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AN INVESTIGATION OF THE NATURE OF THE SPATIAL-RELATIONS AND VISUALIZATION FACTORS IN TWO HIGH SCHOOL SAMPLES[1]

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THE existence of at least two spatial abilities has been shown in several factorial analyses completed by workers in the psychological research units of the AAF (3) and in other analyses reported by Fruchter (1) and by the writers (10, 11, 18). However, in most of these studies samples consisting of adult males have been employed, and some degree of selection has been present. Thus, most samples have consisted of either college students, who in most instances had attained at least a specified minimum score on an aptitude test, or of aviation cadets, who were required to place at or above a specified cutting score on the *Army General Classification Test*. For these samples, nearly all factors isolated and identified, including the two factors of spatial relations and visualization, have appeared to be relatively independent.

The Problem

The primary purpose of this investigation was to test the validity of two hypotheses concerning the psychological nature of the spatial-relations and visualization abilities in two high-

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school samples differing in sex. In addition, attention was to be directed toward finding answers to the following questions:

(1) Given identical test batteries, are the same factors isolated for boys and girls of high-school age?

(2) Is the factor pattern in each of the tests comparable for the two sexes?

With respect to these two questions, several empirical studies have indicated that despite the presence of rather marked differences in the characteristics of samples, the same common factors are usually isolated, although the magnitude of the weights in the same factors may vary considerably in identical tests (9, 12).

A secondary purpose of the study was to ascertain whether certain factors found in the AAF investigations were the same as those identified as Thurstone's primary mental abilities. Tests of the type employed in the AAF psychological research units constituted one-third of the battery.

Plan of Investigation

The general plan followed in the investigation was to incorporate within the batteries those tests thought to represent the psychological processes hypothesized for the spatial-relations and visualization abilities. Additionally, other reference tests known to be representative measures of factors such as verbal comprehension, numerical facility, general reasoning, and perceptual speed were included for three reasons: (1) to gain additional information regarding the over-all nature of mental organization in male and female subjects of high-school age, (2) to identify other possible sources of common-factor variance in the spatial-visualization tests, and (3) to have an indication of the relative degree of purity of various tests incorporated within the battery—especially the degree of purity of the tests loaded in the spatial-relations and visualization factors.

Positive evidence for each of the hypotheses was to be considered attained if each group of tests thought to be associated with the psychological process hypothesized defined a factor. The evidence for the hypothesis would tend to be more nearly clear-cut if each test within a given group was weighted negligibly in the factor defined by the other group of tests. However, the fact that a given test might be loaded substantially in each of the two factors defined by the groups of tests would not necessarily negate the validity of the hypotheses, since it is likely that the successful completion of certain items in many tests requires the use of both psychological functions hypothesized. Moreover, certain individuals in the samples studied may tend to use primarily one of the processes to the exclusion of the other, despite differences in the nature of the tasks required.

Hypotheses

In a recent article by the writers, two hypotheses concerning the nature of the spatial-relations and visualization factors were developed in considerable detail (11). The essential aspects of the two hypotheses may be quoted as follows:

> The factor of spatial relations was hypothesized to represent the ability to comprehend the arrangement of elements within a visual stimulus pattern primarily with reference to the human body. Thus, an important implication in the ability to perceive spatial arrangements is that the subject is able to distinguish whether one object is higher or lower, left or right, or nearer or farther than another within the same field. Through the presentation of two simulated views of a stimulus pattern, a test item may be constructed such that there is a systematic relationship between the order of elements within the first spatial pattern (the stimulus component of a test item) and the order of elements within the second pattern (the response component of a test item).
>
> The factor of visualization was hypothesized to represent an ability that requires the mental manipulation of visual images. In contrast to another factor identified as visual memory (3), which appears to be a static or reproductive form of visualization, the factor referred to as visual manipulation, or simply visualization, is dynamic. This visual manipulative ability appears to be present in the solution of problems in which the individual finds it necessary mentally to move, rotate, turn, twist, or invert one or more objects. Following the performance of the presented manipulation the individual is required to recognize the new position, location, or changed appearance of the object or objects.
>
> Another important difference in the nature of the psychological processes hypothesized for the spatial-relations and visualization factors was that of speed of response. As indicated by findings in the AAF Aviation Psychology Program,

the tests thought to measure the spatial-relations factor were administered with fairly short time limits, but those tests thought to measure visualization were given with fairly liberal time allowances. The spatial relations factor was considered to demand a fairly rapid decision on the part of the examinee as to the spatial position of objects with reference to his own location; whereas, the visualization factor was believed to be represented in problems requiring a more deliberate and less automatic approach. In part such a distinction may be a function of the complexity of a task (i.e., the number of steps entering into the performance of an item), the more complex tasks requiring visualization for their solution.

The Tests

The tests selected to represent the psychological processes hypothesized for the spatial-relations factors were Guilford and Zimmerman's *Spatial Orientation*, Thurstone's *Cubes*, and *Space* from Thurstones' *Chicago Tests of Primary Mental Abilities*. The test *Space* consists of three sub-tests: *Flags*, *Figures*, and *Cards*. A composite score was employed for the test *Space*.

To yield evidence regarding the validity of the second hypothesis, four tests were used: Guilford and Zimmerman's *Spatial Visualization*, Thurstone's *Form Board*, Thurstone's *Punched Holes*, and the sub-test *Spatial Relations* in Wrightstone and O'Toole's *Prognostic Test of Mechanical Abilities*. The Spatial Relations sub-test is actually a modification of Thurstone's *Form Board* test in that the examinee is required to tell which of five sets of two-dimensional pieces, when assembled, will make up the "total" figure placed to the left of the various "multiple-choice" sets of pieces. This test differs from Thurstone's *Form Board* with respect to the method of response. In the latter, the examinee is required to fill in his response by drawing dotted lines in the total figure to show how the pieces fit together; in the former, the method of recognition appears most likely to be used.

In line with the hypotheses it would be expected that this sub-test might be factorially complex in that the format of the items would tend to favor an examinee's using two psychological processes associated with the factor spatial-relations. However, the mental manipulation of the figures would appear simultaneously to facilitate the fulfillment of the correct solution.

Since the other tests in the battery have been described in detail elsewhere, no attempt will be made to give an account of them except for the necessary details presented in Table 1. The six Guilford-Zimmerman tests and the four by Thurstone (*Cubes*, *Form Board*, *Punched Holes*, and *Identical Forms*) have been described at some length by the writers in a recent article (11) and in other places in the literature (7, 8). The four tests taken from the *Chicago Tests of Primary Mental Abilities* (*Number*, *Verbal Meaning*, *Space*, and *Reasoning*) are discussed in detail in the *Manual of Instructions* to the *Chicago Tests of Primary Mental Abilities* (15).

TABLE 1
The Test Battery: Descriptive Data

	Name of Test	Number of Items	Timing Plan (Speed or Power)	Working Time (Minutes)	Scoring Formula
1.	Guilford-Zimmerman Verbal Comprehension	72	Power	25	R-W/4
2.	Guilford-Zimmerman General Reasoning	28	Power	35	R-W/4
3.	Guilford-Zimmerman Numerical Operations	180	Speed	8	R-W
4.	Guilford-Zimmerman Perceptual Speed	72	Speed	5	R-W
5.	Guilford-Zimmerman Spatial Orientation	60	Speed	10	R-W/4
6.	Guilford-Zimmerman Spatial Visualization	60	Power (limited)	25	R-W/4
7.	Thurstones' Number (Addition, Multiplication, Three-Higher)	238	Speed	17	R-W
8.	Thurstones' Verbal Meaning (Sentences, Vocabulary, Completion)	135	Power (limited)	15	R
9.	Thurstones' Space (Flags, Figures, Cards)	168	Speed	15	R-W
10.	Thurstones' Reasoning (Letter Series, Letter Grouping, Pedigrees)	100	Power (limited)	16	R
11.	Thurstone's Form Board	28	Power	7	R
12.	Thurstone's Punched Holes	10	Power	7	R
13.	Thurstone's Identical Forms	40	Speed	3	R-W
14.	Thurstone's Cubes	42	Speed	5	R-W
15.	Wrightstone-O'Toole Prognostic Test of Mechanical Abilities: IV Spatial Relationships	15	Power (limited)	8	R

The Samples

Two high school samples of twelfth-grade students at John Muir Junior College in Pasadena, California, were used. One sample consisted of 151 boys; the other, of 139 girls. For each group the age of the subjects ranged from 15 to 20 years. The median age was 16 years 10 months for the girls, and 16 years 11 months for the boys.

Although more than 200 boys and 200 girls participated in the testing project during the spring semester, complete results could not be obtained, either because of absences or because of changes in the number enrolled at the grade level at any one testing session. For the first six tests, results were available for a variable number of boys and of girls; for the remaining tests complete results were obtained for 151 boys and 139 girls respectively. Intercorrelations among several of the tests for the two groups were based upon numbers slightly smaller than 151 and 139, the two smallest N's for a group correlation coefficient in the samples being 129 and 113 respectively.

In Table 2 are means and standard deviations of test scores for the two groups and *t* ratios reflecting the degree of significance of the difference between means. It is apparent that an hypothesis that the two samples were drawn from the same population with respect to performance on each of the tests must be rejected in most instances. Since the two samples may be said to come from different populations, the problem of factorial invariance is one of considerable interest.

Factor Analyses

Thurstone's centroid method of factoring was employed in the analyses of the two matrices of intercorrelations (Tables 3 and 4). For each matrix two sets of extractions were required to obtain estimates of communalities that deviated from the obtained communalities by less than $|.10|$. Following the second set of extractions the largest discrepancies between obtained and estimated communalities were .058 for the group of boys, and .053 for the group of girls.

An arbitrary criterion was employed as to the number of factors that should be extracted. For the two matrices, centroid factors were extracted up to the point that the highest loading of any test in that factor was less than $|.200|$. Accordingly, seven factors were initially extracted and used in the rotation process. As an aid to subsequent rotations two additional centroid factors were computed. These centroid factor loadings are available from the authors on request.

TABLE 2

Means and Standard Deviations of Test Scores for Two Groups of Boys and Girls along with T-Ratios for Testing Significance of Differences between Means

Test	Boys			Girls			Difference between Means	*t* ratio
	M	σ	N	M	σ	N		
1. Verbal Comprehension	21.64	11.65	149	19.37	10.07	139	2.27	1.76
2. General Reasoning	7.92	4.95	132	5.16	4.02	126	1.76	3.10
3. Numerical Operations	44.72	18.08	148	45.92	16.92	134	−1.20	.57
4. Perceptual Speed	44.11	10.43	143	45.90	11.80	134	−1.79	1.33
5. Spatial Orientation	24.52	9.48	141	17.81	9.43	125	6.71	5.75
6. Spatial Visualization	24.60	14.66	141	15.08	11.06	137	9.52	5.31
7. Number	56.79	17.83	151	59.40	18.32	139	−2.61	1.20
8. Verbal Meaning	69.81	15.29	151	68.22	14.36	139	1.59	.91
9. Space	56.25	20.03	151	43.71	19.69	139	12.54	5.35
10. Reasoning	30.28	9.85	151	32.63	9.28	139	−2.35	2.08
11. Form Board	19.13	6.22	151	16.12	5.74	139	3.01	4.26
12. Punched Holes	7.34	2.30	151	7.08	2.49	139	0.26	.91
13. Identical Forms	28.74	7.83	151	28.35	6.78	139	0.39	.44
14. Cubes	11.76	8.50	151	6.79	6.27	139	4.97	5.60
15. Spatial Relations	9.30	2.32	151	8.19	2.20	139	1.11	1.32
16. Mechanical Knowledge*	27.26	9.46	148	11.37	5.08	133	15.89	17.20
17. Fluency*	54.50	14.20	151	54.72	15.64	139	−0.22	.125

* The tests *Mechanical Knowledge* by Guilford and Zimmerman and *Fluency* in the *Chicago Tests of Primary Abilities* were not factor-analyzed with other tests, since there were no other tests in the battery thought to contain substantial portions of variance in the factors these two tests represent. Their inclusion would seem to make less clear-cut a meaningful factorial solution.

TABLE 3

*Intercorrelations of Tests Administered to Boys**

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Verbal Comprehension	—	567	231	125	201	336	290	750	093	327	178	259	182	205	179
2. General Reasoning	567	—	367	136	210	501	340	514	264	508	460	519	211	353	197
3. Numerical Operations	231	367	—	072	291	302	695	312	098	359	028	134	443	167	160
4. Perceptual Speed	125	136	072	—	131	280	009	255	191	359	396	229	417	235	145
5. Spatial Orientation	201	210	291	131	—	343	210	222	421	401	336	356	286	308	382
6. Spatial Visualization	336	501	302	280	343	—	273	376	456	529	549	619	384	427	412
7. Number	290	340	695	009	210	273	—	412	142	380	128	139	267	121	148
8. Verbal Meaning	750	514	312	255	222	376	412	—	212	434	292	299	268	221	225
9. Space	093	264	098	191	421	456	142	212	—	357	391	338	181	426	373
10. Reasoning	327	508	359	359	401	529	330	434	357	—	463	477	427	518	290
11. Form Board	178	460	028	396	336	549	128	292	391	463	—	626	297	438	405
12. Punched Holes	259	519	134	229	356	619	139	299	338	477	626	—	271	410	418
13. Identical Forms	182	211	443	417	286	384	267	268	181	427	297	271	—	318	243
14. Cubes	205	353	167	235	308	427	121	221	426	518	438	410	318	—	238
15. Spatial Relations	179	197	160	145	382	412	148	225	373	290	405	418	243	238	—

* Decimal points omitted.

TABLE 4

*Intercorrelations of Tests Administered to Girls**

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Verbal Comprehension	—	404	156	—011	211	392	179	714	313	381	394	291	271	320	305
2. General Reasoning	404	—	331	149	263	391	343	426	295	546	309	471	264	259	245
3. Numerical Operations	156	331	—	306	221	105	681	266	155	346	111	200	307	112	160
4. Perceptual Speed	—011	149	306	—	164	095	165	094	324	074	243	215	332	138	111
5. Spatial Orientation	211	263	221	164	—	457	232	261	418	538	445	307	391	563	208
6. Spatial Visualization	392	391	105	095	457	—	225	442	300	515	560	514	291	566	478
7. Number	179	343	681	165	232	225	—	366	151	399	266	217	290	266	268
8. Verbal Meaning	714	426	266	094	261	442	366	—	364	428	408	325	278	249	308
9. Space	313	295	155	324	418	300	151	364	—	392	523	402	274	403	314
10. Reasoning	381	546	346	074	538	515	399	428	392	—	526	451	357	536	375
11. Form Board	394	309	111	243	445	560	266	408	523	526	—	596	421	605	435
12. Punched Holes	291	471	200	215	307	514	217	325	402	451	596	—	384	424	356
13. Identical Forms	271	264	307	332	391	291	290	278	274	357	421	384	—	339	284
14. Cubes	320	259	112	138	563	566	266	249	403	536	605	424	339	—	392
15. Spatial Relations	305	245	160	111	208	478	268	308	314	375	435	356	284	392	—

* Decimal points omitted.

TABLE 5

*Final Rotated Factor Loadings for the Group of Boys**

Test	Factors† I (V)	II (S)	III (N)	IV (R)	V (Db)	VI (P)	VII (Res 1)	VIII (Vz)	IX (Res 2)	h²
1. Verbal Comprehension	86	06	16	21	00	04	−08	09	−05	843
2. General Reasoning	41	00	29	58	24	03	−04	18	−14	706
3. Numerical Operations	06	16	89	08	02	10	−11	03	00	830
4. Perceptual Speed	07	03	−01	06	08	62	16	17	−02	464
5. Spatial Orientation	11	59	20	10	23	09	01	10	01	485
6. Spatial Visualization	18	22	18	33	20	21	00	60	00	675
7. Number (PMA)	15	04	77	12	07	−06	24	06	05	711
8. Verbal Meaning (PMA)	77	04	26	16	04	19	22	14	02	800
9. Space (PMA)	04	50	02	21	07	06	24	36	−11	511
10. Reasoning (PMA)	16	31	24	51	10	37	05	16	08	631
11. Form Board	08	14	−03	34	49	34	20	41	−16	749
12. Punched Holes	14	16	03	41	50	16	−10	47	09	730
13. Identical Forms	07	21	36	03	−02	59	−06	20	02	585
14. Cubes	04	39	04	48	00	23	07	21	−11	506
15. Spatial Relations	13	39	07	−02	32	09	08	33	07	416

* Decimal points omitted. Centroid Factor Loadings for this group are available on request from the authors.

† The meanings of the symbols identifying the factors (designated by Roman numerals) are as follows: V, verbal comprehension; S, spatial relations; N, numerical facility; R, general reasoning; Db, doublet; P, perceptual speed; Vz, visualization; Res, residual.

TABLE 6

*Final Rotated Factor Loadings for the Group of Girls**

Test	Factors† I (Vz)	II (N)	III (V)	IV (P)	V (S)	VI (R)	VII (Tp)	VIII (Res)	IX	h^2
1. Verbal Comprehension	05	05	79	−03	11	17	−01	−16	20	733
2. General Reasoning	10	20	31	06	04	61	18	−02	09	560
3. Numerical Operations	−01	82	05	26	04	23	01	02	00	798
4. Perceptual Speed	−04	15	−02	52	17	00	27	−01	−02	402
5. Spatial Orientation	16	10	06	14	65	30	01	14	19	634
6. Spatial Visualization	61	00	34	02	33	26	12	14	02	706
7. Number (PMA)	19	74	19	11	08	15	00	−02	00	664
8. Verbal Meaning (PMA)	02	22	85	02	16	17	09	14	00	852
9. Space (PMA)	02	01	26	25	46	17	39	−14	04	544
10. Reasoning (PMA)	25	26	23	−08	38	58	12	−04	21	729
11. Form Board	43	02	32	15	36	10	40	00	41	771
12. Punched Holes	40	00	24	28	05	37	35	07	25	621
13. Identical Forms	19	16	15	46	21	15	−02	03	35	484
14. Cubes	48	04	15	02	59	18	03	−11	23	696
15. Spatial Relations	47	10	27	09	14	10	19	−15	01	403

* Decimal points omitted. Centroid Factor Loadings for this group are available on request from the authors.

† The meanings of the symbols below the Roman numerals are the same as those stated in the previous table. The new symbol Tp. stands for a triplet factor.

For the orthogonal solutions given, rotations of pairs of centroid axes were achieved by a graphical method devised by Zimmerman (17). To achieve the final rotated orthogonal loadings presented in Table 5 and Table 6, forty-two and forty rotations were required respectively. Thurstone's criteria of positive manifold and simple structure served as the bases for cessation of the rotation process. For each group an orthogonal reference system appeared to suffice, especially in view of the relatively small number of tests in each battery.[2]

General Results

In both analyses, six rotated factors were meaningfully identified as verbal comprehension (V), general reasoning (R), numerical facility (N), perceptual speed (P), spatial-relations (S), and visualization (Vz). One other factor appeared—a doublet in one analysis and a triplet in the other—for which a tentative interpretation may be offered. In the final rotated factor matrix for the boys, two *residual* factors emerged; in the corresponding matrix for the girls one *residual* factor appeared and a second factor came out for which an interpretation seems improbable, if not impossible. With the exception of S and Vz the factors identified are common with those of Thurstone.

In the main, the two hypotheses regarding the nature of spatial relations and visualization were upheld as they were in a previously cited study by the writers in which a college sample was employed (11). Each of the two groups of tests placed in the test batteries to represent the psychological processes hypothesized for the spatial-relations and visualization abilities served to define a distinct factor. With respect to the two factors identified as spatial relations and visualization, the larger portion of the variance in the tests within each of the two groups of tests was in the factor that the group defined, with

[2] Nevertheless, a separate paper is being prepared to ascertain estimates of the degree of correlation among primary factors in order that comparisons may be made not only between these two groups, but also with an oblique solution derived from a college sample (11). For this forthcoming paper, the three oblique solutions will be presented to yield evidence regarding whether a change in correlation among factors occurs with an increase in age or with an advancement in academic level. Garrett and his associates have found that the amount of correlation among factors decreased markedly as age increased (2). Swineford, on the other hand, found no systematic change in the degree of correlation among factors throughout a range of six grade levels—fifth through tenth (12, p. 32).

one exception. This exception occurred in Wrightstone and O'Toole's *Spatial Relations* sub-test in which the final rotated factor loading for the boys was .39 in S and .33 in Vz.

In the following list of four tests, the first three of which were chosen to yield evidence regarding the first hypothesis, loadings of .39 or higher were found in four rotated factors designated as S, Vz, R, and Db:

Tests	Factor defined as S		Other Factors	
	Boys	Girls	Boys	Girls
(5) Spatial Orientation	.59	.65	(.10Vz)	(.16Vz)
(9) Space (PMA)	.50	.46	(.36Vz)	.39Db (.02Vz)
(14) Cubes	.39	.59	.48R (.21Vz)	.48Vz
(15) Spatial Relations	.39	(.14)	(.33Vz)	.47Vz

It is evident that tests (5), (9), and (14) serve to identify a spatial-relations factor, although tests (9) and (14) are factorially complex.

Attainment of positive evidence for the second hypothesis regarding the nature of visualization is apparent in the following list of tests for which factor weights of .39 or higher are given (along with weights on the S factor for comparative purposes):

Tests	Factor defined as Vz		Other Factors	
	Boys	Girls	Boys	Girls
(6) Spatial Visualization	.60	.61	(.22S)	(.33S)
(11) Form Board	.41	.43	.49Db. (.14S)	.40Tp., .41(?), (.36S)
(12) Punched Holes	.47	.40	.50Db., .41R, (.16S)	(.05S)
(15) Spatial Relations	(.33)	.47	.39S	(.14S)
(14) Cubes	(.21)	.48	.48R, .39S	.59S

Of special interest is the factor labeled Db (an abbreviation for "doublet") which occurred more distinctly in the boys' matrix of rotated factor loadings than it did in the girls' matrix. In fact, in this latter matrix, the factor which may be called a triplet received a moderate weight (.39) in the test *Space* in addition to weights .40 and .35 in the *Form Board* and *Punched Holes* tests. In the boys' matrix, the weights in the *Form Board* and *Punched Holes* tests were .49 and .50, but only .07 in *Space*.

A similar-appearing factor was present in the previously mentioned study with a college sample in which the hypothesis was offered that the "drawing in" or "filling in" nature of the response to the test item might represent the psychological process corresponding to the factor (11). This factor may reflect a specialized aspect of the reasoning processes. In terms of the requirements of the task represented by the *Form Board* and *Punched Holes* tests, the factor may be tentatively referred to, for lack of a better name, as *visual-motor reasoning*.

In general, the factor patterns of each test were similar for the two rotated factor matrices. A few differences did appear which may be worthy of comment, although it is, of course, not possible to make a test of the statistical significance of the difference between factor loadings:

Girls had higher weights than boys in the verbal-comprehension factor on the tests measuring essentially the spatial-relations and visualization abilities, with one exception—namely, *Spatial Orientation*. This finding was consistent with the fact that correlations between scores on the verbal tests and scores on the spatial-visual tests were higher for the girls in every instance than they were for the boys. It may be that since the mean achievement level of girls was slightly lower on the verbal tests (contrary to the expected finding) than that of boys, the reading comprehension level of the girls might have been at a point far enough below that of the boys to make the vebal comprehension factor of greater importance to the girls in understanding the rather complex verbal directions to many of the spatial and visualization tests. On the other hand, it might be that the girls tended to verbalize their performance on the items in such tests as *Form Board*, *Spatial Visualization*, and *Space* to a relatively greater degree than did the boys.

Several miscellaneous and more or less non-systematic differences occurred in the sizes of factor loadings (many of which can probably be attributed in large measure to sampling error):

(1) a weight in the perceptual-speed factor of .37 for boys and −.08 for girls in the test *Reasoning* (PMA);

(2) a weight in the perceptual-speed factor of .34 for boys and .15 for girls in the test *Form Board*;

(3) a weight in the numerical-facility factor of .36 for boys and .16 for girls in the test *Identical Forms*;

(4) weights in the general-reasoning factor of .34 and .48 for boys and .10 and .18 for girls in the tests *Form Board* and *Cubes* respectively;

(5) a weight in the general-reasoning factor of .10 for boys and .30 for girls in the test *Spatial Orientation* (but variances in the reasoning factor were about the same for the two samples in all the other spatial and visualization tests);

(6) weights in the spatial-relations factor of .39, .39, and .14 for boys and .14, .59, and .36 for girls in the tests *Spatial Relations*, *Cubes*, and *Form Board* respectively; and

(7) weights in the visualization factor of .36, .21, and .33 for boys and .02, .48, and .47 for girls, in the tests *Space*, *Cubes*, and *Spatial Relations* respectively.

One anticipated finding confirmed by this study is that in general tests in the spatial and visualization factors tend to be factorially complex. Until relatively pure tests in the spatial-relations and visualization abilities are developed, estimates may be obtained of univocal factor scores through use of suppression formulas, provided that relatively pure tests can be found for suppressing portions of unwanted variance (5). However, the use of such formulas should not serve to discourage the formulation of new hypotheses regarding the nature of spatial-visualization abilities and the development of more nearly pure measures of identified abilities through efforts on the test-construction front. The attainment of such objectives is essential for the success of a test-development program in which factor analysis is employed (4).

Summary

The primary purpose of the study was to test the validity of two hypotheses regarding the psychological properties of the spatial-relations and visualization factors. Within a battery of fifteen tests, two groups of tests (four tests in one group and three in the other) were included which appeared to differentiate between the essential psychological processes associated with the spatial-relations and visualization abilities. Reference tests of fairly well known factorial content composed the remainder of the battery.

Two high-school samples numbering 151 boys and 139 girls participated in taking the same battery of 15 tests. A Thurstone

centroid analysis was completed, followed by rotations of axes to positions such that the criteria of simple structure and positive manifold were satisfied.

For both groups, the same six orthogonal factors were identified: S (spatial-relations), Vz (visualization), V (verbal comprehension), N (numerical facility), P (perceptual speed), and R (general reasoning). Positive evidence was considered attained for the two hypotheses regarding the factors S and Vz, in that each group of tests clearly defined a distinct factor. Moreover, the factor pattern in each test was approximately the same for the two groups studied.

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