancelling the better the test is as a measure of perceptual speed.

3) A test involving manipulation of a symbolic system is a better measure of number ability the more familiar the symbolism.

4) A test involving operations according to a set of rules becomes a better measure of number ability with practice.

5) The data are in disagreement with the hypothesis that number ability is in the nature of a serial response.

6) The results are in agreement with the hypothesis that number ability is characterized by a facility in manipulating a symbolic system according to a specified set of rules.