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AN INVESTIGATION OF TWO HYPOTHESES REGARDING THE NATURE OF THE SPATIAL-RELATIONS AND VISUALIZATION FACTORS

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Primarily as a consequence of the factorial analyses of tests of interest to the construct of a spatial- and/or visual ability amenable to psychological measurement has received increasing attention in recent years. During the past twenty-one years, at least a score of investigators have included in their writings a space factor. In a pioneer study, Thurstone (16) included among his seven primary mental abilities a factor labelled S, which he characterized as a "facility in spatial and visual imagery,"—a factor which he likened to the spatial or visual group factor found by Kelley (15) in earlier experiments. The same factor was identified in other studies carried out subsequently by Thurstone (17) and by Thurstone and Thurstone (18).

During World War II members of the psychological research units of the Army Air Forces devoted a considerable amount of time and effort to the development of tests of the "spatial-visual" type to be used in the selection of men for air-crew positions. Several factorial studies which have been described in a research report of the AAF Aviation Psychology Research Program, edited by Guilford (3), have indicated that the variety of intellectual abilities, the construct of a spatial and/or visual factor found by Kelley (15) in earlier experiments is included as one of the factors which he described as being spatial-relations and visualization. In fact, in addition to these two factors (abbreviated by the symbols S, and V), two other less definite space factors, S, and S, and a factor tentatively identified as visual memory, also appeared in several analyses.

In two recent studies both Fruchter (2) and Dudek (1) have found separate factors of spatial relations and visualization. In his investigation as to the nature of verbal fluency Fruchter reanalyzed a sub-matrix of twenty tests selected from the battery of fifty-seven variables employed by Thurstone in his classical study previously cited (16). He found two apparently independent factors which he described as being spatial-relations and visualization.

Referring to the same Thurstone study, Zimmerman (3) pointed out that further rotations of the residual axis (Number XII) with other axes which defined meaningful factors would produce a promising factor of visualization. Just recently, Zimmerman (in his unpublished doctoral dissertation) has reanalyzed the twelve centroid axes for all fifty-seven variables and has confirmed his initial belief that both a spatial-relations and a visualization factor would appear.

**Problem**

The purpose of the investigation was to test the validity of two (apparently unrelated) hypotheses that purport to represent differences in the psychological properties of the factors of spatial-relations and visualization. The authors believe that both a spatial-relations and a visualization factor will appear.

SPATIAL-RELATIONS AND VISUALIZATION FACTORS

A sufficient, though not necessary, condition for the tenability of both of the hypotheses would be that in the factor-analysis procedure each of the two groups of three tests would define a factor. Moreover, this factor should not appear to be weighted in other tests of the battery that were selected to measure other factors. If one or more tests within either group should be weighted substantially in variance associated with another factor, the evidence for the corresponding hypothesis would be less clear-cut, but not necessarily lacking. It would be quite possible, if not almost certain, that one or more of the tests within a given group might be factorially complex. At the same time, however, all three tests within a given group might contain substantial amounts of variance in one factor that did not appear in any of the other eleven tests.

**Hypotheses**

The factor of spatial relations was hypothesized to represent the ability to comprehend the arrangement of elements within a visual stimulus pattern, primarily with reference to the human body. Thus, an important implication in the ability to perceive spatial arrangements is that the subject is able to distinguish whether one object is higher or lower, left or right, or nearer or farther than another within the same field. Through the presentation of two simulated views of a stimulus pattern, a test item may be constructed such that there is a systematic relationship between the order of elements within the first spatial pattern (the stimulus component of a test item) and the order of elements within the second pattern (the response component of a test item).

For example, in Thurstone's Cubes test, the examinee is asked to recognize that the stimulus-cubes and cube may hold the same relationship to one another as they do on the first cube. By noticing within each cube the left-right, top-bottom, and front-back interrelationships of the faces, the subject is able in each item to refer the locations of the three exposed faces of one cube to the locations of designs on the faces of the other cube. In Thurstone's Flanges test, the examinee is required to determine the relative amount and direction of movement of the boat corresponding to changes in the two movement boundaries (outlines) of the two flags appears to be an important aspect of the solution. Similarly, in Guilford and Zimmerman's test of Spatial Orientation a premium is placed upon the examinee's maintaining the correct relationship of objects to one another in background scenery that has been viewed twice from a more or less static-first before and then second after—which has moved up or down and/or left or right. In the test the examinee is asked to determine the relative amount and direction of movement of the boat corresponding to changes in the two views of its background setting.

The factor of visualization was hypothesized to represent an ability that requires the mental manipulation of visual images. In contrast to another factor identified as visual memory (3), which appears to be a static or reproductive form of visualization, the factor referred to as visual manipulation to be an essentially visual, dynamic form. This visual manipulative ability appears to be present in the solution of problems in which the in-
Spatial-relations and visualization factors

The spatial-relations factor was considered to demand a fairly rapid decision as to the spatial position of objects with reference to his own location; whereas, the visualization factor was believed to be represented in problems requiring a more deliberate and less automatic approach. In part, such a distinction may be a function of the complexity of the task (i.e., the number of steps entering into the performance of an item), the more complex tasks requiring visualization for their solution.

Concerning the psychological properties of spatial-relations and visualization factors, one other important difference has been suggested by one of the psychological research units of the AAF, as follows:

The idea for Flight Orientation [a test] was proposed at the time Aerial Orientation (another test) was being developed. It was hypothesized (4) that the ability visually to maneuver an airplane as if from a position outside the cockpit is a manipulatory-visualization ability and (2) that the ability to imagine maneuvers taking place as if the examinees were within the cockpit is a spatial-orientation ability.

The Aerial Orientation test utilized cockpit views of outside terrain to be matched with depicted plane attitudes; the visualization-of-maneuvers tests involved only views of airplanes seen from a position outside of the cockpit. Flight Orientation was designed to fulfill the requirements of the indicated variation—a test that would utilize only cockpit views of outside terrain. From hypotheses given above, it follows that Aerial Orientation should measure a combination of manipulatory-visualization and spatial-orientation abilities, while Flight Orientation should be a purer measure of the ability to orient in space (3).

That the two groups of tests selected for investigating the validity of the hypotheses may actually contain variance in both the spatial-relations and visualization factors will not be surprising, inasmuch as many subjects on the basis of their own introspective reports revealed that they made use of the two psychological processes associated with the respective hypotheses in tests selected to represent the implications of only one hypothesis. For example, if in the Flags test the subject is able, so to speak, to pick up the flag, move it, turn it about as if he actually has a model in his hands, then visualization is believed to be dominant. On the other hand, if the subject is

Table 1

<table>
<thead>
<tr>
<th>Test Battery: Description Data</th>
<th>Name of Test</th>
<th>Number of Items</th>
<th>Time of Test</th>
<th>Size of Paper</th>
<th>Working Distance</th>
<th>Scoring Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Guilford-Zimmerman Verbal Comprehension</td>
<td>Guilford-Zimmerman General Reasoning</td>
<td>30</td>
<td>10 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2. Guilford-Zimmerman Numerical Reasoning</td>
<td>Guilford-Zimmerman Perceptual Speed</td>
<td>15</td>
<td>5 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3. Guilford-Zimmerman Spatial Orientation</td>
<td>15</td>
<td>4 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Thurstone (Verbal) Comprehension</td>
<td>Thurstone Series Number</td>
<td>30</td>
<td>5 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5. Thurstone (Verbal) Comprehension</td>
<td>15</td>
<td>3 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Thurstone (Verbal) Comprehension</td>
<td>20</td>
<td>15 sec</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Thurstone (Verbal) Comprehension</td>
<td>30</td>
<td>4 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Thurstone (Verbal) Comprehension</td>
<td>45</td>
<td>6 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Thurstone (Verbal) Comprehension</td>
<td>60</td>
<td>10 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Thurstone (Verbal) Comprehension</td>
<td>75</td>
<td>15 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Thurstone (Verbal) Comprehension</td>
<td>90</td>
<td>20 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. Thurstone (Verbal) Comprehension</td>
<td>105</td>
<td>30 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Thurstone (Verbal) Comprehension</td>
<td>120</td>
<td>45 min</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. Thurstone (Verbal) Comprehension</td>
<td>135</td>
<td>1 hour</td>
<td>R/W</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The scores among five words, all matched with respect to difficulty, the one word which most closely approximates the meaning of the stimulus word. Items increase in difficulty progressively from the beginning to the end. Even numbered items were omitted. Responses were recorded on a separate answer sheet. Most examinees attempted all.

Each Guilford-Zimmerman General Reasoning—This test is composed of a combination of the stimulus word. Items increase in difficulty progressively from the beginning to the end. Even numbered items were omitted. Responses were recorded on a separate answer sheet. Most examinees attempted all.

Each Guilford-Zimmerman General Reasoning—This test is composed of a combination of arithmetical-reasoning problems similar to those encountered in courses in general mathematics, elementary algebra, and intermediate algebra. Diagrams accompany a few of

choose among five words, all matched with respect to difficulty, the one word which most closely approximates the meaning of the stimulus word. Items increase in difficulty progressively from the beginning to the end. Even numbered items were omitted. Responses were recorded on a separate answer sheet. Most examinees attempted all.

Each Guilford-Zimmerman General Reasoning—This test is composed of a combination of arithmetical-reasoning problems similar to those encountered in courses in general mathematics, elementary algebra, and intermediate algebra. Diagrams accompany a few of
Spatial-relations and visualization factors

the problems. Numerical work is kept to a minimum. Five multiple-choice responses are presented with each problem statement. Items increase in difficulty level progressively from the beginning to the end. Even-numbered items were omitted. Responses were recorded on a separate answer sheet. Most examinees attempted all assigned items.

3. Guilford-Zimmerman Numerical Operations.—This test is in four parts, consisting of numerous simple problems (of about the same difficulty level) involving respectively the four fundamental operations of addition, subtraction, multiplication, and division. Emphasis is placed in the directions upon the need for both accuracy and speed of work. Items were told to begin with the part upon addition, to work every item, and to go as far as possible in the allotted time. Only a few subjects reached the fourth section upon division. Responses to the items were printed in spaces on the test booklet adjacent to the problems.

4. Guilford-Zimmerman Perceptual Speed.—This test requires the examinee to match a visual object of a familiar shape and of detailed design with one of five other visual objects of a common category (e.g., automobiles, boats, hats, shoes). Four of the five possible pairs of visual objects—four stimulus and five response objects—are arranged in two parallel columns. To each column of pictures in the first column corresponds one of the five response objects. Thus, for each response, four responses are scored for each item of homogeneous content. All items represent a low level of difficulty. Answers to the items were marked on the test booklet in spaces adjacent to the row of response objects. Items were told to go as far as possible in the allotted time. No examinee finished.

5. Guilford-Zimmerman Spatial Orientation.—This test requires the examinee to determine how the position of a boat has changed in a second picture from its initial position in a first picture. Each picture consisted of a boat, in which the examinees were told to pretend to be riding, is shown along with background scenery consisting of water, or a silhouetted shore line, and in some instances of other boats intervening between the immediate and the boat. Movement is also indicated by accompanying shifts in ground scenery. The boat is actually stationary with respect to any forward-backward motion. To each set of two pictures there are four stimulus objects, two of which resemble radial designs to the stimulus object, but differ from it in certain minor details of shape and/or design. For each common category two parallel sets of visual objects—four stimulus and five response objects—are arranged in two parallel columns. To each column of stimulus pictures in the first column corresponds one of the five response figures. Thus, four responses are scored for each item of homogeneous content. All items represent a low level of difficulty. Answers to the items were marked on the test booklet in spaces adjacent to the row of response objects. Items were told to go as far as possible in the allotted time. No examinee finished.

6. Thurstone Identical Forms.—This test resembles rather closely the fourth test, Perceptual Speed, in that the examinee selects from a row of five similar appearing figures that one which is exactly the same as the stimulus figure. Slight differences in color design and in shape appear among the five response figures. In this test the items are also homogeneous with respect to difficulty. The number corresponding to the sequential position of the response selected was recorded on the test page in a box to the right of the row of response objects. Only a few examinees reached the last few items.

7. Thurstone Cubes.—In this difficult test the subject is asked whether two drawings can represent the same cube on each face of which the initial letter of the defined word is supposed to be a different design. In each of the two drawings the sides of three cubes are always exposed. If the two drawings can represent the same cube, a plus sign is placed in a blank square to the right of the two drawn cubes. If, on the other hand, the second drawing cannot represent the cube of the first drawing, then a negative sign is placed in the adjacent square. In the short time allowed no one attempted all items.

8. Thurstone Flags.—On this test two flag pictures, of the same size and of identical design, are presented occasionally in the same position, but generally in different positions. If the two drawings represent the same face of the flag, a plus sign is placed in a square on the test sheet just to the right of the two flags. If the two drawings represent opposite faces of the same flag, a minus sign is placed in the adjacent square. As in the test of Cubes the items were homogeneous with respect to difficulty. However, they were easy for most subjects. A few of the subjects attempted all items during the short period of time allowed.

9. Thurstone Number Series.—Found to be loaded in a factor identified by Thurstone as the factor of number series, this test requires the subject to determine a rule for each item. Numbers are presented in a row with two blanks inserted. The task is to find the mathematical principle by which the number series is formed and to insert in the blank that number which is appropriate. The difficulty level of items increases in relation to the position assigned to the item from the beginning to the end. Responses were recorded on the test sheets in the blanks inserted at various positions within the different number series. Most of the subjects attempted all items. One point of credit was given to each blank correctly filled (two points per item being maximum score).

10. Thurstone Punched-Hole Tests.—Each of these tests consists of a series of figures representing a square sheet of paper that has been folded by steps (as indicated by dotted lines) into smaller squares, rectangular, or triangular sizes. One or more holes are punched in the final folded form. The task for the subject is to imagine where the holes will be when the sheet is unfolded. As an aid to the subject's performance in the more difficult items one or more figures representing the appearance of the sheet of paper at intermediate stages of unfolding are presented. Presented on the unfolded (square) sheet the subject indicates by drawing small circles where the holes will be. In the scoring of the item all holes must be properly spaced in relation to one another if credit is to be given. An item was scored right or
Subjects. These individuals appeared to be a representative sample of the University student body in light of biographical information obtained from each student. Consisting of 220 freshmen and sophomores and 140 juniors and seniors majoring in virtually every department of the University, the sample was deemed satisfactory. Approximately 34 per cent of the subjects were veterans of World War II. The ages of the subjects ranged from 16 to 34, the median age being 22 years.

In order that a satisfactory degree of interest might be sustained throughout the duration of the study, all students were told that they would be given their scores upon completion of testing in profile form. In fact, most subjects received scores on those tests completed during the first two class periods at the beginning of the third period. It was thought that additional motivation might be provided if the temporal interval between taking the tests and receiving the scores was not too long.

The Factor Analysis

The matrix of test intercorrelations (all product-moment) presented in Table 1 was factor-analyzed by Thurstone's centroid method in the usual manner with one minor exception (13). In the reflection of signs the criterion was that used by the workers in several of the psychological research units of the United States Army Air Forces during World War II. The algebraic sum of a column, with the diagonal entry disregarded, was employed instead of the mere number of negative signs appearing in a column. This procedure not only tends to guarantee positive sums but also appears to approximate more closely the maximizing of table totals than does the criterion involving number of negative signs.

Because of marked discrepancies between obtained communalities in the first set of centroid extractions and the estimated communalities in the diagonals, a second set of extractions was required. Following the second extraction (of seven centroid factors) the obtained communality of no test differed more than .07 from the second estimated communality.

The criterion employed for cessation of extraction of the centroid factors was also that used by workers in the psychological research units of the AAF; namely, that factoring should not cease until the product of the two highest factor loadings is at least less than the standard error of the corresponding correlation. Such a criterion tends to yield a greater number of factors than do most other criteria. The rationale underlying this less stringent criterion is that the maximum contribution which the factor makes to the scalar product of two test vectors, or to the correlation between two tests, is no greater than the chance relationship expressed by the standard error of the correlation coefficient.

Following the completion of a set of trial rotations, it was considered advisable to extract two more centroid factors as an aid to further rotations. It was known that probably only six factors would be meaningfully identified. However, previous experience has indicated that use of additional centroid axes in the rotation process frequently brings about, more readily, a psychologically meaningful solution. The superfluous factors eventually appear as mere residuals (factors containing insignificant amounts of communality) to which no interpretation can be dependably given. Moreover, the presence of residual factors seldom interferes at the conclusion of the rotation procedure with the interpretation of those principal factors which account for most of the common-factor variance.

Fifty-six rotations of pairs of axes were required to satisfy Thurstone's criteria of positive manifold and simple structure. Each rotation was achieved graphically according to the method devised by Zimmerman (19). In general the structure determined the direction and magnitude of each new rotation. Information concerning the content of tests was put to use only to the extent that the tests were rotated. In view of the large number of rotations the differences between the communalities of centroid factors and final rotated factors were negligible, the largest two discrepancies being .017 and .013. An orthogonal reference frame appeared to suffice for the interpretation of the factors. The final rotated factor loadings are shown in Table 4.

Interpretation of Factors

Inspection of the final rotated factor loadings in Table 4 reveals that on the whole the criteria of positive manifold and
simple structure have been fulfilled. Six rotated factors were meaningfully identified as visualization (V2), verbality (V), numerical facility (N), general reasoning (R), spatial-relations (S), and perceptual speed (P). Two other factors (V1 and V3) appeared that could not be satisfactorily defined, although their weights in certain tests were suggestive of possible interpretations. A ninth factor turned out as a residual with loadings ranging from -0.08 to +.13.

As much as the primary purpose of the study centered about the investigation of the factors of spatial relations and visualization, the discussion relating to the identification and meaning of the other four factors will be kept to a minimum. The factors, V, N, and P are actually doublets. However, since the factorial content of the pairs of tests weighted in these three factors was well known in advance of their inclusion within the factorial content of the pairs of tests weighted in these three factors, there is little reason to doubt the correctness of the identification given.

It should be pointed out that the major loadings in some tests describing these three factors tended to be somewhat smaller than those reported in other studies or in manuals. This is due to the fact that many of the tests were shortened in order that they might be given within the time period available for testing. However, in view of the size of the sample (N = 360), loadings of .35 or greater are probably indicative of the presence of a significant amount of variance in a factor. Somewhat greater attention should probably be given to the interpretation of the factor R. Two tests, General Reasoning and Number Series, are loaded in this factor to the extent of .54 and .42, respectively. In view of the small number of items contained in the shortened form of the first test (fourteen in all) and of the consequent limitation imposed on the reliability of the test, the magnitude of first loading is substantial. Although the factor may be tentatively described as relating to some type of reasoning function, it is not clearly defined. That it may represent an ability to grasp the essential steps involved in the solution of problems presented in quantitative or symbolic terms appears to be a plausible interpretation.

Interesting to note is the fact that factor V1 is loaded .39 and .41 in the two tests Number Series and Pattern Analogies, respectively. A highly speculative interpretation would suggest that this factor may be that of induction previously identified by Thurstone (16). When the possible existence of an induction factor is taken into account along with the fact that the test of Pattern Analogies received an insignificant loading of .09 in the factor R, it appears even more plausible that the factor R may represent an ability to diagnose a problem expressed in quantitative terms. If the interpretation of the R factor is correct, a significant finding is that a test (General Reasoning) can be constructed to measure quantitative thinking without the introduction of substantial amounts of variance in the numerical factor.

Examination of the loadings for the final rotated factors I and VII in Table 4 reveals positive, though not conclusive, evidence for the existence of two reference variables which may be meaningfully identified as spatial-relations and visualization. In short, the two hypotheses as set forth are, in the main, upheld—at least to the extent that the factorial composition of the two groups of selected tests differs.

In the following list of four tests, the first three of which were selected to test the hypothesis relating to the psychological processes involved in visualization, loadings of .35 or higher in all rotated factors including I (V2), VII (S), and VI (V1) may be summarized as follows:

<table>
<thead>
<tr>
<th>Test</th>
<th>Factor I (V1)</th>
<th>Other Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Spatial Visualization .65</td>
<td>44</td>
</tr>
<tr>
<td>18</td>
<td>Punched Holes .59</td>
<td>39</td>
</tr>
<tr>
<td>1c</td>
<td>Form Board .58</td>
<td>40</td>
</tr>
<tr>
<td>1j</td>
<td>Spatial Orientation .47</td>
<td>51</td>
</tr>
</tbody>
</table>
In view of the presence of weights of .52 or higher in three tests Spatial Visualization, Punched Holes, and Form Board, (all of which made up the group intended to represent a measure of visualization), factor I can be identified as visualization, even though the test of Spatial Visualization is loaded to the extent of .44 in factor VII (S). That the spatial-relations and visualization abilities may be required in one or more tests in either of the two groups of tests as indicated in the battery was mentioned previously as a definite possibility. After taking the test of Spatial Visualization, many of the subjects reported that in addition to manipulations mentally the stimulus figure (an alarm clock) into the final position called for, there were difficulties in the location of various parts of the stimulus object (hands, numerals, top, base, winding and setting mechanisms of the clock) to the location of corresponding parts of one or more response figures (five alarm clocks in different positions). In the easier items which required only one manipulation the role of spatial cues is undoubtedly important. On the other hand, in those items requiring two or three movements of the clock, it would appear that a greater dependence was placed upon manipulations of the clock; in fact, in the most difficult items variance as associated with reasoning, verbal, and memory factors would possibly be important. However, only four items requiring a sequence of three movements were scored. Nevertheless, a small, though perhaps insignificant, loading of .25 appeared in the R factor. In short, the presence of items associated with the drawing (filling in) response required in the test of spatial-relations and visualization abilities that were hypothesized. Positive evidence for the hypotheses was to be considered attained if the two groups of tests defined separate factors and if none of the other eight tests was substantially weighted in factors unique to either group of tests. Within a battery of fourteen tests, two groups of tests (three tests in each group) were included which appeared to reflect differences in the psychological nature of spatial relations and visualization abilities. In addition to the six tests expressly included to yield evidence regarding the validity of the hypotheses, eight reference tests of fairly well-known factorial content were included to aid in the identification of two hypotheses regarding the psychological nature of spatial relations and visualization abilities that were hypothesized. Despite their relatively high saturations in the visualization factor, those two tests appear to involve additional unknown factors.

The magnitude of the weights in factor VII for the tests of Spatial Orientation, Flags, and Cubes indicates that identification of the factor as spatial relations is psychologically meaningful. With the substantial loading of the visualization factor in the test of Spatial Orientation—a fact which has been rationalized previously—the first hypothesis regarding the psychological nature of spatial relations appears to be upheld. Of passing interest is the loading of .34 in the spatial-relations factor appearing in the test of Pattern Analogies. In this factorially complex test, the presence of variance in the spatial-relations factor may have been due to the role of those changes in the design of complex figures or patterns, which depended upon a rule involving the spatial order of parts. In the more difficult items of complex design it was usually helpful, if not necessary, to give specific attention to the spatial organization of the various geometric properties within each of the patterns appearing in the row.

A second source for possible variance in the spatial-relations factor was that of the format of each item. Pattern A and pattern B, which stood in a left-right order on the page, corresponded to the order of pattern C and one of the five alternative responses. Having been exposed to spatial tests administered earlier, many of the subjects may have transferred techniques previously learned in solving other items to the task required in the test of Punched Holes. Thus, the influence of mental set may have been an important reason for the appearance of the loading in the spatial-relations factor.

The results of the factor analysis seem to indicate that in the main, the two hypotheses have been upheld. Two of the final rotated factors may be readily interpreted in terms of their weights in two groups of tests administered earlier, and the presence of variance found in the six tests and to answer questions of identity of the Thurstone and AAF factors. Having evaluated the hypotheses that the two tests were to be considered attained if the two groups of tests defined separate factors and if none of the other eight tests was substantially weighted in factors unique to either group of tests. Moreover, none of the three tests in one group should contain large amounts of variance in common with tests of the other group except to the extent that a given test might consist of items that reflected the presence of that factor which was defined in the main by use of suppression tests (8).
tests of the other group. If a test did appear in one group that contained variance in the factor associated primarily with tests of the other group, a satisfactory rationalization of this finding would be required.

Product-moment correlations computed from sets of scores of 350 students in the introductory course in psychology at Rutgers University were factored by Thurstone's centroid method. Eight of these factors were rotated by graphical means to positions satisfying the criteria of positive manifold and simple structure.

In the orthogonal system six factors were identified as verbal comprehension, numerical facility, perceptual speed, reasoning, visualization, and spatial relations. In the main, the variabilities associated with factors identified as spatial relations and visualization were confined to the respective groups of tests initially placed within the battery to bring out the factors. In only one test in each of three groups of tests were substantial amounts of variance found in both the visualization and spatial-relations factors, although the larger portion of variance was in the factor common to the group in which that test appeared.

The presence of variance in these two factors was rationalized for each of the tests. Introspective reports of the subjects revealed that in many items the psychological processes used involved both spatial-relations and visualization abilities as described in the hypotheses. The range of difficulty level of test items in one test also appeared to be an important reason for the occurrence of two factors.

In short, it may be concluded that the two hypotheses regarding the psychological nature of visualization and spatial relations were confirmed. However, other research projects need to be carried out with a variety of samples before a dependable generalization can be made regarding the nature of these two abilities. Since there is some evidence of still other spatial abilities (4), some or all of which may be correlated, it is recommended that a conscientious attempt be made to formulate in the hypotheses, attention can be directed toward building for the appearance of two factors.

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