

# THE FACTORIAL INVARIANCE OF PURE-FACTOR TESTS<sup>1</sup>

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Current practice in assembling batteries of pure-factor tests has been to select, for each factor, one or two tests that have been found by factor analysis to have high loadings on that factor and low loadings on all others. Factorial invariance is assumed. That is, if several tests were to be selected to represent each of a number of factors and tried out with a new group of subjects, it would usually be assumed that the factors would reappear clearly defined by the expected tests. The analysis described in this paper was carried out to check on the invariance of a set of pure-factor tests selected for possible use in making educational predictions.

The tests making up the experimental battery of pure-factor tests were suggested by tests used in the studies covered in a survey of the factor-analysis literature (2). In general, three tests were selected as the purest and most reliable measures of each of thirteen of the best-established aptitude and general achievement factors.

Deviating from this pattern, Reading Comprehension and Mathematics Reasoning were used as measures of two factors each: Verbal Comprehension and General Reasoning for the former, Number and General Reasoning for the latter. Although these two are not pure-factor tests, they were included because of their known validity for a number of purposes. The Carefulness factor represents another deviation from the pattern of three tests for each factor. Since not much confidence was felt in the way that the scores for the Carefulness factor would turn out, five tests were tried. Four are used in the analysis; the other was eliminated, since eighty-five per cent of the subjects received a zero score. It should be pointed out that the Carefulness scores were derived from the tests of other factors. However, it was believed that there would be little distortion of the factors produced by experimental

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<sup>1</sup> Grateful acknowledgment is made to the staff of the U. S. Military Academy for their coöperation in scheduling and administering the tests and in supplying data on course grades. Without this kind of coöperation the study would not have been possible.

dependence, because the four Carefulness scores bore no consistent relationship to the scores on the four tests from which they were derived.

#### THE TESTS

The tests are described in the order in which they were administered. The first four were a part of the entrance tests given five months earlier than the experimental battery. The expected factor content is given, as are the number of items, the time limit, and an estimate of speededness. The word "speeded" is used to indicate that very few or no subjects finished. "Some speeding" indicates that it was estimated that twenty-five per cent to seventy-five per cent finished. Most of the tests in the experimental battery were given in two separately-timed parts in order to make possible the computation of alternate-form reliability coefficients. The coefficients corrected by the Spearman-Brown formula are given with the test descriptions. A full description of the test is not given when the name of the test or the kind of test is familiar to most readers (2).

#### For Verbal Comprehension

##### *Verbal*

- (1) *Antonyms (Entrance)*. 40 items, 15 minutes. 93% finished.
- (2) *Reading Comprehension (Entrance)*. 20 items, 25 minutes. 93% finished. This test was intended to measure both Verbal Comprehension and General Reasoning.
- (3) *Verbal Relations (Entrance)*. Verbal-analogies items and verbal-completion items. 40 items, 30 minutes. 88% finished.

#### For General Reasoning

*Reading Comprehension*. (Described above.)

- (4) *Mathematics Reasoning (Entrance)*. 20 items, 30 minutes. 61% finished. This test was intended to measure both General Reasoning and Number.

Variables 5 to 44 were scores from the experimental battery.

- (5) *Figure Analogies*. Two parts, each 20 items, 10 minutes. Some speeding.  $R = 0.64$ .

#### For Space

- (6) *Similar Figures*. From four squares containing black and white areas the one was to be selected which was not simply

rotated but was the mirror image of the other three. Two parts, each 40 items, three minutes. Speeded.  $R = 0.89$ .

- (7) *Block Counting*. In pictures of large piles of similarly shaped blocks, the subjects were to count the shaded blocks that touched each numbered, unshaded block. The contacts between the blocks had to be inferred from the parts of the blocks shown. Two parts, each of five piles and 50 items, five minutes. Some speeding.  $R = 0.82$ .
- (8) *Revised Minnesota Paper Form Board*. Series MA (Psychological Corporation). One of five patterns was to be selected which could be produced by putting together given outlined parts. 60 items, 25 minutes. Speeded.

#### For Speed of Judgment

- (9) *Visual Preferences*. Each item consisted of a pair of somewhat similar, simple line drawings or figures. The subject checked the one he liked best. The score was the total number checked. No account was taken of the kinds of figures preferred. (For this test and the following two tests, approximately every fifth item presented a very easy choice so that subjects responding at random could be detected. No such subjects were found.) Two parts, each 50 items, two minutes. Speeded.  $R = 0.88$ .
- (10) *Social Judgment*. Each item consisted of pairs of names of personal qualities. The subject checked the quality he would prefer to characterize someone with whom he had to associate closely. The score was the number checked. No account was taken of the kinds of qualities preferred. Two parts, each 70 items, two minutes. Speeded.  $R = 0.92$ .
- (11) *Size Judgment*. Each item consisted of pairs of descriptions of objects. The subject checked the larger (assuming standard or average sizes). The score was the number checked. No account was taken of the correctness of the judgments. Two parts, each 50 items, two minutes. Speeded.  $R = 0.92$ .

#### For Perceptual Speed

- (12) *Picture Discrimination*. Each item consisted of three simple drawings of faces, two exactly alike, and one different in some respect. The subject checked the different one in each item. Two parts, each 56 items, two minutes. Speeded.  $R = 0.89$ .
- (13) *Cancellation*. Each part consisted of a page of typed, ran-

dom, capital letters reproduced in red. The task was to draw an X over every A, starting at the top of the page and working down. The score was the number of A's cancelled. Two parts, each two minutes. Speeded.  $R = 0.96$ .

- (14) *Number Checking*. This test is similar to the Minnesota Clerical Aptitude Test. Two parts, each 48 items, two minutes. Speeded.  $R = 0.89$ .

#### For Number

*Mathematics Reasoning* (Described above.)

- (15) *Checking Addition and Division*. This was an alternation of five items, each of simple addition and division of 1- and 2-digit numbers, with an answer given. The answer was to be checked as correct or wrong. Two parts, each 50 items, two minutes. Speeded.  $R = 0.89$ .
- (16) *Subtraction and Multiplication*. This was simple subtraction and multiplication of 1- and 2-digit numbers in spiral, omnibus form with five response alternatives. Two parts, each 42 items, two minutes. Speeded.  $R = 0.91$ .

For Carefulness (with these variables the signs of the correlations were reversed, so that high scores would represent favorable performance)

- (17) *Picture Discrimination (Wrong)*. This was the wrongs score on Variable 12.  $R = 0.56$ .
- (18) *Checking Addition and Division (Wrong)*. This was the wrongs score on Variable 15.  $R = 0.42$ .
- (19) *Subtraction and Multiplication (Wrong)*. This was the wrongs score on Variable 16.  $R = 0.38$ .
- (20) *Cancellation (Omit)*. This was the number of uncanceled A's prior to the last A cancelled in Variable 13.

#### For Visualization

- (21) *Similar Rotations*. The same solid form is shown in two different rotational positions in each item, labeled 1 and 2 respectively. A new form, labeled 3, was to be visualized as going through the same rotation in one or two planes as that represented by drawings 1 and 2. The resulting position of form 3 was to be selected from drawings A, B, C, D, or E. Two parts, each 20 items, five minutes. Some speeding: 53% finished Part 1; 41% finished Part 2.  $R = 0.69$ .
- (22) *Folded Paper Test*. This test was similar to Thurstone's

Paper Folding Test (2). Two parts, each 20 items, five minutes. Some speeding.  $R = 0.71$ .

- (23) *Mechanical Movements*. Drawings were presented of different kinds of mechanical apparatus involving gears, belts, eccentrics, etc. One or two items based on each drawing asked about the direction, speed, or nature of the movement within the apparatus pictured. Two parts, each 10 items, eight minutes. Some speeding: 25% finished Part 1; 31% finished Part 2.  $R = 0.68$ .

#### For Induction

- (24) *Inducing Rules*. This test presented, in spiral omnibus form, items requiring the subject to discover a rule and then to answer the item by applying it. Included were analogies, series, and belonging items drawn from verbal, social studies, science, numerical, and graphic material. Variety was introduced to prevent the subjects from using a system for solving the problems. Two parts, each 24 items, seven minutes.  $R = 0.72$ .
- (25) *Letter Sets*. The set of four letters not belonging with other sets of four letters was to be chosen. Two parts, each 15 items, five minutes. Considerable speeding: 7% finished Part 1; 20% finished Part 2.  $R = 0.43$ .
- (26) *Locations*. This test was adapted from Thurstone's Marks Test. Each item presented five groups of dashes with spaces between them. An X replaced one dash in each of the first four lines, following some rule for placement. The subject was to indicate where the X should appear in line 5. Two parts, each 24 items, five minutes. Speeded.  $R = 0.72$ .

#### For Fluency of Expression

- (27) *Metaphors*. This was similar to Thurstone's Similes. The subject was required to complete a simile in three different ways. The score was the number of completions regardless of their quality; no entirely senseless ones were made. Two parts, each 10 items, three minutes.  $R = 0.84$ .
- (28) *Double Opposites*. This was similar to Thurstone's test of the same name. Each item presented a word and called for writing two words opposite to it in meaning and beginning with specified letters. The score was the total number of correct responses. Two parts, each 25 items, four minutes.  $R = 0.72$ .

- (29) *Multiple Expressions*. Part I required the subject to write down as many ways as he could of requesting a second helping of meat. Part II required the subject to write down as many ways as he could of asking someone for his name. In each case the score was the number of expressions written down regardless of their quality; no senseless ones were made. Two parts, each four minutes.  $R = 0.82$ .

For Aiming or Eye-hand Coördination

- (30) *Dotting*. The subject placed a dot inside as many typewriter-sized capital O's as he could. The O's were irregularly spaced along crooked lines. The score was the number of O's correctly marked. Very few were incorrectly marked. Two parts, one minute each.  $R = 0.92$ .
- (31) *Tracing-Easy*. A line was to be drawn as rapidly as possible through lines  $\frac{1}{4}$  inch apart having  $\frac{3}{16}$ -inch openings. The score was the number of lines correctly passed. Very few were passed incorrectly. Two parts, one minute each.  $R = 0.92$ .
- (32) *Tracing-Difficult*. A line was to be drawn as rapidly as possible through lines  $\frac{1}{10}$  inch apart having  $\frac{1}{8}$ -inch openings. The score was the number of lines correctly passed. Very few were passed incorrectly. Two parts, one minute each.  $R = 0.94$ .

For Motor Speed

- (33) *Writing X's*. This score was the number of X's drawn on or near regularly spaced dots. Two parts, one minute each.  $R = 0.92$ .
- (34) *Writing "lack."* This score was the number of times the word "lack" could be written (or printed) on or near regularly spaced lines. Two parts, one minute each.  $R = 0.92$ .
- (35) *Writing Digits*. This score was the number of times the digits 1 to 9 could be written on or near given lines. Two parts, one minute each.  $R = 0.91$ .

For Mechanical Knowledge

- (36) *Mechanical Information*. This was a multiple-choice factual information test in practical mechanics. Two parts, each 12 items, five minutes. Power conditions.  $R = 0.74$ .
- (37) *Tool Information*. Each item presented the picture of an object used in some mechanical operation and the pictures

of three tools or other objects. The task was to select the one of the three tools which was commonly used with the first object. Two parts, each 15 items, five minutes. Power conditions.  $R = 0.80$ .

- (38) *Mechanical Comprehension*. This was patterned after the Bennett test. Two parts, each 15 items, five minutes. Power conditions.  $R = 0.64$ .

Variables 39-44 were scores taken from a ninety-item multiple-choice information test designed to measure interest in six areas on the assumption that a subject who has been sufficiently interested in an area to participate in it or read much about it will have picked up factual information about it. The scores are the number of items correct among the 15 (placed in random order) representing each area. Total time, 50 minutes. Unspeeded. The areas covered were:

- (39) *Interest Information (Science)*. Common-sense science.  
(40) *Interest Information (Social)*. Games, dancing.  
(41) *Interest Information (Financial)*. Economics, banking, stock-market.  
(42) *Interest Information (Athletic)*. Baseball, sailing, bowling, etc.  
(43) *Interest Information (Art and Music)*. Painting, great music, technical information.  
(44) *Interest Information (Mechanics)*. Home shop tools, auto mechanics.

#### THE SUBJECTS AND THE ADMINISTRATION

The battery, five and one-half hours long including directions and administrative time, was given to about seven hundred and sixty plebes shortly before the end of their first summer at the United States Military Academy at West Point. With a ninety-minute history examination and a twenty-minute biographical inventory, it was given in two half-day sessions on successive days. In order to include some of the entrance test variables in the study, it was necessary to restrict the group to those who had been "Regular" and "Validating" candidates. This left out the "Certificate" candidates, i.e., those Congressional appointees who are certified by their schools. The number of subjects with scores on all forty-four variables was four hundred and ten.

## THE CRITERION VARIABLES

Five additional variables as follows were obtained when first-year course grades became available in the summer of 1951:

- (45) *Language Grades*. Choice within quotas of French, German, Portuguese, Russian, or Spanish. Fundamentals of grammar, reading, writing, and particularly oral practice.
- (46) *English Grades*. Includes review of fundamentals, composition, reading of literature with class discussion and themes, and speech making.
- (47) *Military Topography and Graphics Grades*. Use of instruments, engineering drawing, freehand and design sketching, map projections, descriptive geometry, and charts and graphs.
- (48) *Mathematics Grades*. Includes algebra, trigonometry, solid geometry, analytic geometry, and an introduction to calculus.
- (49) *Aptitude for Service Ratings*. Ratings on leadership potential. These are made by an officer advisor and by all fellow cadets in the cadet's company including those from all four classes. The officer's rating counts one-third; the cadets' ratings count two-thirds.

The product-moment intercorrelations were computed for the 44 variables on the four hundred and ten cases. For the tests which were divided into two parallel parts, the score used for the intercorrelations was the sum of the part scores.

For all correlations with the criterion variables the number of subjects was three hundred and sixty-one. This reduction was caused by normal attrition that took place during the first year at the Academy. The intercorrelations and the means and standard deviations may be obtained from ADI (8). Attrition at the Academy during the first year tended to raise the means and decrease the standard deviations a little, as might be expected. For tests given in two parts the alternate-form reliability coefficients corrected by the Spearman-Brown formula are given in the diagonal of the table.

## THE FACTOR ANALYSIS

Factor analysis of the 49-variable intercorrelation matrix was carried out by means of Thurstone's Grouping Method (5). The



highest value in each column was used as an estimate of the communality.<sup>2</sup>

Sixteen factors were extracted before the residuals seemed to be reduced to near the chance level. The groups of variables used for the extraction of the sixteen reference factors may be obtained from ADI (8).

After the sixteen factors had been extracted, no pattern that seemed significant could be found among the residuals. The largest value among the residuals was 0.079. The distribution of the residuals and the unrotated factor matrix may also be obtained from ADI (8).

#### THE ROTATION

Rotation to oblique simple structure was carried out by the method described by Thurstone (5, p. 194), where all pairs of factors are plotted on paper and the new positions for the hyperplanes are drawn on the paper, so that the direction cosines for the new rotation may be read from the graph. Seven sets of rotations were carried out, including a total of one hundred and twenty rotated pairs. A new set of graphs was assembled after each set of rotations. The final transformation matrix may be obtained from ADI (8). The final oblique solution is presented in Table I. Loadings of 0.20 or higher are printed in parenthesis. Communalities derived from the unrotated matrix are given in the right-hand column. Table II shows the intercorrelations of the factors.

In the process of rotation it was borne in mind that the infinite number of rotational solutions simply represent alternate ways of explaining the intercorrelations that were found. No one solution is more mathematically correct than another. While simple structure was the general goal, this concept is so variously or vaguely defined that there is much freedom for variation within its limits. Rotation in this study was not done blind, but was carried out so that the result would be of maximum value to the broad purpose

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<sup>2</sup> After the extraction of the ninth factor, the values used for the communalities were the highest values in the columns of the residual table. Up to the extraction of the ninth factor the highest values in the columns of the original table were inserted in the diagonals of the residual tables. Since no groups had overlapped up to this point, it made little difference whether the communalities of members of the groups were estimated from the original or the residual tables.

TABLE I—ROTATED FACTOR MATRIX AND COMMUNALITIES  
(Decimal points omitted. Loadings of 0.20 or higher are printed in parenthesis.)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	h <sup>2</sup>
1 Antonyms (entrance)	(66)	-.03	-.04	-.02	.09	.07	-.04	-.10	.08	-.01	-.01	-.02	-.02	-.03	.04	.19	.79
2 Reading comprehension (entrance)	(43)	.05	.06	-.01	-.09	-.07	.10	.06	-.12	-.02	-.03	-.01	.04	.08	-.05	(-.23)	.68
3 Verbal relations (entrance)	(55)	-.03	-.03	.04	.00	.01	-.07	.04	.04	.03	.05	.03	-.02	-.05	.02	.07	.76
4 Mathematical reasoning (entrance)	.04	.04	-.08	-.03	-.03	.01	.05	.02	.05	-.10	.09	(.32)	-.04	.02	.00	.07	.54
5 Figure analogies	.02	.04	-.01	-.04	.09	.06	-.05	.14	.15	.01	-.04	.18	.06	.12	-.07	-.01	.52
6 Similar figures	.01	-.05	-.01	.06	-.09	-.03	-.02	(.23)	.00	.03	.00	-.06	.00	(.30)	.04	.06	.45
7 Block counting	-.10	-.08	.11	.01	-.01	-.00	-.00	(.28)	-.11	-.07	.08	.03	.11	(.22)	-.03	.07	.54
8 Minnesota paper form board	-.10	.04	.18	.04	-.01	-.03	.14	(.32)	.00	-.01	-.00	-.12	-.03	(.24)	-.05	-.17	.57
9 Visual preferences	-.12	-.07	-.01	-.02	(.59)	.04	.05	.03	.06	.04	-.10	-.01	-.01	.04	.01	-.06	.66
10 Social judgment	-.01	-.02	-.03	.02	(.65)	-.01	-.00	-.02	.02	-.04	-.00	-.07	.04	.02	-.00	.05	.75
11 Size judgment	.04	.07	.01	-.01	(.48)	-.03	-.05	-.02	-.05	.00	.18	.07	-.03	-.05	-.01	.00	.72
12 Picture discrimination	.06	.19	.01	.02	.19	.02	-.05	.13	.05	.05	-.01	-.07	-.03	.16	-.01	.04	.65
13 Cancellation	.03	(.36)	.02	.00	-.02	-.02	.04	-.03	-.18	-.06	.05	.05	.01	.01	-.05	.05	.49
14 Number checking	-.03	(.32)	-.03	-.02	.03	.01	-.03	.05	-.01	.04	-.01	-.04	-.06	.03	.04	-.07	.64
15 Checking addition and division	.02	-.01	-.01	-.02	.01	.03	.01	.00	-.06	.01	-.01	-.01	-.02	-.01	(.43)	.04	.85
16 Subtraction and multiplication	-.00	.02	-.02	.03	-.02	-.04	-.02	-.01	.03	-.01	-.02	-.00	.02	.02	(.42)	-.05	.84
17 Picture discrimination (wrong)*	-.04	.02	.04	-.03	-.05	-.12	.01	.05	(.37)	.07	.01	-.03	-.01	.01	-.04	(.26)	.41
18 Checking addition and division (wrong)*	.03	.05	.02	-.04	-.01	.03	.03	-.08	(.46)	-.01	-.05	.06	-.01	-.04	-.04	-.04	.37
19 Subtraction and multiplication (wrong)*	-.03	-.03	-.04	-.03	.05	.07	-.03	-.04	(.43)	-.05	.05	.01	-.08	.04	.03	(-.22)	.37
20 Cancellation (omit)*	-.08	-.09	.01	.09	.01	-.06	.04	-.01	(.31)	-.03	.01	-.00	.12	-.01	-.01	.07	.23
21 Similar rotations	-.06	.08	.02	-.06	.10	-.01	.02	-.04	.15	-.05	.05	-.01	.02	(.31)	.01	-.04	.40
22 Folded paper test	-.04	.01	.07	-.07	.02	.10	.03	.06	.16	.01	-.06	.11	-.05	(.27)	-.01	-.04	.53
23 Mechanical movements	.02	.08	.11	-.00	.03	-.02	-.03	-.09	-.01	-.09	.10	.13	.01	(.28)	-.03	.13	.60

24 Inducing rules	05	-02	-07	-03	05	03	-04	(30)	03	11	(20)	06	-03	01	-06	06	64
25 Letter sets	-03	08	06	-07	04	-02	07	(29)	-02	13	-06	08	09	-07	-03	-03	40
26 Locations	03	05	-08	-00	06	07	-03	(30)	-04	-01	-06	(21)	-06	01	-06	03	49
27 Metaphors	-11	-06	02	02	-02	00	-02	-02	-05	(56)	04	-00	01	-00	05	02	54
28 Double opposites	(25)	07	-01	-01	06	-01	-02	-09	06	(28)	01	-02	02	00	03	03	36
29 Multiple expressions	08	03	-01	-00	02	02	05	04	01	(51)	-07	03	-04	-00	-05	-05	47
30 Dotting	-06	-04	-00	09	01	(41)	02	02	03	02	03	-15	05	10	04	-01	58
31 Tracing-easy	01	07	-08	02	-02	(56)	-08	01	-04	-03	05	14	-10	-09	-06	08	70
32 Tracing-difficult	-05	-05	08	-11	-02	(60)	06	-03	02	01	-08	01	05	-02	01	-06	61
33 Writing X's	13	02	04	(47)	-04	05	-03	-03	-07	-04	-01	-01	00	-02	04	18	51
34 Writing "lack"	01	04	-02	(60)	05	01	03	-07	08	02	-01	04	-00	-01	-07	-16	68
35 Writing digits	-06	-07	-01	(68)	-01	-06	01	10	-01	03	01	-03	01	03	02	-05	71
36 Mechanical information	06	-02	(44)	-02	-01	00	-01	-09	11	-04	(29)	-02	-05	-01	03	02	73
37 Tool information	-01	-01	(62)	01	05	-05	10	02	-01	-03	02	01	11	03	-01	01	75
38 Mechanical comprehension	06	-11	(39)	-03	-00	07	-01	06	07	08	-03	-09	02	(23)	10	-03	57
39 Interest information (science)	-04	03	02	-04	-05	02	00	-05	-01	-02	(48)	07	-04	-02	-09	01	50
40 Interest information (social)	06	-02	03	-01	01	02	-04	-07	-01	05	-04	-03	(48)	10	02	05	49
41 Interest information (financial)	08	-14	09	02	02	-07	-03	-01	-01	05	15	01	05	09	10	00	29
42 Interest information (athletic)	-06	01	-05	05	04	-03	06	09	-03	-02	04	08	(29)	-11	-07	-04	27
43 Interest information (art and music)	14	01	02	-05	-00	04	-07	04	09	-03	(25)	-18	11	03	01	-04	37
44 Interest information (mechanics)	-09	01	(43)	05	-02	-05	-02	01	02	03	(30)	05	01	-08	-09	01	63
45 Language grades	05	-04	-08	01	-02	01	(51)	04	-02	-03	-01	-03	03	-02	01	06	60
46 English grades	(28)	12	-03	-04	06	-02	(37)	02	04	06	-01	18	-04	-15	-10	02	69
47 Military topography and graphics grades	-12	-02	(24)	05	01	04	(25)	12	01	04	06	11	-05	(22)	-03	-07	74
48 Mathematics grades	-06	-01	02	01	-01	02	(45)	05	-04	-03	-03	(30)	07	04	-02	-05	71
49 Aptitude for service ratings	-06	05	(26)	-04	-00	-06	(38)	-06	-09	06	-17	04	(27)	-01	-02	-02	24

\* For these variables the signs of the correlations have been reversed, so that these variables represent Carefulness.

TABLE II—INTERCORRELATIONS OF THE FACTORS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		-.02	.05	.01	.01	.05	.27	.35	.13	.24	.60	.46	.31	.39	.04	-.20
2	-.02		.03	.42	.38	.55	.26	.58	.16	.18	.27	.11	.33	.23	.74	.05
3	.05	.03		.05	.17	.18	-.23	.06	-.10	-.03	.24	.19	-.22	.37	-.11	.14
4	.01	.42	.05		.35	.54	.18	.13	-.07	.30	.18	.07	.11	.06	.36	-.01
5	.01	.38	.17	.35		.34	.13	.25	-.32	.49	.27	.07	.20	.08	.41	.03
6	.05	.55	.18	.54	.34		.18	.36	-.11	.19	.24	.07	.24	.27	.34	.07
7	.27	.26	-.23	.18	.13	.18		.23	.09	.15	.33	.08	-.02	.09	.34	.14
8	.35	.58	.06	.13	.25	.36	.23		.37	.16	.47	.46	.43	.48	.48	.01
9	.13	.16	-.10	-.07	-.32	-.11	.09	.37		-.28	.12	.25	.26	.15	.18	-.04
10	.24	.18	-.03	.30	.49	.19	.15	.16	-.28		.31	.13	.26	.07	.24	-.09
11	.60	.27	.24	.18	.27	.24	.33	.47	.12	.31		.51	.38	.43	.41	-.15
12	.46	.11	.19	.07	.07	.07	.08	.46	.25	.13	.51		.30	.58	.36	-.20
13	.31	.33	-.22	.11	.20	.24	-.02	.43	.26	.26	.38	.30		.08	.41	-.18
14	.39	.23	.37	.06	.08	.27	.09	.48	.15	.07	.43	.58	.08		.08	.16
15	.04	.74	-.11	.36	.41	.34	.34	.48	.18	.24	.41	.36	.41	.08		-.11
16	-.20	.05	.14	-.01	.03	.07	.14	.01	-.04	-.09	-.15	-.20	-.18	.16	-.11	

of the research, that of identifying a useful set of pure-factor tests by proving that an expected factor structure would appear. The criterion of simple structure and a positive manifold of loadings was responsible for a great majority of the decisions made in the rotation, but two examples may be mentioned to show the way in which other conditions played a part.

(1) This study does not reveal Space, Induction, and Visualization as three separate factors. Rotation could have been such that any two of these three factors would appear as primaries with the third being a combination of the other two. The final rotation was made to show the spatial tests as loading both Induction and Visualization, because this situation had the readiest psychological interpretation and, therefore, served as the best framework for the development and understanding of pure-factor tests.

(2) The second group of variables used in the extraction of factors consisted of the three "perceptual speed" tests and the two "number" tests. The first two rotations left the fifteenth reference factor as a residual, while the second was distinctly divided into two clusters, the perceptual and the number tests. The hyperplanes were then rotated so as to pass through these two clusters. Thus, Factor 2 became perceptual only, and Factor 15, which was a residual factor, became the Number factor. Although the two new factors correlate 0.74, the clear division between them provides convincing evidence that factorial invariance has occurred. These two

factors can, therefore, be relied upon to appear and may well be useful dimensions in a testing program.

#### INTERPRETATION OF THE FACTORS

The factors are named to indicate their identification with factors found by other investigators. Most of them are now included in the *Kit of Selected Tests for Reference Aptitude and Achievement Factors* (4), which was developed by committees of investigators for the purpose of defining some of the better established factors. The selection of some tests for the *Kit* was based partly on the findings of this analysis.

Following each factor name there appears the "Universal Index Number" of the factor from a numbering system for factors which was initiated by Cattell (1).

Because this study was undertaken to confirm the establishment of familiar factors, which have been described elsewhere (2 and 3 and original sources referred to therein), the factor descriptions and the discussions are brief.

Factor I is *Verbal Comprehension*, Universal Index No. T13: Knowledge of the English language, including the skill necessary both to read and to write it.

Factor II is *Speed of Symbol Discrimination*, Universal Index No. T12: The ability to find a known symbol amidst a mass of distracting stimuli. Tests 12, 13, and 14 were included to define the "perceptual speed" factor. However, "perceptual speed," thought by the writer at the time of testing to be a single entity, actually consists of at least two factors, one of which is speed of symbol discrimination. Another "perceptual speed" factor calls for speed in perceiving detail. Variable 12 is a test of this other factor. Its low loading seems to confirm that fact.

Factor III is *Mechanical Knowledge*, Universal Index No. T8: A knowledge stemming from instruction or experience in the field of mechanics. This factor is unique in this battery of so-called aptitude factors, since it depends on variance in achievement. It was included in this aptitude battery because mechanical knowledge plays such an important part in our culture that it is likely to be of consequence in predicting success in various academic or vocational fields.

Factor IV is *Motor Speed*, Universal Index No. T9: Speed in

moving the fingers in complex patterns that are well fixed by habit. This may be the same as the U. S. Employment Service's Finger Dexterity factor (2), and seems certainly to be the same as Bechtoldt's "Motor Speed" factor (2), and Woodrow's "Speed" factor (7). It is not known to what extent this factor may be generalized beyond tasks carried out with the hand holding a pencil.

Factor V is *Speed of Judgment* (not indexed): Speed or lack of compulsiveness in making minor decisions. The large individual differences on tests of this sort are probably affected rather strongly by the situation and the test directions. However, it seems reasonable to assume that a part of this variance represents a mode of response habitual to the individual.

Factor VI is *Aiming* or eye-hand coördination, Universal Index No. T1: The three variables having high loadings on Factor VI in this study are all performed with pencil and paper. These tests occur in several studies in the literature without evidence that the factor encompasses more than the paper and pencil situation (2).

Factor VII is the *Grades* factor (not indexed): The variance that is common to course grades but not to any tests. This variance probably covers a wide variety of characteristics including such things as classroom attitude and parental pressure.

Factor VIII is a combination factor of *Induction* and *Space*, Universal Index Nos. T5 and T11: *Induction* involves the findings of general concepts that fit sets of data, the forming and trying out of hypotheses. *Space* is the ability to comprehend the nature of the arrangement of elements within a visual stimulus pattern primarily with reference to the location of the examinee. The zero hyperplane of this factor could have been rotated to a different position with respect to Factor XIV, but the present rotation is the one which is most meaningful. The *Space* factor appears between *Induction* and *Visualization* and lies in the plane determined by these two. This makes good psychological sense, as suggested by the term "Spatial Reasoning" so often used instead of "Space." The three variables (6, 7, and 8) put into this battery to represent the "Space" factor are shown here to be a combination of a reasoning factor and the factor called *Visualization*.

Factor IX is *Carefulness* (not indexed): The ability or disposition to proceed cautiously on speed tests. In all of these tests the nature of the task itself is easy, but the directions call for work at top speed. The table of intercorrelations indicates that there was

no consistent correlation between the carefulness scores and the speed scores derived from the same tests. Because this is true, there can be no serious distortion of the factor pattern resulting from use in the analysis of two scores derived from each of four tests.

Factor X is *Fluency of Expression* or possibly *Idea Fluency*. (The latter would be Universal Index No. T6): The three tests were placed in the battery to define *Fluency of Expression*, the ability, once an idea is given, to express that idea rapidly in many different ways. However, the distinction between new ideas and new expressions of an idea is not clear and is not established here, because no recognized test of *Idea Fluency* was included in the battery. Such a test, for example, might call for writing down as many ideas as possible about some such topic as "a house." The score on this kind of "Idea Fluency" test would be the number of ideas written down within a given short time limit.

Factor XI appears to be *Interest in Science* (not indexed) or an information acquisition factor. Even if it is considered an interest factor, it is not known whether it, defined as it is by tests of *information* about science, is the same as that found in numerous other studies loaded by questions asking about *preference* for science. The Inducing Rules test includes questions involving a knowledge of general science, and so its loading is reasonable, but there is no explanation for the loading of Interest in Art.

Factor XII is probably *General Reasoning*, Universal Index No. T34: A factor which has been associated with arithmetic reasoning tests and a rather varied list of other reasoning measurements. Figure Analogies was included in the battery to help define this factor. Its loading of 0.18 is lower than expected, but it is the highest loading for Figure Analogies on any of the factors.

Factor XIII is an interest or personality factor. It could be called *Interest in People* or it may be similar to the *Sociability* factor found in personality inventory analyses. Factors having both of these interpretations have been found in many studies (3).

Factor XIV is a combination of *Visualization* and *Space*, Universal Index Nos. T14 and T11: As noted for Factor VIII, the inference here is that the tests used for the "Space" factor actually require the abilities represented by both *Visualization* and *Induction*. *Visualization* may be defined as the ability to comprehend objects in space so as to manipulate them in the imagination.

Factor XV is the *Number* factor, Universal Index No. T10: Speed in carrying out simple numerical computations involving the four arithmetical processes. Mathematics reasoning tests ordinarily have a loading on this factor as well as on *General Reasoning*. In this analysis the mathematics variables do not load on *Number*, because *Number* and *General Reasoning* were rotated to a correlation of 0.36 with each other.

Factor XVI has no apparent psychological meaning. It can best be considered a residual factor.

#### CONCLUSION

Tests of each of thirteen well-established factors and some tests of general information were administered to an entering class at West Point. Together with scores from the entrance examinations and first-year grades, these were factor-analyzed to confirm the identification of useful pure-factor tests by finding factorial invariance when such tests are factored.

All thirteen of the sets of tests used in the battery for the designation of reference factors did in fact turn out to fall into clusters within the factor space. Twelve out of thirteen of the clusters defined factors as expected. Those three variables intended to represent *Space* did not load a separate factor for themselves but did form a thirteenth cluster located in the plane defined by *Induction* and *Visualization*. The *General Reasoning* factor is included among the twelve that were separated successfully. However, the Mathematics Test (No. 4) and Figure Analogies (No. 5) do not define the factor well by themselves. The factor does become adequately clear, however, because of the presence of Mathematics Grades (No. 48).

The other three of the fifteen non-residual factors make good psychological sense: (1) The *Grades* factor has often appeared in studies where several course grades and tests were included. (2) The *Interest in Science* factor is one that has been found several times in the literature (3). Leadership ratings and amount of information on social activities combine to define a factor. The implication here is that the leadership ratings are heavily weighted with sociability.

From the standpoint of the study's purpose, it may be concluded that factorial invariance held up, and, therefore, the selection of tests to represent certain factors was successful. The clusters are



sufficiently tight to make it reasonable to represent factors in a battery by one test from each set. Thus, the study served as a pre-test yielding factorial, reliability, and level-of-difficulty data upon which to base the selection of tests.

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