A SPEED FACTOR IN MENTAL TESTS

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A Speed Factor in Mental Tests

CHAPTER I

THE PROBLEM

In the evaluation of a mental task, the most important consideration is often difficulty or level. A task so complex that it can be performed by but one intellect in a thousand tends to have a high social value. As in the case of a scientific discovery, a mechanical invention or a piece of literature, the worth of a mental task often depends upon the rarity of achievement, rather than on the time taken in its formulation.

Sometimes, however, the social value of mental work is in terms of extent or amount done, difficulty remaining approximately the same throughout. Such is the case with the making of routine computations and records. An employer is interested in speed when time is used as a basis for compensation, but his primary concern is that the work be done completely and accurately. Nevertheless, the speed at which a mental task is performed is of considerable theoretical and practical significance. Of two intellects which are able to solve problems of the same difficulty, or the same number of problems at a given level of difficulty, the one which is able to accomplish its tasks in less time is properly considered the better (22).

In any mental act, level of performance and speed of performance are indissolubly connected. Time is consumed in all performances and all performances have at least some complexity. With motivation and all other factors held constant, it is possible, however, to minimize the influence of speed in producing differences in mental product by allowing sufficient time for even the slowest of the subjects to complete the task if it is possible for him to do so.

Similarly by presenting tasks of the same level of difficulty to different individuals and securing from each the same quantity of product, the differences among the individuals can be ascribed to speed, varying inversely with the time consumed in the performance.

Differences in speed of mental performances may be measured still better, perhaps, by presenting a series of tasks of the same general level of difficulty and holding time constant. The number of tasks completed within the time set will afford a basis for the comparison of different individuals in regard to speed in that type of performance.

It has been shown by experiment that the ability to perform difficult tasks possesses considerable unity in verbal and numerical materials, the manipulation of which comprises much of mental life (20). Tests of verbal ability and numerical ability are not mere measures of general ability, however, as the group factors which they represent have been shown to be independent of at least one other mental trait, immediate memory (1).

The question arises whether there is a unitary ability to perform mental tasks quickly. The problem is complicated because with an increase of difficulty there is usually a decrease of speed, the variables changing simultaneously. On the higher levels of difficulty there is a greater tendency to make errors, which undoubtably represent mental work of a sort, but of a quality that differs from mental work done correctly.

To demonstrate speed as a unitary characteristic of mental behavior, an ideal method might be to find a speed factor on a number of representative levels of difficulty, accuracy being held constant at each level. If these abilities at the various levels could then be shown to be one and the same, a general speed factor could be taken as demonstrated. Such a procedure, however, would require a very large expenditure of time and effort.

This experiment seeks to discover a common factor of mental speed on a single level—and that a low level—of difficulty. It is not only necessary to locate such a factor but also to establish its independence of other unit-abilities. The tests which best measure the ability must also be determined.

In planning the present experiment two distinct types of tests were selected: "Level" tests in which the difficulty in the latter parts of the tests was so great and the time limits were so long that an increase in the time allowed would not have added appreciably to the score, and "speed" tests composed of very simple mental test material so arranged that the level of difficulty was approximately the same throughout and in which alterations in the time limits would have been reflected in score fluctuations of considerable size.

If it can be shown that there is a factor common to the "speed" tests when difficulty or level is uniform throughout and that this factor is not large in the level or power tests, a speed factor in mental test material may be taken as demonstrated.

THE LITERATURE

In the literature no experiment is reported that attempts to investigate the existence of a factor of mental speed by using the materials and plan of attack outlined above. Many investigations of speed are reported, but as a rule these are concerned with speed in sensori-motor skills such as tapping and reaction time or else employ stock intelligence tests, in which level of difficulty, number of items on any one level of difficulty and speed are factors that vary in unknown proportions. Studies made on such tests can hardly be expected to answer the questions proposed in this study. Few of the studies employ adequate groups of subjects and many of the investigators fail to determine the reliability of their tests.

A number of studies bearing on the rôle of speed in mental ability are summarized by McFarland (16). Tinker also discusses the significance of speed in test response (23).

On what seems to be inadequate evidence, Bernstein (2) concludes that there is no general speed ability. He bases this statement on the low correlations, varying from $-.228$ to $+.193$ between estimates of "slowness" made on two groups of subjects and their "slowness" scores obtained by subtracting scores on haste tests from scores on leisure tests of sentence completion, directions, concomitants, analogies and moral classifications. The two groups of subjects consisted of 70 boys averaging 11 years 9 months in age and of 60 boys averaging 13 years. All of Bernstein's tests were short, the leisure tests having fewer items than the haste tests. A constant time allowance of 30 seconds a page was observed. In the leisure tests this allowed ample time for all subjects to complete the test, but this time allowance on the haste tests was too brief for all but the fastest subjects to complete. The reliability of all the tests averaged .70, the reliability of the estimates of slowness, .55.

Bernstein's intercorrelations of leisure tests, of haste tests, and the correlations of leisure with haste tests are all about the same, averaging from .66 to .69 in one group, and from .69 to .73 in the other. If leisure and haste tests measure different abilities, their respective intercorrelations should be higher than the correlations with each other. Since this is not true, the procedure of subtracting haste scores from leisure scores is of doubtful value. Furthermore, there is no evidence that the slower individuals excelled, relatively, in the leisure tests; and the leisure tests entailed more

difficulty and a greater range of difficulty than the haste tests, but to unknown extents.

McFarland (15) reports evidence of a speed factor through such tests as simple auditory reaction time, free association, opposites and the pencil maze. He gave the tests individually to one group of four subjects and another of five subjects, holding accuracy constant experimentally. The rank orders of the subjects in speed of performance on the various tests showed considerable consistency. Ten of the tests were given to 34 subjects, and the intercorrelations, ranging from .00 to .87, fell into a hierarchy, which McFarland took as evidence of a speed factor involving Spearman's *g*. The size and importance of the factor, however, are not determined. No reliabilities of the tests are given.

Kennedy (13) obtained intercorrelations ranging from .02 to .70 with a group of 32 adults in such speed tests as Woodworth-Wells substitution, Woodworth-Wells simple directions, color naming, three reaction tests and the Otis S. A. and Terman Group tests given individually. Kennedy's tests had reliabilities between .74 and .91. The intercorrelations ranged from .02 to .70 and averaged .34, and were not greatly affected when the variability due to intelligence was held constant. The relationship between speed ratings and intelligence was .14. There is little evidence of a speed factor in these results. If present, it must have been small.

In studying the relationship between rate and ability, Hunsicker (11) used four groups of subjects, two groups of college students and two of grade children, the total being 153. The speed tests were administered individually and consisted of two sheets of I. E. R. arithmetic problems, 20 elements in all, of which the reliabilities for the various groups were .83, .86, .95 and .79, and two sheets of I. E. R. completions, 22 elements in all, of which the reliabilities were .91, .76, .75 and .80. The correlations between rate in arithmetic and rate in sentence completion for the groups were .71, .46, .56 and .50, and, corrected for attenuation, .81, .57, .67 and .63. The correlations between rate and level in arithmetic were .29, .46, .49 and .29, and in sentence completion .50, .19, .49 and .41. These results indicate a low positive relation between speed of performance and level of performance in mental tasks. No evidence on the existence of a speed factor is presented.

Freeman (9) used the method of giving various intelligence tests twice, once with the regular time limits, once with unlimited time, in order to determine the importance of speed in affecting

scores on the tests. High correlations show that speed does not appreciably affect the score, low correlations that speed is of considerable importance. He found a correlation of .58 between the two trials of the Otis Advanced Examination with a group of 51 school children, a correlation of .88 between two trials of the Dearborn (N = 100), a correlation of .83 between two trials of the National Intelligence Test (N = 56) and a correlation of .93 between two trials of the Terman Group Test (N = 42). From these results he concludes that speed of reaction is of some importance in the Otis Advanced Examination, but not particularly important in the Dearborn, N. I. T., or Terman Group Test.

Peak and Boring (18) using two men and three women subjects timed each item separately on the Army Alpha, Forms 5 and 6, and the Otis S. A. Higher Examination, Forms A and B. Simple reaction times were also taken. High correlations, ranging from .70 to .90, were found between scores on intelligence tests, speed on intelligence tests and reaction time, but the small number of subjects alone makes the conclusions of very slight importance. Lemmon's (14) more adequate study, with 100 subjects, resulted in a correlation of but .13 between the Thorndike Intelligence examination and 200 discriminative reactions from each subject. Farnsworth, Seashore and Tinker (7) also failed to confirm Peak and Boring's study. Using 34 subjects, correlations of −.16 to −.24 were found between simple reaction time and intelligence, and from .07 to .53 between serial reaction times and various intelligence scores. These studies indicate little relation between speed of reaction and intelligence test results.

Using a group of 180 "pupils," Clark (4) gave two speed tests, arithmetic and sentence completion, taking as the score the median number of seconds to complete the items of the tests. Except in nine cases, records were discarded unless accuracy was 80% or better. The reliability of the arithmetical speed test was .79, of the sentence completion speed test, .88. The correlation between the two speed tests was .42, corrected for attenuation, .50. The intercorrelations were:

	Binet M. A.	*Otis S. A.*	*Terman Group*
Arithmetic speed			
Obtained coefficients	.55	.71	.67
With variability due to arithmetic level and range held constant by partial correlation	.27	.23	.25

	Binet M. A.	*Otis S. A.*	*Terman Group*
Sentence completion speed			
Obtained coefficients	.49	.49	.32
With variability due to sentence completion level and range held constant by partial correlation	.31	.29	.02

The correlation between general level and intelligence was estimated at .97, between general range and intelligence at .91 and between general speed and intelligence .62. From Clark's data it is not possible to draw any conclusions regarding the existence of a general speed factor, other than that speed does not enter into intelligence test scores to the same degree as level or range.

In attempts to isolate speed as a general factor in intelligence tests, R. E. Clarke (5) used three forms of Scale B of the N. I. T. on 166 undergraduates and Sanford (19) used levels F, G and H of Thorndike's CAVD scale with 100 undergraduates, both groups being homogeneous as to sex and race. In both cases the evidence for the existence of such a factor was suggestive as all correlations were positive, but not conclusive, as no factor was isolated. Clarke found a considerable number of errors made by adult subjects even on tests designed for use in the grades, so that difficulty was not constant. Tetrad analysis showed overlapping group factors which obscured any general factor of speed. Sanford was not able to get measures of reliability.

CHAPTER II

PROCEDURE

In the present experiment tests were given to 233 male subjects. The records of only 139, however, were used in computing our results, for reasons to be given later (p. 20). Our tests included an arithmetic reasoning test of 44 items ranging from fairly easy to decidedly difficult; a vocabulary test of 160 items, the difficulty of which becomes progressively greater; two batteries of five tests designed to reflect differences in speed; the Minnesota paper form board tests, the two forms of which were administered in the speed batteries; and Test 3 of the Haggerty reading examination, Sigma 3, Form A, given at the end of the second speed battery.

The arithmetic reasoning test is a revision of a similar test found by Schneck (20) to be the best measure of "numerical ability." The vocabulary level test is similarly a revision of Schneck's best measure of "verbal ability."

Four speed tests, each having two forms, were constructed especially for this experiment. These tests were arithmetic computation, analogies, directions and arithmetic problems. All contain material similar to that used in standard intelligence tests, but of a low difficulty level. Preliminary forms were first constructed and administered to groups of college students averaging 68 in number. The tests were given with an amount limit, each subject being allowed to finish. In some cases the subjects read their own scores from a stop clock in the front of the room; in other cases approximate time scores were put on a blackboard by the administrator, using a stop watch and changing the time readings each ten seconds.

The general purpose of the preliminary tests was to discover the relative difficulty of the various items, using the number of errors made on each item as the criterion of difficulty. The most difficult items were eliminated and the material on each of the tests was rearranged so that the difficulty would be approximately the same throughout. By using the new tests with the time limit instead of an amount limit, the amount done would represent the subject's speed in the particular function at the low level at which the tests were designed. No item was retained on which more than nine errors had been made by the preliminary subjects.

TABLE I

CONSTRUCTION OF REVISED FORMS OF FOUR SPEED TESTS

	Computation		*Analogies*		*Directions*		*Problems*	
	Form A	Form B	Form A	Form B	Form A	Form B	Form A	Form B
Preliminary subjects	61	62	74	73	73	69	66	67
Fastest time in minutes	4:33	4:31	7:10	5:20	5:40	7:07	7:20	7:15
Slowest time (13 incomplete)	12:30	12:12	14:33	15:00	12:00	14:00	22:00	14:40
Fewest errors for any subject	0	0	0	0	0	0	0	0
Most errors for any subject	25	23	44	33	27	16	18	27
Average errors per subject	7.6	6.3	9.6	9.6	4.3	4.7	3.3	3.7
Fewest errors on any item	0	0	0	0	0	0	0	0
Most errors on any item	12	12	36	28	28	38	17	18
Average errors per item	2.8	2.4	5.5	5.4	3.2	3.2	1.9	2.2
Total items, preliminary forms	164	164	130	130	100	100	112	112
Number of items, revised forms, with:								
0 errors	27	33	19	14	23	32	30	29
1 error	25	25	25	27	20	19	31	32
2 errors	28	29	12	13	17	9	24	23
3 errors	27	21	10	11	9	12	7	7
4 errors	16	23	12	12	8	6	9	10
5 errors	13	12	12	10	7	7	0	0
6 errors	9	5	2	6	3	2	2	2
7 errors	3	4	6	3	3	1	1	1
8 errors	2	0	2	4	0	2	0	0
9 errors	2	0	0	0	0	0	0	0
Total items, revised forms	152	152	100	100	90	90	104	104
Total errors, revised forms	400	354	257	273	187	167	155	158
Average errors per item	2.6	2.3	2.6	2.7	2.1	1.9	1.5	1.5
Items in each cycle	4	4	5	5	5	5	4	4
Average errors per cycle	10.5	9.4	12.9	13.7	10.4	9.3	6.0	6.1
Time limit, revised forms	4:00	4:00	3:30	3:30	4:30	4:30	6:30	6:30

Both preliminary and final forms of the four specially constructed tests had cover pages with eight samples of the material. Time was allowed to do the samples so that the subjects would have a clear idea of what was expected of them. The same cover was used on the two different forms of each test, although the covers differed from preliminary to final forms, as the instructions were different, because of different methods of administration. In general, the sample items were more difficult than those which made up the test proper, that the subjects might be prepared for more rather than less difficulty than was later encountered.

The subjects used in the preliminary trials varied from test to test. All, however, were students in courses in elementary psychology. There were two mixed groups in University Classes, Columbia University, two mixed but predominately male groups in the college department of the John Marshall College of Law, Jersey City, and two classes of women students in Adelphi College, Garden City. Groups used in setting the time limits of the final forms included a class in psychological testing in the graduate school of Columbia University, a class in elementary psychology in Reshiva College, New York, and a group of eight adults of superior background, most of them holding advanced degrees.

The amount of writing involved in indicating the answers in the tests was very small and fairly constant. In the arithmetic computation and arithmetic problems tests, the answers were indicated by writing a number, always of one or two digits. In the analogies and I. E. R. Easy Vocabulary test answers were indicated by drawing a line under the appropriate word. In the directions test, various responses were called for, but all involved a minimum of writing, the most being the writing of three letters.

Four tests were mimeographed, arithmetic computation, analogies, directions and arithmetic problems, three were printed, the I. E. R. Easy Vocabulary test, the Minnesota paper form board test and the Haggerty reading test, and two were multigraphed, the vocabulary level test and the arithmetic reasoning test. When there were two forms of the same test, the texture of the paper was always the same for the two forms. In the final tests the use of pencil was required for all subjects for the tests of the speed battery.

DESCRIPTION OF THE SPEED TESTS

1. The Computation Test

Each of the two preliminary forms of the computation test consisted of 164 simple computation examples such as the following:

(1)	6 times	14	is
(2)	15 minus	4	is
(3)	11 plus	4	is
(4)	48 divided by	8	is

In each form there was an equal number of examples involving the four fundamental operations, addition, subtraction, multiplication, and division.

In each of the addition examples there were two numbers to be added, one of which was not greater than 19 nor less than 7, and the other of which was not greater than 9 nor less than 3. The 41 examples for each form were selected from a table in such a way that, using the size of the numbers as a criterion, the two sets of numbers were approximately equal in difficulty. After the pairs of numbers were selected, they were arranged so that sometimes the greater number would be in first place and sometimes in second place.

The subtraction examples were, in general, the sums obtained in the addition examples less the smaller number. The larger number varied from 12 to 28 and the smaller from 3 to 9, both inclusive. The combinations that appeared on a given form as addition examples appeared again on the same form as subtraction examples, the sum of the numbers appearing as the number from which the subtrahend was to be taken.

The multiplication examples were taken from a multiplication table in which one number varied from 2 to 8 inclusive and the other from 3 to 19 inclusive. Combinations involving the higher numbers of both series did not appear. Sometimes the larger number was given as the multiplicand and sometimes as the multiplier. The selection of numbers from the tables was such that, using size as a criterion, the two sets to be used on the two forms would have approximately the same difficulty.

The division examples on each form were the multiplication examples in division form, the product appearing as the dividend and with the multiplicand or the multiplier, selected through chance, appearing as the divisor.

The preliminary forms of the computation test had 41 cycles of examples, each cycle consisting of an example of each of the four fundamental operations, the order of the operations varying from cycle to cycle.

Table 1 gives the data of the preliminary trials and the number of errors made on each item that was retained. The items were rearranged in cycles, similar to those of the preliminary forms. The order of operations within the cycle was repeated only in different halves of the test and the same operation never occurred in succession. The same number or numbers rarely occurred in successive examples. The items were so arranged that the number of errors made by the preliminary subjects was approximately the same from cycle to cycle.

2. The Analogies Test

The analogies test was made up of material such as the following:

BLACK : WHITE	:: UP	:	(high, down, hill, lift)
COLOR : RED	:: ANIMAL	:	(dog, wild, big, circus)
STATE : NEBRASKA	:: CITY	:	(subway, Los Angeles, rural, metropolitan)
GENERAL : ARMY	:: ADMIRAL	:	(sailor, sea, navy, car)

About half of the 260 analogies used in the preliminary forms were devised by the author. Others were taken from the easier parts of the several forms of the Army Alpha and from other standard intelligence tests. Only those analogies were included that were judged to be quite simple. The material from different sources was arranged in two forms in random order.

About half of the preliminary subjects took the two forms of the test on the same day and the others took the two forms a week apart.

Table I gives information on the results of the preliminary trials and the construction of the new forms. The number of errors in each cycle in the new test was approximately the same throughout. A few of the items were made easier by changing the word in the fourth part of the analogy on which the greatest number of mistakes had been made.

3. The Directions Test

The original forms of the directions test consisted of 100 items such as the following:

1. Put a square around the number that is less than five — 3 5 7 9
2. Draw a line through the letter in the margin that is also in the word CASH — d k c e
3. Underscore the number eight twice — y 8 3
4. Make a word meaning not polite by putting a vowel in the blank: — R.....DE

Material for the directions test was taken from various sources, including the Woodworth-Wells Simple Directions Test, and various intelligence and research tests. A number of original items were added. Some of the material involved the use of numbers, but verbal material predominated.

Results of the preliminary trials, which were from three days to a week apart for the two forms, are tabulated in Table I, together with the number of errors made on items used in the revision. As with the other tests, the cycles or groups of five items had approximately the same difficulty value, using the number of errors made by the preliminary subjects as the criterion.

4. Arithmetic Problems Test

The test of simple arithmetic problems was constructed systematically. Combinations of the eight digits from 2 to 9 were arranged in a table. Twenty-eight multiplication examples were arranged for each form, the only difference being that the order of presentation of the two numbers always varied between the two forms. Sometimes the smaller number appeared first and sometimes the larger number.

The division examples consisted of the multiplication examples arranged backwards. The product became the dividend, and the two numbers from the table became the divisors for either form.

The addition examples consisted of the original 28 combinations of numbers plus 28 new combinations involving numbers as high as 16, but with the other number 9 or less.

The subtraction examples consisted of the sums of the original 28 combinations with one of the original numbers as the subtrahend on one form and the other number as the subtrahend on the other form.

The arithmetic examples were used as the basis of the 112 simple arithmetic problems on each form. These problems varied in length from two to four lines, but most of them were of three lines. They were arranged in cycles of four, each cycle involving the four fundamental operations. Here is one cycle as a sample:

1. In a gymnastic competition, 5 teams, each composed of the same number of boys, took part. There were 40 boys in all taking part. How many boys were there on each team? (Ans. boys)
2. A real estate operator sold 9 houses in a development project of the 11 originally built. How many houses remained to be sold? (Ans. houses)
3. A certain rectangular table is 4 feet wide and 6 feet long. What is its area in square feet? (Ans. sq. ft)
4. A child who had 7 cents was given 5 cents more. How much money did he have then? (Ans. cents)

The preliminary forms were given several days apart. After the elimination of the two most difficult items on each form involving each of the four fundamental operations, the material was rearranged in cycles of four, using the same principles that had been employed in the construction of the computation test. The results of the preliminary trials and the difficulty of the retained items are given in Table I.

5. I. E. R. Easy Vocabulary Test

The I. E. R. Vocabulary Test (Easy Words) is a test devised by the Institute for Educational Research of Teachers' College for use with children in the lower grades. It consists of 100 items, such as the following:

1. large	big	wrong	up	near	black
2. big	red	poor	up	not any	large
3. small	little	down	not now	right	more
4. bad	above	far	not good	long	big

In each line the subjects are instructed to look at the first word and then find the word of the other five that means the same or nearly the same and draw a line under it.

The test was administered to 68 preliminary subjects, allowing all to finish and taking the times necessary for completion. Time scores varied from 3 minutes 25 seconds to 10 minutes. Errors made by each subject varied from 0 to 8, the mean number being 1.3. The number of errors on each items varied from 0 to 8, the average being .87. The errors for each group of ten items were as follows: 6, 6, 1, 3, 1, 11, 12, 11, 9, 27. Applying the criterion of number of errors reveals that the second half of the test is somewhat more difficult than the first half. As the test was found to have fewer errors than the specially constructed tests, it was retained in its original form. As there is only one form of the test, it was administered three times to each subject, the results from the first trial being disregarded in the final computations.

Administration of the Final Tests

The final tests were given in four classroom periods of fifty minutes each in the following order:

First period:	1. I. E. R. Easy vocabulary	(2 minutes 30 seconds)
	2. Arithmetic reasoning test	(Approximately 45 minutes)
Second period:	Vocabulary test	(Approximately 50 minutes)
Third period:	Speed Battery A	
	1. Arithmetic computation. Form A	(4 minutes)
	2. I. E. R. Easy vocabulary	(2 minutes 30 seconds)
	3. Minnesota paper form board. Form A	(10 minutes)
	4. Directions. Form A	(4 minutes 30 seconds)
	5. Arithmetic problems. Form A	(6 minutes 30 seconds)
	6. Analogies. Form A	(3 minutes 30 seconds)
Fourth period:	Speed Battery B	
	1. Analogies. Form B	(3 minutes 30 seconds)
	2. Arithmetic problems. Form B	(6 minutes 30 seconds)
	3. Directions. Form B	(4 minutes 30 seconds)
	4. Minnesota paper form board. Form B	(10 minutes)
	5. I. E. R. Easy vocabulary	(2 minutes 30 seconds)
	6. Arithmetic computation. Form B	(4 minutes)
	7. Test 3 of Haggerty reading examination, Sigma 3. Form A	(6 minutes)

It will be noted that the six repeated tests appear in Battery A in one order and in Battery B in the reverse order. This was done on the assumption that if there were a tendency for fatigue to affect the results, this tendency might be partly counteracted.

All tests were given by the group method, the size of the sections varying from 12 to 35. In each section all tests were given within four weeks. Intervals between first, second and third testing periods varied somewhat, but all final subjects took Battery B exactly one week after Battery A. At Seth Low Junior College the speed batteries were administered by the author with the assistance of Mr. Solomon E. Asch. At Brooklyn City College and at the College of the City of New York, Mr. Asch was the administrator of the speed tests, assisted by the instructors of the sections. At Seth Low the vocabulary and arithmetic reasoning tests were administered by the author, and at the other institutions by the instructors.

TABLE II

Analysis of Subjects

Age		*Birthplace*		*Mother's Birthplace*	
Mean	233.1439 months	New York City	108	Russia	65
S.D.	19.9149 months	U. S. A. (Ex. N. Y. C.)	13	New York City	21
Range	195 to 293 months	Russia	7	Austria	18
Sk	−4.4683	Poland	3	Poland	11
σ_{sk}	2.1745	Austria	3	U. S. A. (Ex. N. Y. C.	7
Sk/σ_{sk}	−2.0549	Palestine	2	Hungary	5
(piling at lower end.)		England	1	Roumania	3
		Not given	1	Palestine	2
				Europe (country not specified)	2
				Latvia	1
				France	1
				Lithuania	1
				Scotland	1

Father's Birthplace		*Foreign Languages in Home (Plural in Some Cases)*		*Father's Education*	
Russia	73	Yiddish	91	College	14
Austria	17	Not specified	10	High school or equivalent	48
Poland	13	Russian	6	Elementary school only	41
New York City	10	Hebrew	5	None or not stated	35
Roumania	6	German	4		
Hungary	5	Hungarian	2		
U. S. A. (Ex. N. Y. C.)	5	Polish	2		
England	2	French	1		
Lithuania	2	Lithuanian	1		
Europe (country not specified)	2				
Latvia	1				
Palestine	1				
Scotland	1				

Father's Occupation

Merchants and shopkeepers	35	Manufacturers	11
Tailors, furriers, clothiers	28	Salesmen and real estate	6
Laborers, mostly skilled	24	Clerical	2
Professional	12	Not given, retired or not living	20

The speed batteries were distributed clipped together, the subjects being instructed to remove the clip before beginning the first test. Timing was by stop-watches and time scores were recorded for subjects who finished any of the speed tests before time was called.

Instructions for the various tests were uniform throughout, with time allowed on the practice pages of the four specially constructed tests of the speed batteries. For the Minnesota paper form board

test, oral directions were given and illustrative samples were drawn on the blackboard, in accordance with the procedure suggested by the authors of the test (17).

The Subjects.

The final tests were given to 233 male white college students at Seth Low Junior College, Brooklyn; Brooklyn City College; and the College of the City of New York. The records of 54 subjects were incomplete, either through absence at one or more of the testing periods or through misunderstanding of directions on some test. With 19 subjects the interval between the administration of Battery A and Battery B was greater than one week. The averages of these subjects on several tests were compared with the averages of the main group of 139 subjects. The differences in averages divided by the sigmas of the differences showed the probability that the results were affected by the variation in interval. Similarly, in certain of the tests the scores attained by the 22 non-Hebrews were found to average considerably lower than the scores of the Hebrew subjects. The records of all 233 subjects were used in determining the errors on the various items of the speed tests, but the records of only 139 were used in computing the intercorrelations of the tests and their reliabilities. These 139 subjects took all the tests, with the regular interval of one week between Battery A and Battery B. All were Hebrews.

A questionnaire answered by all but one of the 139 final subjects showed something of their cultural background. The replies are tabulated in Table II.

The Scoring

All the scoring was done by the author. Each test was scored twice to insure accuracy, an interval of several weeks usually elapsing between the scorings. It was found, however, that few changes were necessary during the re-scoring, as the original scoring had been quite accurate. Scoring was facilitated by the use of keys and stencils.

As in the preliminary tests, omissions were counted as errors. The number of omissions was, however, very small. In all tests except Minnesota paper form board the scoring was objective, each item being either right or wrong. The score entered for each subject on each test was the total number of items performed correctly. For the five speed tests, records were made of the number and type of errors on each item.

Considerable difficulty was encountered in scoring the Minnesota paper form board. The system finally adopted consisted in rating each diagram on a basis of five, allowing proportional credit for diagrams partly completed and taking off varying credit for various types of errors. It was impossible to eliminate subjective judgment in scoring the test, even when careful measurements were made of the deviations of the lines from their correct positions.

In order to compute reliabilities, scores for the arithmetic reasoning test, the vocabulary level test and the reading test were obtained on alternate items. The correlation between the two sets of scores was computed and this was raised by the Spearman-Brown prophecy formula to get the reliability of the whole test. With computation, analogies, directions, problems and Minnesota paper form board, reliabilities were computed by correlating the score on Form A with the score on Form B, and applying the prophecy formula. Similarly, the reliability of the I. E. R. Easy vocabulary test was obtained by correlating the score obtained the second time it was given, in Battery A, with the score obtained the third time, in Battery B. The score obtained the first time it was given, during the first period of testing was disregarded, as it did not seem that conditions were sufficiently controlled to warrant including the results with the other tests. The use of this test during the first testing period served as an introductory test to the various tests of the speed batteries.

There were 12 subjects on the arithmetic computation test, two on the directions test, one on Minnesota and 12 on arithmetic problems who completed one form (Form B in all cases except one) in less than the time allowed. Their scores were increased proportionally to the score that they would probably attain had the test been longer and had they worked at a uniform rate throughout. Through an error on the part of the administrator, the time limit of Form A of the Minnesota test was 10 minutes 30 seconds instead of 10 minutes, for one section of 14 subjects. These scores were proportionally reduced.

Scores used for the intercorrelations were the total scores made on the vocabulary level and arithmetic reasoning tests, and the sums of the scores made on the two forms for all the other tests except I. E. R. Easy vocabulary, the score for which was the sum of the scores made when it was administered as a part of Battery A and when as a part of Battery B.

CHAPTER III

THE RESULTS

1. Level of Difficulty

As it has been described previously, the items of the final forms of the four speed tests that were constructed for this experiment were arranged in cycles, in each of which items were included on which the preliminary subjects had made approximately the same number of errors. In the computation and problems tests, each cycle consisted of four items, involving the four fundamental arithmetical operations. The cycles of the analogies and directions tests were simply groups of five items equated for difficulty. The I. E. R. Easy vocabulary test was arranged in groups of five.

If tests are truly tests of speed, difficulty must be approximately the same throughout. From Table III it is possible to get some idea of the degree to which difficulty was uniform in speed tests. Since practically no subjects completed any of the tests and since the number of subjects who completed any one item varied enormously, the tenth percentile was taken as an arbitrary point up to which the number of errors in each cycle was computed. If a further study had been made, it would have been necessary to have computed the number of subjects who worked the items of any one cycle and the percentage of error. For the purposes of this experiment, the items up to the tenth percentile seem to be sufficient, as they include about half of all the items printed on the tests and perhaps three-quarters of all the items actually attempted by the various subjects.

In making records of difficulty, the results of all subjects were used, including the 94 whose records were not used for the intercorrelations because they were incomplete, or because conditions were not sufficiently uniform, or because of race. The number of subjects for the I. E. R. test is 632 because the same test was given three times.

Table III shows that the number of errors is very low, being lowest on the I. E. R., and fairly consistent on the other four speed tests. In none of the tests is the amount of error greater than four per cent of the items attempted. The test in which difficulty is most consistent is Form B of the problems test, in which the average number of errors in each cycle is 11 with a P. E. of 3.3.

TABLE III

Errors Made on Final Forms of Speed Tests

	Number of Subjects	Average Number of Errors Made by Each Subject	Tenth Percentile of Gross Score	Items in Each Cycle	Total Responses in Each of Early Cycles	Average Correct Responses in Each Cycle to Tenth Percentile. (Decreases Due to Falling Out of Subjects Disregarded.)	Percent Correct Responses	Average Number of Errors in Each Cycle to Tenth Percentile
Computation—								
Form A	219	3.0	78.8	4	876	851.5	97.2	24.5 ± 6.7
Form B	211	4.2	84.8	4	844	813.0	96.3	31.0 ± 8.0
Analogies—								
Form A	220	1.0	42.1	5	1100	1080.8	98.3	19.2 ± 5.5
Form B	211	1.3	45.0	5	1055	1029.9	97.6	25.1 ± 8.3
Directions—								
Form A	220	1.6	36.7	5	1100	1065.0	96.8	35.0 ± 18.0
Form B	210	2.1	49.5	5	1050	1011.8	96.4	38.2 ± 17.5
Problems—								
Form A	219	1.3	48.8	4	876	858.0	97.9	18.0 ± 7.1
Form B	209	1.1	57.3	4	836	825.0	98.7	11.0 ± 3.3
I. E. R.	632	.24	49.4	5	3160	3151.3	99.7	8.7 ± 6.9

Since the four tests were constructed with the same principles throughout, and since difficulty has been shown to be fairly uniform in the first half of the tests, it is highly probable that difficulty is fairly uniform throughout.

In this experiment the reading test and the Minnesota paper form board test conform neither to the standards of a power test nor of a speed test. The short time limits used with both tests, so short that practically no subject had time to complete all the items, make it highly probably that had longer time limits been used the subjects would have changed their relative positions. Therefore they cannot be thought of as tests of difficulty. They do not conform to our standards for speed tests, as the percentage of errors is too high, being 20 per cent in the reading test, 18.2 per cent in Form A of the Minnesota and 24.1 per cent in Form B of Minnesota.

2. Evaluation of the Tests

Means, standard deviations, reliabilities and the degree of skewness of all tests are shown in Table IV. The reliabilities of the arithmetic reasoning, vocabulary level and Haggerty reading tests were computed by correlating the scores on the odd-numbered items against the scores on the even-numbered items, as the percentage of error on these tests was high. For the other tests, scores attained on different forms of the same test, or with the I. E. R., two different trials with the same test, were correlated.

TABLE IV

Means, Standard Deviations, Reliability and Skewness of All Tests. (N = 139)

	Mean	*S.D.*	*Reliability*	*Skewness*	σ_{sk}	$\frac{Skewness}{\sigma_{sk}}$
Age	233.1439	19.9149	1.0000	−4.4683	2.1745	−2.0549
Arithmetic Reasoning	30.1870	7.5777	.9079	1.9908	1.0060	1.9789
Vocabulary Level	107.0000	16.5551	.9165	3.3463	1.8977	1.7633
Minnesota Paper Form Board	285.1223	72.3313	.9180	1.9166	8.6015	.2228
Haggerty Reading Test 3	15.0719	2.7135	.6988	−.1854	.2187	−.8477
Speed Battery:						
Arithmetic Computation	220.8849	43.4069	.9180	−2.2793	5.1351	−.4439
Analogies	117.5396	21.7602	.9294	−.4687	1.4472	−.3239
Directions	112.3022	19.3822	.8985	−1.7338	2.3354	−.7424
Arithmetic Problems	152.2518	29.2285	.9236	−.2641	3.6542	−.0723
I. E. R. Easy Vocabulary	149.9712	18.1129	.9073	.5769	2.0597	.2801

Since results were combined for the intercorrelations, the reliability of the whole was obtained by the Spearman-Brown prophecy formula (10). It will be noted that with the exception of the reading test, all reliabilities are .90 or more.

Skewness was obtained by the formula:

Skewness = Median $-\frac{1}{2}(P_{10}+P_{90})$, and the sigma of the skewness from the formula $\sigma_{sk}=.5185\frac{D}{\sqrt{N}}$, D being the difference between the ninetieth and tenth percentiles (12). When the skewness divided by the sigma of the skewness is greater than three, it is usually taken that the distributions are significantly skewed. From the table it will be seen that the greatest skewness is with

age, arithmetic reasoning and vocabulary level, but that in no case is it significant. With the two level tests, the piling is towards the top of the distribution; with age, it is towards the bottom.

3. Intercorrelations of the Variables

Product-moment coefficients of correlation for nine variables, including age, are given in Table V, the raw coefficients being above the diagonal and the coefficients corrected for attenuation below. The corrections for attenuation range from .0003 to .0720, averaging .0260. For the ten intercorrelations of the "speed" tests, the average correction for attenuation is .0527.

Probable errors for the intercorrelations range from .0310 for the correlation of .6872 between analogies and I. E. R. to .0572 for the correlation of .0031 between age and arithmetic reasoning. All of the correlations with age are well within 4 P. E. and may be regarded as insignificant.

4. Evidence for the Existence of a Speed Factor

(a) *Average Intercorrelations.*

The first evidence for the existence of a factor running through the speed tests but not entering the power tests to any great extent is shown in Table VI. Here the various intercorrelations of the speed tests among themselves and with arithmetic reasoning and vocabulary level are averaged by Fisher's z-scale method. The average of ten intercorrelations among the speed tests is .58, while the average of the correlations of arithmetic reasoning with four speed tests (the fifth correlation is negative) is .18, and the average of the correlations of vocabulary with five speed tests is .27. This fact may be taken as an indication that there is something common to the five speed tests which does not enter into the level tests to the same degree. It will be noted, however, that the speed tests are not entirely independent of the abilities which enter into the level tests. The average correlation between arithmetic reasoning and the two speed tests using numbers, computation and problems, is .10 higher than the correlations between arithmetic reasoning and two tests using mostly verbal material, analogies and directions. On the other hand the correlations between vocabulary level and three verbal speed tests, analogies, directions and I. E. R., are .12 higher than the correlations with the two speed tests using numbers. The intercorrelations of the three speed tests of verbal

TABLE V

INTERCORRELATIONS OF EIGHT TESTS AND AGE, RAW AND CORRECTED COEFFICIENTS. *N = 139

	1	2	3	4	5	6	7	8	9
1. Age	*1.0000*	.0031	-.0046	.0646	.0878	-.1627	-.0219	-.0320	-.0979
2. Arithmetic Reasoning	.0034	*.9079*	.1889	.4455	.2532	.0641	.1968	.2094	-.0647
3. Vocabulary Level	-.0050	.2071	*.9165*	.2690	.0738	.3539	.3474	.3164	.2559
4. Minnesota	.0704	.4880	.2933	*.9180*	-.0226	.2536	.2881	.0358	.1885
5. Arithmetic Computation	.0956	.2773	.0805	-.0246	*.9180*	.3242	.4777	.3561	.4256
6. Analogies	-.1751	.0698	.3835	.2746	.3510	*.9294*	.6187	.5812	.6872
7. Directions	-.0244	.2179	.3828	.3172	.5260	.6768	*.8985*	.6692	.6695
8. Arithmetic Problems	-.0346	.2287	.3439	.0389	.7125	.6057	.7346	*.9336*	.5741
9. I. E. R. Easy Vocabulary	-.1079	-.0713	.2809	.2066	.4663	.7484	.7415	.6272	*.9073*

* Raw coefficients above the diagonal, corrected coefficients below the diagonal.
Reliability coefficients in italics.
The raw coefficients were computed by the Columbia University Statistical Bureau.

material and the correlation of the two speed tests employing numbers are about .15 higher than the intercorrelations of the three verbal speed tests with computation and problems speed. These facts suggest that it may be well to determine the intercorrelations of the various speed tests, with the variability due to factors entering arithmetic reasoning and vocabulary level held constant. Should these factors be of great importance, the intercorrelations among the speed tests ought to drop considerably when the partial correlation technique is employed.

TABLE VI

AVERAGE INTERCORRELATIONS OF SPEED TESTS WITH ARITHMETIC REASONING AND VOCABULARY. (FISHER'S z-SCALE METHOD.) (8)

		Number of Correlations Averaged
Speed tests: Average of 10 intercorrelations	.5764	10
Computation and problems	.6585	1
Analogies, directions and I. E. R.	.6594	3
Analogies, directions and I. E. R. with computation and problems	.5140	6
Arithmetic reasoning with 4 speed tests	.1817	4
with I. E. R.	-.0647	1
with computation and problems	.2313	2
with analogies and directions	.1311	2
Vocabulary level with 5 speed tests	.2721	5
with computation and problems	.1982	2
with analogies, directions and I. E. R.	.3198	3

(b) *Partial Correlation.*

Table VII shows the raw intercorrelations of the various speed tests when the variability due to several factors is held constant by means of partial correlation. In each group of six coefficients, the first one is the obtained coefficient; the one below is the coefficient with age variability held constant; the next, with arithmetic reasoning held constant;[1] the fourth with vocabulary level held constant, the fifth with arithmetic reasoning and vocabulary level held constant simultaneously and the last, the coefficient with age, arithmetic reasoning and vocabulary level held constant at the same time.

[1] It is understood that holding a test "constant" means holding its variability constant.

In no case is the correlation between any two speed tests changed materially when partial correlation is applied, the greatest change being a lowering of the correlation between analogies and problems from .5612 to .5063, or about .06, when vocabulary level is held constant. In fact vocabulary level may be said to be the only variable that affects the intercorrelations to any degree at all, and that very little. In two cases, both involving arithmetic computation, the correlation is raised very slightly with vocabulary level held constant, while in eight cases it is lowered an average of .03. With arithmetic reasoning held constant, the correlation

TABLE VII

INTERCORRELATIONS OF SPEED TESTS WITH VARIABILITY DUE TO AGE AND FACTORS IN ARITHMETIC REASONING AND VOCABULARY LEVEL HELD CONSTANT BY PARTIAL CORRELATION. RAW COEFFICIENTS ABOVE THE DIAGONAL, COEFFICIENTS CORRECTED FOR ATTENUATION BELOW

	Computation	*Analogies*	*Directions*	*Problems*	*I. E. R.*
Computation		.3242	.4777	.6561	.4256
Age constant		.3444	.4816	.6619	.4380
Arithmetic constant		.3190	.4511	.6375	.4578
Vocabulary constant		.3196	.4834	.6688	.4219
Arith. and vocab. constant		.3303	.4674	.6578	.4684
Age, arith. and vocab. constant		.3528	.4716	.6640	.4821
Analogies	.3510		.6187	.5612	.6872
Age constant	.3734		.6236	.5638	.6836
Arithmetic constant	.3460		.6194	.5614	.6942
Vocabulary constant	.3478		.5653	.5063	.6599
Arith. and vocab. constant	.3639		.5715	.5135	.6643
Age, arith. and vocab. constant	.3925		.5764	.5157	.6584
Directions	.5260	.6768		.6692	.6695
Age constant	.5305	.6829		.6690	.6707
Arithmetic constant	.4965	.6795		.6550	.6973
Vocabulary constant	.5378	.6212		.6287	.6405
Arith. and vocab. constant	.5217	.6303		.6202	.6690
Age, arith. and vocab. constant	.5285	.6382		.6199	.6703
Problems	.7125	.6057	.7346		.5741
Age constant	.7190	.6092	.7344		.5717
Arithmetic constant	.6940	.6073	.7207		.6022
Vocabulary constant	.7317	.5464	.6951		.5377
Arith. and vocab. constant	.7225	.5565	.6869		.5682
Age, arith. and vocab. constant	.7307	.5618	.6884		.5680
I. E. R.	.4663	.7484	.7415	.6272	
Age constant	.4803	.7456	.7433	.6275	
Arithmetic constant	.5072	.7572	.7777	.6627	
Vocabulary constant	.4638	.7229	.7151	.5888	
Arith. and vocab. constant	.5245	.7285	.7523	.6276	
Age, arith. and vocab. constant	.5433	.7249	.7542	.6286	

between the two tests involving numbers, computation and problems, is lowered .02, while the correlations between computation and I. E. R. and directions and I. E. R. are raised .03.

The lower part of the table presents the same general data using coefficients corrected for attenuation. The same general set-up obtains and it may be concluded that the variability due to factors that enter into the two power tests cannot explain the intercorrelations of the five speed tests. The explanation would seem to be in some factor or factors running through the speed tests largely independent of factors in the level tests.

(c) *Tetrad Analysis.*

The high average intercorrelations among the speed tests and the fact that these intercorrelations are not greatly changed by partialling out level tests give evidence pointing towards the existence of a speed factor. The next steps are to apply the tetrad difference criterion, and to measure the size and importance of the factor. The intercorrelations of four or more tests may be explained in terms of a general factor running through them all and factors specific to each test when the tetrads arising from them equal zero within their probable errors. It is necessary to find the correlation between each test and the central factor, however, in order to determine the importance of the central factor (1).

TABLE VIII

TETRAD DIFFERENCES OF FIVE SPEED TESTS. RAW COEFFICIENTS ABOVE AND CORRECTED COEFFICIENTS BELOW

1. Computation
2. Analogies
3. Directions
4. Problems
5. I. E. R.

Variables	t_{abcd}	t_{abdc}	t_{acdb}	*Probable Group Factors*
1234	−.0511	−.1890	−.1378	14 or 23
	−.0604	−.2244	−.1639	
1235	−.1112	−.0463	.0650	13 or 25
	−.1334	−.0553	.0781	
1245	−.2648	−.0527	.2120	14 or 25
	−.3131	−.0623	.2498	
1345	−.1650	−.0106	.1545	14 or 35
	−.1984	−.0126	.1858	
2345	−.0205	−.1047	−.0842	25 or 34
	−.0246	−.1253	−.1006	

Table VIII shows the fifteen tetrads arising from the five speed tests, using both raw and corrected coefficients. With only one group of tests, the group without the computation test, is the tetrad criterion even approximately satisfied. If a common factor exists, it is obscured by the presence of group factors. This may account for those tetrads which in the case of the raw coefficients run as high as .26, clearly beyond the limits of 4 P. E.$_t$. The probable group factors are indicated at the right of the table. The combination of computation and problems occurs three times and the combination of analogies and I. E. R. occurs three times. Overlapping factors common to these test combinations may explain the failure of the tetrads to satisfy the criterion.

In order to eliminate, if possible, overlapping group factors, the computation and problem tests were combined with equal weight by the formula

$$r_{(A+B)C} = \frac{r_{AC}+r_{BC}}{\sqrt{2+2r_{AB}}}$$

and the tetrads again computed. The combination of computation and problems was selected in preference to the combination of analogies and I. E. R. because the tetrad differences in which they are involved are the greater.

Table IX shows the intercorrelations of the speed tests, and the tetrad differences with arithmetic computation and arithmetic problems combined with equal weights. Raw and corrected coefficients are given, both the obtained coefficients and those with variability due to age, arithmetic reasoning and vocabulary level held constant by partial correlation.

With the raw coefficients, the tetrads fall within 4 P. E.$_t$, the largest tetrad being .1074 or 3.22 times its P. E. with obtained coefficients and .0748 or 2.34 times its P. E. when variability due to age, arithmetic reasoning and vocabulary level is held constant. Hence no one of the three tetrads is statistically significant. The fact that the tetrad criterion is satisfied, at least according to conventional standards, may be taken as evidence that there is some factor common to the speed tests. However, the fact that two of the three tetrads remain fairly large and approximately equal suggests strongly that there are small group factors in analogies and I. E. R. which we have not been able to eliminate.*

* Kelley (12), Proposition 16, p. 69.

TABLE IX

INTERCORRELATIONS OF SPEED TESTS WITH ARITHMETIC COMPUTATION AND ARITHMETIC PROBLEMS COMBINED WITH EQUAL WEIGHTS. RAW COEFFICIENTS ABOVE THE DIAGONAL, CORRECTED COEFFICIENTS BELOW

	Computation and Problems	*Analogies*	*Directions*	*I.E.R.*
Computation and problems—obtained r's		.4865	.6302	.5493
Age, Arith. & Vocab. variability constant		.4761	.5983	.5756
Analogies—obtained r's	.5169		.6187	.6872
Age, Arith. & Vocab. variability constant	.5129		.5764	.6584
Directions—obtained r's	.6811	.6768		.6695
Age, Arith. & Vocab. variability constant	.6541	.6382		.6703
I. E. R.—obtained r's	.5909	.7484	.7415	
Age, Arith. & Vocab. variability constant	.6299	.7249	.7542	

Tetrads

	t	P.E.$_t$	t/P.E.$_t$
Raw Obtained Coefficients			
t_{1234}	−.1074	.0333	3.22
t_{1243}	−.0141	.0103	1.37
t_{1342}	.0932	.0345	2.70
Raw Coefficients. Age, Arith. & Vocab. variability constant			
t_{1234}	−.0748	.0319	2.34
t_{1243}	−.0127	.0184	.69
t_{1342}	.0621	.0323	1.92
Corrected Obtained Coefficients			
t_{1234}	−.1265		
t_{1243}	−.0166		
t_{1342}	.1098		
Corrected Coefficients. Age, Arith. & Vocab. variability constant			
t_{1234}	−.0873		
t_{1243}	−.0152		
t_{1342}	.0722		

(d) *Triad Analysis.*

In order to eliminate, if possible, the factors common to analogies and I. E. R., which seem to be responsible for the large tetrads in Table IX, analogies and I. E. R. were combined with equal weight. The resulting correlations for raw and corrected coefficients, obtained and with variability due to age, arithmetic reasoning and vocabulary level held constant, are shown in Table XI. All three resulting coefficients are greater than .50 and therefore, according to Brown and Thompson (3), must contain a general

TABLE X

CORRELATIONS OF SPEED TESTS WITH CENTRAL FACTOR. VARIANCE IN TESTS DUE TO THE FACTOR

	Raw Obtained Coefficients	*Raw Coefficients; Age, Arith. & Vocab. Variability Constant*	*Corrected Obtained Coefficients*	*Corrected Coefficients; Age, Arith. & Vocab. Variability Constant*
Computation and Problems	.6822 (46.5%)	.6883 (47.4%)	.6989 (48.8%)	.7106 (50.5%)
Analogies	.7426 (55.1%)	.7226 (52.2%)	.7841 (61.5%)	.7530 (56.7%)
Directions	.8493 (72.1%)	.8173 (66.8%)	.8943 (80.0%)	.8674 (75.2%)
I. E. R.	.8357 (69.8%)	.8567 (73.4%)	.8775 (77.0%)	.9071 (82.3%)

TABLE XI

INTERCORRELATIONS OF SPEED TESTS: COMPUTATION AND PROBLEMS COMBINED, AND ANALOGIES AND I. E. R. COMBINED, BOTH WITH EQUAL WEIGHTS. BROWN AND THOMPSON'S TEST. CORRELATIONS WITH CENTRAL FACTOR. VARIANCE IN TESTS DUE TO CENTRAL FACTOR

	Raw Coefficients		*Corrected Coefficients*	
	Obtained r's	*Age, Arith & Vocab. Variability Held Constant*	*Obtained r's*	*Age, Arith. & Vocab. Variability Held Constant*
Directions with Computation and Problems	.6302	.5983	.6811	.6541
Directions with analogies and I. E. R.	.7013	.6845	.7584	.7497
Computation and Problems with analogies and I. E. R.	.5640	.5774	.5924	.6153
Brown and Thompson's Test	1.7056	1.6328	2.0020	1.9678
Correlations with Central Factor:				
Directions	.8852 (78.4%)	.8422 (70.9%)	.9338 (87.2%)	.8927 (79.7%)
Computation and Problems	.7119 (50.7%)	.7104 (50.5%)	.7294 (53.2%)	.7327 (53.7%)
Analogies and I. E. R.	.7922 (62.8%)	.8128 (66.1%)	.8122 (66.0%)	.8398 (70.5%)

factor. Furthermore, when for three variables $r^2_{ab} + r^2_{ac} + r^2_{bc} + 2r_{ab}r_{ac}r_{bc}$ is significantly greater than unity, there must be a common factor running through the tests. Table XI shows that this criterion is satisfied. When the proper values are substituted in the formula given above, the result is 1.7 with raw obtained coefficients and 1.6 with three values held constant, both numbers being considerably greater than one.

Correlations between each test and the central factor running through all four, computation and problems being combined, are shown in Table X. In Table XI are given the correlations of three variables, computation combined with problems, directions and analogies combined with I. E. R., with their central factor. The lowest correlation in Table X is .68 between computation and problems combined with the central factor and the highest .85 between directions and the central factor. These correlations indicate that our speed tests are "saturated" to a varying but in general high degree by their central factor.

5. FURTHER EVIDENCE FOR THE EXISTENCE OF A SPEED FACTOR

As a check upon the influence of arithmetic reasoning and vocabulary level in causing intercorrelations among the speed tests, the two level tests were each combined with the three speed tests correlating highest with them, and the resulting four tests subjected to tetrad analysis. As shown in Table XII, the tetrad criterion is satisfied in both cases. When the correlations with the central factor are computed, however, it is found that the correlation of arithmetic reasoning with the central factor is .29. Hence the central factor accounts for only 8% of the variance of the level test but for 49 to 72% of the variance of the speed tests. With the vocabulary level test, the separation is not so clear cut, but is decidedly significant. Here tetrads were computed involving the tests having the highest correlations with vocabulary, namely, analogies, directions and problems. The correlations between vocabulary level and the central factor is .43, while the correlations between the speed tests and the central factor ranges from .75 to .83. The central factor accounts for 18% of the variance in the vocabulary test, and from 57 to 59% of the variance of the speed tests. It can be taken as demonstrated that the ability measured by the speed tests varies considerably from the abilities measured by the arithmetic reasoning and vocabulary level tests. It is clear that speed accounts for but little of the variability in the level tests.

TABLE XII

A—Intercorrelations of Arithmetic Reasoning and Speed Tests Having Highest Relationship—Raw Obtained Coefficients

	Computation	*Directions*	*Problems*
Arithmetic Reasoning	.2532	.1968	.2094
Computation		.4777	.6561
Directions			.6692

$t_{1234} = .0403 \pm .0395$ $t/P.E._t = 1.02$
$t_{1243} = .0694 \pm .0292$ $t/P.E._t = 2.38$
$t_{1342} = .0291 \pm .0257$ $t/P.E._t = 1.13$

	Correlation with Central Factor	*% Variance*
Arithmetic Reasoning	.2852	8.1%
Computation	.7864	61.8%
Directions	.8464	71.6%
Problems	.7001	49.0%

B—Intercorrelations of Vocabulary Level and Speed Tests Having Highest Relationship—Raw Obtained Coefficients

	Analogies	*Directions*	*Problems*
Vocabulary Level	.3539	.3474	.3164
Analogies		.6187	.5612
Directions			.6692

$t_{1234} = .0419 \pm .0326$ $t/P.E._t = 1.29$
$t_{1243} = .0411 \pm .0337$ $t/P.E._t = 1.22$
$t_{1342} = .0244 \pm .0244$ $t/P.E._t = .04$

	Correlation with Central Factor	*% Variance*
Vocabulary Level	.4326	18.7%
Analogies	.7688	59.1%
Directions	.8318	69.2%
Problems	.7560	57.2%

Another check upon the existence of a factor central to the speed tests is the correlation between directions and the central factor in each of the four set-ups that have been described. The directions test is the only one that is not combined with another test, when the variables are reduced to three. Using the raw obtained coefficients, the correlation between directions and the factor central to four speed variables is .85, with the factor central to the three variables (Table XI) is .89; with the factor central to arithmetic reasoning and three speed tests (Table XII), .85; and with the factor central to vocabulary level and three speed tests, .83. This result indicates a high degree of stability in the central factor and strengthens our belief in its reality.

These various lines of evidence point with considerable certainty to the existence of a speed factor running through the speed tests, all of which were composed of the type of material commonly found in mental tests, but of a low order of difficulty for the subjects for which they were used.

6. Multiple Correlation Between Central Factor and Various Combinations of Speed Tests

Correlations between the speed tests and the factor central to them when the tests are combined into three variables are shown in Table XI. Using the raw obtained coefficients, multiple correlations have been worked between combinations of tests and the central factor as follows:

1—Directions
2—Computation and Problems
3—Analogies and I. E. R.
o—Central Factor
$R_o(12) = .9081$
$R_o(13) = .9173$
$R_o(23) = .8546$
$R_o(123) = .9300$

From Tables X and XI it may be learned that the directions test is probably the best measure of the central factor, the correlation being in the neighborhood of .85. This correlation is quite consistent, even in different arrangements of tests. When computation and problems, and analogies and I. E. R. are combined with equal weights and then the two pairs combined with the best weights, the resulting correlation with the central factor is also .85. The correlation of the whole battery tests with the central factor is .93, which shows that the present set of tests measures the function to a fairly high degree.

This conclusion is supported by the partial coefficients of correlation among the three variables, with the variability due to the influence of the central factor held constant. There were found to be as follows:

$r_{12.o} = .0001$
$r_{13.o} = .0157$
$r_{23.o} = .0147$

These coefficients seem to be insignificant and may be taken to show that group factors have been largely eliminated from the final

combinations of tests and that the resulting intercorrelations are due chiefly to the presence of a factor central to them.

As a final step a regression equation in terms of reduced scores was computed, using the three final combinations of tests, in order to find the relative weights with which each independent variable enters into the dependent variable (x_o). The equation follows:

$$x_o = .5578\ x_1 + .1968\ x_2 + .2900\ x_3 + K,$$

in which 1 is directions, 2 computation and problems, and 3 is analogies and I. E. R. This equation shows that directions contains our speed factor to a somewhat greater extent than analogies and I. E. R., and considerably greater than computation and problems.

Summary

1. A group of 139 adult subjects, homogeneous as to sex, race and educational status were given five speed tests, two level tests and two tests in which speed and level were not clearly separated.

2. Four of the speed tests were constructed so that difficulty was very slight and approximately constant throughout. The fifth was a test designed for children which on experiment proved to be a speed test for adults.

3. The level tests contained items of much difficulty and were so arranged that longer time limits would not have added appreciably to the score.

4. Evidence for the existence of a factor common to the speed tests but not affecting the level tests to any great extent was found in

(1) the high average intercorrelations of the speed tests compared with the low average correlations between speed tests and level tests.

(2) the fact that the intercorrelations of the speed tests remain practically unchanged when the variability due to age and factors in the two level tests is held constant by partial correlation.

(3) the satisfaction of the tetrad criterion when group factors are largely eliminated by combining two of the speed tests.

(4) the satisfaction of criteria for the presence of a factor central to three variables, after two pairs of the original five speed tests are combined with equal weights.

(5) the high correlations, .68 or better, between the speed tests and the factor central to them, after one and after two pairs of tests are combined.

(6) the stability of the correlation of the directions speed test, the one test which is not put into combinations, with the factor central to four set-ups.

5. The independence of the central factor is shown by the low correlations between it and the level tests.

6. The battery of five speed tests correlates .93 with the factor central to them.

7. The intercorrelations of two combinations of speed tests and the remaining speed test drop to approximately zero when the variability due to the effect of the central factor is held constant by partial correlation.

8. A regression equation indicating the relative weights with which each of the three variables enters into the central factor is given.

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