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SPEED AND LEVEL COMPONENTS IN TIME-LIMIT
SCORES: A FACTOR ANALYSIS¹

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WHETHER by force of tradition, or for reasons of expedience, it has been the practice to administer and score group tests of ability, aptitude, and achievement in such a way as to yield only a time-limit score, defined as the number of items correctly answered within a specified length of time. Thus, the time-limit score often becomes the sole measure of the behavior represented in a test. When a test is "validated" with respect to some external criterion, a time-limit score, rather than some other type of score, is most likely to be used as the measure which is correlated with the criterion. Likewise, in making a factor analysis of a battery of tests, one is most likely to use time-limit scores. It is the writers' belief that the indiscriminate use of time-limit scores is one of the more unfortunate characteristics of current psychological testing since the time-limit score of a test frequently represents *two* relatively independent aspects of behavior: (a) the amount the subject knows or can perform (or in certain cases, the level of difficulty which he can reach), and (b) the rate at which the subject works. Somewhat at variance with current usage, we shall identify these aspects of test behavior, respectively, by the terms *level* and *speed*.² By ignoring the possibility that these two aspects of test performance may play different rôles in any given situ-

¹ This paper is a revision of a thesis presented by the first-named author as a candidate for the M.A. degree at Indiana University, 1943.

² These terms were employed by Baxter (1), who was able to show a marked independence of speed and level in a single omnibus test of intelligence. The present study, in effect, extends Baxter's approach to tests of varying content.

ation, the applied psychologist runs the risk of obtaining validity coefficients lower than those which might be obtained if the level and speed components were correctly weighted in the prediction. For example, if the level score on a test has greater validity than the speed score in predicting a criterion, use of the time-limit score may tend to mask the potential validity of the test by introducing the "dead wood" variance of the speed component. In factor analysis there exists a real danger that the primary factors which are found in a set of time-limit scores may themselves be factorially complex, that is, that they may consist of both speed and level components. When these primary factors are correlated, as is frequently the case, one should not consider the hypothesis that the correlations indicate the presence of a general factor of intelligence until it is shown that they are *not* due to the presence of an underlying speed factor.

It is true that a logical distinction between speed and level elements in test performance has long been recognized. However, in practice it has been assumed that since these elements appear to be highly correlated they are merely different aspects of the same underlying entity and that consequently the distinction can be ignored. Furthermore, it has been assumed that in any case the normal exigencies of group test administration preclude any attempt to make separate measurements of speed and level. Without undertaking to review the literature on the problem, we believe that these assumptions will bear analysis.

The assumption that speed and level are different aspects of the same thing has arisen partly through confusion in terms and partly through misinterpretation of the experimental evidence. The most frequent error is that of identifying time-limit scores as "speed" scores and then proceeding to cite correlations between time-limit scores and scores obtained in unlimited time. The point has been missed that these correlations are spuriously high, since they rest on a part-whole relationship. The score obtained in unlimited time is equal to the time-limit score *plus* whatever the subject can accomplish in additional time. Moreover, the correlation between these scores is a function of the length of the time-limit, for obviously as the time-limit is

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obtained in unlimited time. In any case, correlations as high
as even .7 or .8 are still not high enough to rule out the possi-
bility that a speed component, independent of level, exists in
the time-limit scores.

The number of studies in which correlations have been
obtained between rate-of-work scores and level scores is exceed-
ingly small. The obtained correlations are seldom more than
moderately high but even these have occasionally been cited as
showing the fundamental identity of speed and level compo-
nents in test performance.

These misinterpretations have usually occurred in connec-
tion with test performances in which the subjects vary con-
siderably with respect to their ability to answer the items and
in which the scores involve the number of items correctly
answered. A particularly dangerous misinterpretation, how-
ever, is likely to arise in connection with tests in which the sub-
jects vary *not* in their item-passing ability, but only in their
rate of performance. One frequently cited study is that carried
out by Paterson and Tinker (7), who came to the perfectly
sound conclusion that when corrected for attenuation, the cor-
relation between "work-limit" and "time-limit" scores on a
speed-of-reading test is virtually perfect. The work-limit score
was not the number of items correctly performed in unlimited
time but, instead, the time taken to read all the paragraphs in
the test. The time-limit score was the number of paragraphs
read within a time-limit. What Paterson and Tinker showed,
then, was that in the measurement of a *rate* of performance
it makes little difference whether the scores are expressed in
terms of performance-per-unit-of-time or time-per-unit-of-per-
formance. A convenient paradigm is that of a runner's speed,
which can be expressed either in terms of feet per second or in
terms of seconds per foot. It is a mistake, however, to general-
ize the results of the Paterson and Tinker study by inferring
that "work-limit" and "time-limit" scores in the usual sense
will be highly correlated in situations where elements of test
performance other than rate are measured.

With respect to the presumed impracticability of measuring

speed and level components separately within a time-limit, we can only point out that few attempts have been made to explore the problem. With ingenuity, it should be possible to devise relatively simple methods of making separate measurements even within a reasonable time-limit.

We would by no means assert that speed and level components of test performance are invariable entities from test to test. Rate of performance in one task may be completely independent of rate of performance in another task. Similarly, level components undoubtedly vary from test to test. The investigation reported here establishes the independence of several distinct types of speed, in addition to a general speed factor, and previous factorial investigations have isolated several types of level components (such as vocabulary knowledge, ability to solve problems expressed verbally, etc.).

We conclude this general introduction by making several recommendations in the fields of test construction and factor analysis. First, we suggest that persons responsible for the standardization and validation of tests experiment with the differential validities of speed and level scores and incorporate any significant findings in the directions for administering, scoring, and interpreting the tests. Investigations should be made of the possibility of restandardizing various published tests in terms of speed and level. Persons charged with selecting tests for use in given situations should give preference to tests which have been so standardized. Collateral experiments should meanwhile be directed towards discovering more efficient and reliable methods of measuring speed and level than, say, those employed in the present investigation.

Our second major recommendation is that in factorial studies aimed at discovering unitary abilities, tests should be represented by speed scores, level scores, or both, and that if time-limit scores are to be studied at all they should be treated in the manner exemplified in the investigation reported here.

The Experiment

In order to establish the linear independence of speed and level scores it was decided to study by factor analysis a matrix

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of correlations between speed, level, and time-limit scores in a number of short mental ability tests. As in Baxter's study (1), speed scores were obtained as the number of seconds taken by the subject to work from the beginning to the end of the test, attempting every item once. Level scores were defined as the number of items correctly answered when the subject is allowed to take all the time he desires to try every item and to check over his work. Time-limit scores were defined as the number of items correctly answered within a prescribed time-limit.

The test battery consisted of the eight subtests of the *Revised Alpha Examination*, Form 5; the *Minnesota Speed of Reading Test for College Students*, Form A; and several tests which had been specially constructed for previous factorial investigations. These included Letter Grouping and Scattered X's, studied by Thurstone (8); and Phrase Completion and Disarranged Morphemes, constructed by Carroll (2, 3). The *Revised Alpha Examination* was used because its subtests appear to measure verbal, numerical, and reasoning factors, to judge from Guilford's analysis of the original Army Alpha test (4), and because it is somewhat more practicable to administer and score than the original Army Alpha. Letter Grouping and Disarranged Morphemes were included to aid in defining the domain of reasoning ability. Scattered X's was included to test the hypothesis that the Perceptual Speed factor (*P*) as measured by the test might be involved in some of the speed scores studied here. The *Minnesota Speed of Reading Test* was included because it was believed that speed of reading might be related to speed scores on mental tests which contain reading material.

Speed, level, and time-limit scores (as defined above) were obtained for each test or subtest in the battery, with three exceptions. For the *Minnesota Speed of Reading Test*, the only score obtained was the number of paragraphs marked, correctly or incorrectly, in the prescribed 6-minute time-limit. This score measures the speed aspect of performance on the test. The score on Scattered X's was the number of x's found and marked in 4 minutes; again, this score is primarily a measure of rate of performance. Phrase Completion had no time-



limit and was scored by means of a key the construction of which has been described in a previous article (3). Special instructions and procedures were devised to obtain speed, level, and time-limit scores on the same test.

A large clock with a sweep-second hand was placed in view of the subjects. First the subjects worked on a test for the prescribed time-limit, marking an "x" in the margin after the last answer written within the time-limit. They were then instructed as follows: "Continue working rapidly on the test to the end, but do not yet change any answers you have already written. As soon as you each individually finish the test, quickly look up at the clock and record at the bottom of the page the minutes and seconds you required to do the remainder of the test below the X. Do not stop too long on any one problem. You may guess at answers you don't know or leave blanks. Write the time before you go back to fill blanks or make corrections. After you record your time, you may take your red pencil and make any additions or corrections, but do not erase present answers." The students were allowed to work on each test until all had finished, except on the Disarranged Morphemes and Letter Grouping tests, where a few students were not able to finish within 23 and 18 minutes, respectively, after the time-limit. This procedure yielded a *time-limit* score, which resulted from the application of the prescribed scoring formula to all answers written in ordinary pencil up to the X marked by the subject. The *speed* score was the number of minutes and seconds recorded at the bottom of each test. The *level* score was the score on the entire test; if an answer in black was followed by a different one in red, the latter was taken as the answer for purposes of arriving at the level score.

The time-limits used for the subtests of the *Revised Alpha Examination* were those recommended by Tinker and Baker (9) for use with college students. For experimental purposes scores for two time-limits—2 and 4 minutes—were obtained for subtest 2, Arithmetical Reasoning.

The subjects were undergraduate students in elementary experimental psychology at Indiana University. The analysis of test scores was based upon 91 complete cases—12 men and 79 women.

The markedly skewed distributions of certain speed scores (on the subtests Addition, Common Sense, Same-Opposite, and Disarranged Sentences in the Revised Alpha) were made more

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nearly normal by converting them to the reciprocals of the number of seconds. Scores on all variables were coded in ten or fewer class intervals. Hollerith procedures were used to obtain Pearsonian product-moment coefficients. No corrections for grouping or attenuation were applied to the coefficients. Before the correlation matrix was assembled for factor analysis, the level and time-limit scores on subtests Addition, Common Sense, and Disarranged Sentences were discarded, first, because most of the students made perfect scores in unlimited time, and second, because time-limit scores were very highly correlated with speed scores. Scattered X's was omitted from the analysis because it was little correlated with any other variable in the battery. It was therefore concluded that the Perceptual Speed factor as measured by Scattered X's is not significantly involved in the speed variables studied here.

The Factor Analysis

Level and time-limit scores, as defined here, are overlapping measures since the level score on a test can be regarded as equal to the time-limit score plus whatever additional correct answers the subject can give in time beyond the time-limit. There is likewise an obvious overlap between speed and time-limit scores since the faster the subject works the more items he has an opportunity to pass within the time-limit. It was believed that these factors of overlap would introduce spurious dimensions in the factor analysis if the correlation matrix were analyzed in the usual fashion. To put the matter differently, insertion of the time-limit scores would spuriously raise the communalities of the speed and level scores. A special method of factoring the matrix was suggested by Dr. L. R. Tucker. The *main* matrix, involving only speed and level variables, was analyzed in the usual way by the centroid method. All correlations between time-limit scores and speed scores or between time-limit scores and level scores were placed in a *subsidiary* matrix and factored separately. (See Table 1 for the correlations represented in the main and subsidiary matrices.) Correlations *among* time-limit variables were not analyzed at all. Essentially, the procedure involved locating the time-limit variables in the factor space

TABLE 1
The Correlation Matrices*

	"Main" matrix										"Subsidiary" matrix																
	5	6	7	8	9	10	11	12	13	14	16	18	20	21	22	23	24	25	35	27	28	30	32	33	34	36	38
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9	31	13	62	44																							
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14	10	33	30	18	29	34	29	42	36	21																	
16	23	44	30	18	29	34	29	42	36	21	13																
18	03	16	33	33	35	17	30	25	31	03	13	13															
20	26	16	34	29	34	48	42	56	32	18	22	22	22														
21	00	-02	30	15	24	17	34	29	12	05	24	34	37	37	29												
22	08	13	32	25	41	35	37	42	46	13	35	34	64	29	56	07											
23	-07	03	00	11	01	21	11	14	21	07	20	21	17	13	07	29	29										
24	12	20	16	26	29	18	30	33	26	14	33	38	43	25	43	44	29	19									
25	06	30	52	45	50	36	44	43	40	30	15	40	24	27	18	35	10	19									
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* This table presents only the correlations used in the factor analysis. The names of the variables are given in Table 4. All entries have been multiplied by 100 to eliminate the decimal point.

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11	25	34	58	59	49	43	58	40	42	30	42	34	37	34	11	30	44	46	36	52	47	83	52	49	37
12	27	33	60	47	48	64	58	40	42	25	56	29	42	47	14	33	43	50	46	42	52	62	56	51	41
13	07	26	50	45	38	28	38	40	36	31	32	12	46	56	21	26	40	42	28	36	35	35	56	60	30
14	10	33	33	20	19	23	36	28	21	03	18	05	13	18	07	14	30	20	14	16	26	38	28	20	45
16	23	44	30	18	29	34	29	42	36	13	51	24	35	42	20	33	15	80	71	11	45	25	32	46	29
18	03	16	33	33	35	17	30	25	31	03	13	34	34	36	21	38	40	25	21	62	23	28	35	34	34
20	26	16	34	29	34	48	42	56	32	18	51	22	37	64	55	17	43	47	48	26	77	41	40	48	48
21	00	-02	30	15	24	17	34	29	12	05	24	34	37	29	33	13	25	30	32	27	30	39	23	32	30
22	08	13	52	25	41	35	37	42	46	13	35	34	64	56	07	43	18	39	33	27	51	35	57	48	45
23	-02	15	41	30	37	18	34	47	56	18	42	36	55	33	56	29	44	48	41	39	42	34	52	78	42
24	-07	03	00	11	01	21	11	14	21	07	20	21	17	13	07	29	10	25	26	22	19	04	21	33	20
25	12	20	16	26	29	18	30	33	26	14	33	38	43	25	43	44	29	27	30	33	29	31	36	38	73
35	06	30	52	45	50	36	44	43	40	30	15	40	24	27	18	35	10	32	19	47	30	45	50	40	36

* This table presents only the correlations used in the factor analysis. The names of the variables are given in Table 4. All entries have been multiplied by 100 to eliminate the decimal point.

defined by the speed and level variables. Factor loadings for the variables in the subsidiary matrix were obtained by summing the columns of correlations or residuals; the product of the column sum and the value $(\sum r)^{-1}$ used to compute the *m*th factor loadings for the main matrix was the *m*th factor loading for the subsidiary matrix variable. Residuals in the subsidiary matrix were computed and treated in the usual way.

TABLE 2
The Centroid Matrix

Test	I	II	III	IV	V	VI	<i>h</i> ²
5	.32	-.37	-.10	-.18	-.05	.27	.36
6	.43	-.40	-.09	-.35	.18	-.18	.54
7	.71	.05	-.41	-.18	-.17	-.11	.75
8	.62	.10	-.34	-.16	-.04	.23	.59
9	.65	.12	-.28	-.16	-.04	-.05	.52
10	.63	-.36	-.10	-.18	.15	.10	.60
11	.71	-.06	-.21	-.14	.07	-.07	.58
12	.76	-.21	-.05	-.13	.05	-.06	.65
13	.64	.18	-.11	.32	-.32	-.02	.66
14	.39	-.16	-.23	.32	-.04	-.13	.35
16	.56	-.24	.29	.19	-.13	-.10	.52
18	.48	.44	.03	-.05	.32	-.04	.53
20	.67	-.12	.42	-.22	-.18	-.05	.72
21	.40	.18	.22	-.20	.15	-.24	.36
22	.61	.11	.33	-.17	-.22	-.14	.59
23	.65	.34	.28	.16	-.18	-.13	.70
24	.26	.06	.24	.21	.21	.31	.31
25	.51	.14	.35	.05	.10	.12	.43
35	.58	.17	-.34	.05	.22	-.17	.56
27	.69	-.23	.26	.22	-.05	-.22	.70
28	.58	-.25	.36	.07	.02	-.06	.55
30	.62	.44	-.23	-.20	.18	.26	.77
32	.70	-.28	.23	-.25	-.09	-.01	.69
33	.71	-.13	-.20	-.18	.15	-.17	.64
34	.75	.15	-.11	.03	-.01	.02	.60
36	.72	.26	.15	.26	-.07	-.09	.69
38	.62	.13	.25	.18	.17	.10	.54

As shown in Table 2, six centroid factors were extracted. The centroid matrix for the *main* correlational matrix was the basis for the rotation to simple structure, which was accomplished by Tucker's semi-analytical method (10) in five trials. The transformation matrix (Table 3) was used to obtain the final rotated matrix (Table 4) both for the *main* and the *subsidiary* matrix variables. The time-limit scores did not in any way influence the rotational procedures; nevertheless, the vec-

TABLE 3
The Transformation Matrix

	A	B	C	D	E	F
I	.33	.30	.16	.09	.18	.42
II	-.77	.09	-.16	.11	.32	-.11
III	.29	.73	-.24	-.18	.12	-.84
IV	.04	.29	.89	-.03	.20	-.05
V	.38	-.54	.07	.74	.43	-.15
VI	.27	.00	-.30	-.63	.79	.28

tors for these scores were found to fit well in the simple structure already established by the speed and level scores. Table 5 shows the correlations between the primary factors.

TABLE 4
The Rotated Factorial Matrix

Test Variable	A	B	C	D	E	F
<i>Speed Scores:</i>						
5 Addition	.38	-.04	-.13	-.18	.06	.34
6 Arith. Reasoning	.46	.02	.54	.28	-.06	.22
7 Common Sense	-.02	-.06	.04	.10	-.09	.63
8 Same-Opposite	.06	-.08	-.05	-.06	.25	.62
9 Disarr. Sentences	.04	.04	.02	.10	.04	.43
10 Number Series	.53	-.05	-.02	.09	.09	.41
11 Verbal Analogies	.21	-.01	.06	.16	.04	.46
12 Directions	.38	.10	.05	.13	.02	.36
13 Disarr. Morphemes	-.07	.39	.34	-.13	.10	.36
14 Letter Grouping	.13	.04	.46	.12	-.08	.34
<i>Level Scores:</i>						
16 Arith. Reasoning	.38	.50	.26	-.08	-.05	-.01
18 Same-Opposite	-.07	.05	-.01	.34	.32	.06
20 Number Series	.31	.53	-.16	-.14	-.03	-.02
21 Verbal Analogies	.02	.14	-.11	.30	-.02	-.10
22 Directions	.10	.51	-.10	-.04	-.05	-.06
23 Disarr. Morphemes	-.08	.61	.18	-.02	.10	-.04
24 Phrase Completion	.28	.21	.08	-.06	.46	-.06
25 Letter Grouping	.22	.38	.00	.01	.31	-.09
35 Speed of Reading	.02	-.16	.26	.38	.09	.46
<i>Time-Limit Scores:</i>						
27 Arith. Reasoning .. (2')	.43	.46	.32	.08	-.06	.06
28 Arith. Reasoning .. (4')	.50	.46	.10	-.02	.04	-.08
30 Same-Opposite ... (1')	-.08	-.13	-.16	.14	.46	.46
32 Number Series ... (2½')	.48	.34	-.13	-.08	-.04	.14
33 Verbal Analogies .. (2')	.30	-.09	.05	.30	-.04	.42
34 Directions (3')	.10	.18	.16	.10	.18	.38
36 Disarr. Morphemes (8')	.04	.44	.28	.08	.18	.11
38 Letter Grouping .. (8')	.24	.36	.17	.07	.37	.02

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A	1.00
B	-.00
C	-.00
D	-.00
E	-.20
F	.00

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- 20 Numb
- 33 Verbal

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TABLE 5
Correlations between the Primary Factors

	A	B	C	D	E	F
A	1.000	-.044	-.082	-.035	-.242	.002
B	-.044	1.000	-.422	.688	.052	.635
C	-.082	-.422	1.000	-.460	.051	-.401
D	-.035	.688	-.460	1.000	.130	.516
E	-.242	.052	.051	.131	1.000	-.036
F	.002	.635	-.401	.516	-.036	1.000

Interpretations of the Factors

In interpreting the factors we follow the arbitrary rule that a projection larger than .30 indicates a significant loading of a test on a factor.

The variables having projections of .30 or greater on factor A are ranked below in order of size of projection. Significant projections on other factors are also given.

No.	Test Variable	Projections	
		A	Other factors
10	Number Series (speed)	.53	.41F
28	Arithmetical Reasoning (4' time-limit)	.50	.46B
32	Number Series (time-limit)	.48	.34B
6	Arithmetical Reasoning (speed)	.46	.54C
27	Arithmetical Reasoning (2' time-limit)	.43	.46B; .32C
5	Addition (speed)	.38	.34F
12	Directions (speed)	.38	.36F
16	Arithmetical Reasoning (level)	.38	.50B
20	Number Series (level)	.31	.53B
33	Verbal Analogies (time-limit)	.30	.30D; .42F

Most of these tests obviously have to do with simple arithmetical computation; factor A, then, appears to be the number factor N identified in previous studies. The speed scores of Arithmetical Reasoning and Number Series have higher loadings than the corresponding level scores. The interpretation may be offered that for college adult subjects this factor refers to the speed aspect of computational behavior. The level of competence of these subjects is such that accuracy in arithmetic plays only an incidental rôle, although rapid arithmetical ability appears to facilitate the correct performance of the relatively complicated tasks set in Arithmetical Reasoning and Number Series. The presence of Directions (speed) on this factor becomes understandable when it is noted that a con-

rix

D	E	F
.09	.18	.42
.11	.32	-.11
.18	.12	-.84
.03	.20	-.05
.74	.43	-.15
.63	.79	.28

vell in the simple struc-
d level scores. Table 5
ary factors.

strix

C	D	E	F
-.13	-.18	.06	.34
.54	.28	-.06	.22
.04	.10	-.09	.63
-.05	-.06	.25	.62
.02	.10	.04	.43
-.02	.09	.09	.41
.06	.16	.04	.46
.05	.13	.02	.36
.34	-.13	.10	.36
.46	.12	-.08	.34
.26	-.08	-.05	-.01
-.01	.34	.32	.06
-.16	-.14	-.03	-.02
-.11	.30	-.02	-.30
-.10	-.04	-.05	-.06
.18	-.02	.10	-.04
.08	-.06	.46	-.06
.00	.01	.31	-.09
.26	.38	.09	.46
.32	.08	-.06	.06
.10	-.02	.04	-.08
-.16	.14	.46	.46
-.13	-.08	-.04	.14
.05	.30	-.04	.42
.16	.10	.18	.38
.28	.08	.18	.11
.17	.07	.37	.30

siderable share of the items involve numbers and numerical operations.

Factor *B* has the following tests:

	<i>B</i>	Other
23 Disarranged Morphemes (level)61	-----
20 Number Series (level)53	.31A
22 Directions (level)51	-----
16 Arithmetical Reasoning (level)50	.38A
27 Arithmetical Reasoning (2' time-limit) ..	.46	.43A; .32C
28 Arithmetical Reasoning (4' time-limit) ..	.46	.50A; .10C
36 Disarranged Morphemes (time-limit)44	-----
13 Disarranged Morphemes (speed)39	.34C; .36F
25 Letter Grouping (level)38	.31E
38 Letter Grouping (time-limit)36	.37E
32 Number Series (time-limit)34	.48A

In previous factorial studies tests similar to those represented above have been identified as tests of reasoning ability. Of the eleven variables listed, ten are directly or indirectly measures of *level* of ability (time-limit scores being regarded as a function of both level and speed). In the light of these considerations, factor *B* may be identified as a Level of Reasoning factor. The present battery is too limited to indicate the relation of this factor to the *inductive* and *deductive* reasoning factors which have been indicated in previous studies. The presence of the speed score of Disarranged Morphemes on this factor is interesting. In contrast to other tests in this battery, Disarranged Morphemes is of such a nature that it is almost impossible for a subject to be satisfied with an incorrect answer; the subject either solves an item correctly or is forced to skip it. Consequently, speed of performance in this task would be almost perfectly related to ability to answer the items if the subjects did not differ in their willingness to skip items. Because of this inherent connection between speed and level aspects of performance on the Disarranged Morphemes test, it is not surprising to find the speed score present on the Level of Reasoning factor. Parenthetically, we may say that there are several subtle problems in this area which this study has not been designed to handle. For example, one would like to know how the speed-level relationship varies with the difficulty of the task and whether the relationship in the case of multiple-choice tests is essentially different from that in the case of tests where

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the subject is forced by the nature of the test to answer correctly or not at all.

Factor *C* is represented by the following test variables:

<i>B</i>	Other
.61	-----
.53	.31A
.51	-----
.50	.38A
.46	.43A; .32C
.46	.50A; .10C
.44	-----
.39	.34C; .36F
.38	.31E
.36	.37E
.34	.48A

	<i>C</i>	Other
6 Arithmetical Reasoning (speed)54	.46A
14 Letter Grouping (speed)46	.34F
13 Disarranged Morphemes (speed)34	.39B; .36F
27 Arithmetical Reasoning (2' time-limit) ..	.32	.43A; .46B

The tests represented here are reasoning tests also found in factor *B*. Factor *C*, however, is constituted by measures of speed. Level is not independently represented at all. Factor *C* may hence be regarded as a Speed of Reasoning factor. As will be shown later by multiple regression techniques, the time-limit scores of these reasoning tests are much more heavily weighted with level than with speed. It is not surprising that only one time-limit score (from the 2' time-limit on Arithmetical Reasoning) appears on factor *C*. The 4' time-limit score on this test has a loading of only .10 on *C*.

It is of interest to note from Table 5 that there is an appreciable negative correlation between factors *B* and *C*. This probably indicates that when other factors are ruled out, those who are hasty in performing these reasoning tests are likely to be inaccurate.

Factors *D* and *E* lack definition in the present limited battery. They are represented by the following variables:

Factor <i>D</i> :	<i>D</i>	Other
35 Speed of Reading38	.46F
18 Same-Opposite (level)34	.32E
21 Verbal Analogies (level)30	-----
33 Verbal Analogies (time-limit)30	.30A; .42F

Factor <i>E</i> :	<i>E</i>	Other
24 Phrase Completion46	-----
30 Same-Opposite (time-limit)46	.46F
38 Letter Grouping (time-limit)37	.36B
18 Same-Opposite (level)32	.34D
25 Letter Grouping (level)31	.38B

Factor *D* may perhaps be characterized as a verbal reasoning factor which emphasizes formal relationships such as those of antonymity, genus-species, etc. Factor *D* is highly correlated with factor *B*, the Level of Reasoning factor. Were it not for the presence of both the level and time-limit scores of Letter

r to those represented reasoning ability. Of tly or indirectly mea- es being regarded as a he light of these con- s a Level of Reasoning ed to indicate the rela- l deductive reasoning revious studies. The ed Morphemes on this r tests in this battery, ture that it is almost h an incorrect answer; r or is forced to skip it. t this task would be iswer the items if the ss to skip items. Be- tween speed and level ed Morphemes test, it resent on the Level of ay say that there are ch this study has not ne would like to know th the difficulty of the ase of multiple-choice he case of tests where



Grouping on the factor, factor *E* might readily be interpreted as the verbal factor identified in previous studies.

Factor *F* is represented by the following variables:

	<i>F</i>	Other
7 Common Sense (speed)63	---
8 Same-Opposite (speed)62	---
11 Verbal Analogies (speed)46	---
35 Speed of Reading46	.38 <i>D</i>
30 Same-Opposite (time-limit)46	.46 <i>E</i>
9 Disarranged Sentences (speed)43	---
33 Verbal Analogies (time-limit)42	.30 <i>A</i> ; .30 <i>D</i>
10 Number Series (speed)41	.53 <i>A</i>
34 Directions (time-limit)38	---
12 Directions (speed)36	.38 <i>A</i>
13 Disarranged Morphemes (speed)36	.39 <i>B</i> ; .34 <i>C</i>
5 Addition (speed)34	.38 <i>A</i>
14 Letter Grouping (speed)34	.46 <i>C</i>

Every one of these variables involves either a direct or an indirect measure of speed. It is also true that with one exception every speed score in the battery appears in the above list. Only four time-limit scores are absent: those of Arithmetical Reasoning, Number Series, Disarranged Morphemes, and Letter Grouping, and in these tests it can be shown that speed contributes little to the time-limit scores. Hence it may be concluded that factor *F* is a general speed factor involving rate of work in performance of tasks of the sort found in intelligence tests. The factor is similar to a general speed factor found in some of Holzinger's studies (5, 6). The content of a test does not seem to play any rôle in determining the loading of its speed score on factor *F*, since tests of verbal, numerical, and reasoning abilities all appear in the above list. No definite conclusions can be drawn from the present data, however, as to whether this factor extends to both easy and difficult tasks.

The presence of Speed of Reading on factor *F* might lead one to suspect that speed of reading is fundamentally involved in this factor. However, some of the tests whose speed scores measure the factor (e.g., Addition, Number Series, and Same-Opposites) do not have items containing connected text-material where a speed of reading factor could be expected to operate. It appears, therefore, that an individual's reading speed is partly a function of some more general speed factor.

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Multiple-Correlation Analysis

Although it is believed that factorial techniques provide a more complete and concise analysis of the data, multiple-correlation techniques can be used to evaluate the independent rôles of speed and level in determining the variance of time-limit scores. Table 6 presents such an analysis for all tests in the

TABLE 6
Beta-Coefficients and Multiple Correlations in the Prediction of Time-Limit Scores (T) from Speed (S) and Level (L) Scores

Test	Zero-order Correlations			Relative Contributions of:		$R_{T \cdot SL}$
	r_{TS}	r_{TL}	r_{SL}	Speed $\beta_{TS \cdot L}$	Level $\beta_{TL \cdot S}$	
Alpha Examination:						
1. Addition809	.344	.220	.770	.174	.826
2. Arith. Reas. (2')482	.797	.439	.165	.725	.811
2. " " (4')323	.712	.439	.012	.706	.711
3. Common Sense830	.296	.187	.804	.146	.843
4. Same-Opposites734	.616	.327	.596	.421	.835
5. Disarranged Sentences790	.518	.308	.698	.303	.842
6. Number Series673	.766	.485	.394	.575	.840
7. Verbal Analogies831	.392	.338	.788	.125	.840
8. Directions649	.566	.415	.500	.358	.724
Disarranged Morphemes596	.782	.564	.227	.653	.804
Letter Grouping451	.726	.137	.358	.677	.808

battery for which speed, level, and time-limit scores were obtained. The beta-coefficients indicate the relative contributions made by speed and level in predicting time-limit scores. In some tests, such as Arithmetical Reasoning, the time-limit scores are chiefly a function of the level of item difficulty that can be mastered by the subject, while in other tests, such as Common Sense, the time-limit scores are primarily measures of the subject's rate of work. In still other tests, such as Same-Opposite, speed and level are about equally weighted in the time-limit score. These relationships depend to some extent on the particular time-limits which had been set.

Even where the correlation between time-limit and level scores is fairly high the contribution of an independent speed component to the time-limit score is sometimes fairly large (e.g., in the case of Letter Grouping). The multiple correla-

readily be interpreted
s studies.
ving variables:

F	Other
.63	—
.62	—
.46	—
.46	.38D
.46	.46E
.43	—
.42	.30A; .30D
.41	.53A
.38	—
.36	.38A
.36	.39B; .34C
.34	.38A
.34	.46C

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tions in Table 6 are the correlations obtained in the prediction of time-limit scores from level and speed scores. These multiple correlations are in some cases considerably higher than the corresponding zero-order coefficients. Nevertheless, there remains in each case a certain amount of specific (unpredicted) variance in the time-limit score which would militate against the prediction of level scores, for example, from a weighted combination of speed and time-limit scores.

Summary

A number of relatively simple group mental tests were administered to 91 college students in such a way as to yield three types of score: *speed*, *level*, and *time-limit*. Speed scores represented the time required to attempt every item once; level scores represented the number of items correctly answered in unlimited time; and time-limit scores were the number of items correctly answered in a prescribed time-limit. Factor analysis revealed that in all cases speed scores were linearly independent of level scores and that time-limit scores could be represented as factorially complex measures having loadings on both speed and level dimensions of ability. Of the factors which were identified several were similar to verbal, numerical, and reasoning factors isolated in previous factorial studies. In the domain of reasoning ability both level and speed factors were identified. A general speed factor involving nearly all of the speed scores was found. It is concluded that because of their factorial complexity, time-limit scores should be used with considerable caution both in factorial studies and in studies involving the prediction of criteria.

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