

THE CLOSURE FACTORS RELATED TO OTHER COGNITIVE PROCESSES*

CAROL PEMBERTON

UNIVERSITY OF DELAWARE

Twenty-five group tests, assembled with certain hypotheses concerning the first and second closure factors in mind, were administered to 154 subjects, mostly graduate students. The intercorrelations were analyzed factorially, yielding eight factors that were rotated to an oblique simple structure. The factors were interpreted as: speed of closure, C_1 ; flexibility of closure, C_2 ; verbal closure, C_3 ; word fluency, W ; reasoning, R ; perceptual speed, P ; the first space factor, S_1 ; and speed of handwriting, H . Four second-order factors were tentatively described as analytical ability, synthetic ability, speed of perception, and word fluency. Three of the reasoning tests had their highest loadings on C_2 and one on C_3 , which seems to be evidence that flexibility of closure generalizes in the cognitive domain and is associated with analytical ability.

I. Purpose of the Study

Factors named by Thurstone "speed of closure" and "flexibility of closure" have been isolated in several previous studies (1, 2, 17, 19). The purpose of the present study was to ascertain whether abilities on speed of closure and flexibility of closure tests generalize to other domains. The part of the study to be reported in this paper deals with generalization of the closure factors to tasks requiring higher cognitive functions. The generalization of the closure factors to the conative and affective domains will be discussed in a subsequent paper.

The hope of finding meaningful personality variables through studying perception has been expressed by Klein and Schlesinger (9), and by Frenkel-Brunswick (6), among others.

Thurstone (19) writes:

The first closure factor C_1 (speed of closure) seems to facilitate the making of a closure in an unorganized field, the second closure factor C_2 (flexibility of closure) seems to facilitate the retention of a figure in a distracting field. If this . . . interpretation of the two closure factors has any generality beyond the perceptual domain, then one could imagine that the factor C_1 determines the ease with which the subject can unify a complex situation, whereas the second factor determines the ease with which he can keep in mind its essential features against distraction. . . The first closure factor might be associated with inductive thinking, whereas the second closure factor might be more associated with deductive thinking.

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The present investigation deals with a factor analysis of the intercorrelations of twenty-five tests selected to include longer forms of tests for both closure factors; inductive and deductive reasoning tests; verbal closure tests; and key tests for the first space factor, perceptual speed, word fluency, and speed of handwriting.

II. *Description of the Tests*

1. *Figures*. Twenty rows of geometrical figures are presented, with instructions to mark the figures of the six on the right which can be made the same as the one on the left by sliding them around in the plane of the paper.

2. *Cards*. A hundred and twenty diagrams of differently shaped cards are given and the subject asked which cards of the six in a row can be made the same as the model by sliding them around in the plane of the paper.

3. *Gestalt Completion*. This test is similar to Thurstone's adaptation of the Street Gestalt Completion test (12). Two 24-item forms of this test were devised by the writer and the scores for the two combined.

4. *Mutilated Words*. Two 51-item forms similar to Thurstone's test (17) were prepared by the writer and the scores for the two combined.

5. *Concealed Figures*. This 49-row form of the Gottschaldt Figures Test was devised in the Psychometric Laboratory, University of Chicago. A small design at the beginning of each row is followed by four more complex designs. The subject indicates in which of the more complex diagrams the simple figure occurs.

6. *Designs*. In this test of Thurstone's (15), 300 designs are presented, in 40 of which the Greek capital letter "sigma" is concealed. The task is to mark the figures containing the "sigma."

7. *First Letter*. The subject lists all the words he can that begin with the letter "c."

8. *Suffixes*. The subject lists all the words he can that end with the letters "-tion."

9. *Writing Phrase*. The task here is to write the phrase "Now is the time for all good men" as many times as possible within one minute.

10. *X's*. The subject writes as many X's as possible in 30 seconds.

11. *Identical Forms*. The first figure in each of 60 rows of figures is exactly the same as one of the five numbered figures following it. For each row the subject writes the number of the figure which is identical with the model.

12. *Identical Numbers*. The subject is required to mark the number 927 each time it occurs in 18 columns of three-digit numbers.

13. *Number Series*. The subject is to discover the rule underlying each of 22 series of numbers and fill in the blanks.

14. *Letter Series*. This 25-series test is similar to the previous one, except that letters are used.

15. *Figure Classification*. This is a test of Spearman's adapted by Thurstone (14). Two groups of four figures each occur at the beginning of each line, followed by eight more figures. The subject is to discover the rule by which the first two groups were constructed, and then mark those of the remaining eight figures which belong to Group I.

16. *False Premises*. The subject labels as true or false each of 25 syllogisms composed of absurd-sounding premises and conclusions.

17. *Hidden Words*. This test was devised for the present study. A four-letter word appears at the beginning of each of 36 sentences. The subject is to encircle the four-letter word whenever it occurs, either embedded in a longer word, or in two longer words, ignoring spaces and punctuation. This test represents an attempt to produce a verbal test of flexibility of closure. The sentences were made amusing in an effort to strengthen the Gestalt of the complete sentence.

18. *Incomplete Words*. Individual letters are deleted from 50 common English words and the subject is asked to fill in the missing letters.

19. *Anagrams*. Fifty English words are presented, the letters of which, when rearranged, spell another English word. The subject encircles the letter with which the new word begins. The words to be found are well-known ones, the majority being AA or A in Thorndike's (13) word list.

20. *Scrambled Words*. This is a revision of a test constructed by Botzum (2). The letters of 50 common four-letter words are rearranged so as to form nonsense syllables. The subject encircles the first letter of the unscrambled meaningful word.

21. *Copying*. This test was adapted by Thurstone from a test by MacQuarrie (14). The subject is to copy 36 figures in the dotted spaces provided. Each drawing must have the same size, shape, and position as the original.

22. *Four Letter Words*. This test, devised by Bechtoldt (1), and revised by Botzum (2), consists of rows of evenly-spaced typewritten capital letters. In each row there are a few combinations of four letters that spell words, which are to be encircled.

23. *Backward Writing*. In this test, designed by Thurstone (16), 50 words are typed in the normal manner, and below each are four words reversed as in a mirror. The subject underlines the word corresponding to the normally typed one.

24. *Hidden Pictures*. This test of Thurstone's contains items like those found in children's books, in which one can see objects hidden in the lines of a larger picture. Six pictures, in which 28 hidden persons are to be located, constitute the test.

25. *Sentences*. A test similar to this was used by Meili (10). In the present form, devised for this study, the subject is required to write as many sentences as he can containing each of four given words, then as many as he can containing each of five and six given words.

Data on reliability coefficients, means, standard deviations, time limits, and scoring formulas are summarized in Table 1. The reliabilities are corrected split-half coefficients. As the tests are all speed tests, the reliability coefficients based on parallel forms would probably be lower. Distributions were all approximately normal except for the test Backward Writing, where there was a bunching of high scores as a result of a too liberal time limit.

III. *Subjects and Administration of the Tests*

The 154 subjects (122 graduate students and 32 undergraduate students) were volunteers, obtained through announcements on bulletin boards at the University of Chicago. There were 94 men and 60 women. All the divisions and professional schools were represented, but the largest group (43 persons) was from the Social Science Division. Ages ranged from 17 to 59 years, with the majority of cases between 19 and 29, and the average 26.4 years.

TABLE 1
Information Regarding the Tests

Test	Mean	S. D.	r	Time Limits (minutes)		Scoring Formula
				Fore-Ex.	Test	
1. Figures	20.97	9.40	.96	2	3.5	R - W
2. Cards	27.46	12.09	.96	2	4	R - W
3. Gestalt Completion	35.98	6.94	.80	2	3	R
4. Mutilated Words	72.01	14.14	.87	2	5	R
5. Concealed Figures	100.88	28.70	.94	4	10	R - W
6. Designs	26.94	6.28	.94	2	3.5	R
7. First Letter	56.98	13.36	—	2	5.5	Total
8. Suffixes	28.44	9.00	—	1.5	5	Total
9. Writing Phrase	54.05	6.97	—	1	1	Total
10. X's	62.68	8.22	—	.5	.5	Total
11. Identical Forms	44.68	9.11	.98	3	4	R - W/4
12. Identical Numbers	63.74	7.02	.92	3	2.5	R
13. Number Series	11.66	3.53	.82	4	7	R
14. Letter Series	13.55	4.28	.85	4	6	R
15. Figure Classification	66.79	23.22	.94	6	10.5	R - W
16. False Premises	12.33	5.54	.65	3	6	R - W
17. Hidden Words	68.94	15.64	.98	3	5	R
18. Incomplete Words	29.31	9.80	.95	1.5	5	R
19. Anagrams	18.74	7.59	.88	4	6.5	R
20. Scrambled Words	27.21	10.65	.94	4	5	R - W/3
21. Copying	16.36	7.93	.96	4	4	R
22. Four-Letter Words	39.32	9.11	.92	3	4.5	R
23. Backward Writing	40.74	7.98	.98	2	3	R - W/3
24. Hidden Pictures	13.31	3.35	.66	5	14	R
25. Sentences	12.60	3.71	—	1	9	Total

Tests were administered in group form on three consecutive evenings during August, 1950, from 7:30 to 10 p.m.

IV. *The Factor Analysis*

All tests were scored by hand twice, usually by two different people. Pearson product-moment correlation coefficients were calculated from the raw scores by I.B.M. equipment (Table 2). Eight factors were extracted from the correlation matrix by the complete centroid method (18). The matrix was factored twice. During the first factoring, the highest value in each column, adjusted after the extraction of each factor, was used as the diagonal entry. During the second factoring, the communalities resulting from the first factoring were employed. The largest difference between the communality estimates resulting from the two factorings is .06, and the majority of differences are .00 or .01. The eighth-factor residuals (Table 3) range from .08 to $-.10$ with a mean of .00 and a standard deviation of .030. The orthogonal centroid matrix F is reproduced in Table 4.

The F -matrix was rotated to a simple structure using both radial and single-plane methods of rotation (18). Table 5 presents the final transformation matrix and Table 6 the resulting oblique factor matrix, V . Table 7 shows the cosines between the reference vectors of the V -matrix, and Table 8 the correlations between the primary axes. The reference vectors have been identified by the letters A to H, but in Tables 8-12, which refer to the primary vectors, the columns are denoted by letters which refer to the psychological interpretation of the factors. The columns are kept in the same order to facilitate the comparison of tables.

Table 8 shows that some of the correlations between the primary vectors are high. For this reason the A -matrix presented in Table 11 was calculated. This table represents the test vectors as linear combinations of the primary vectors. It is obtained by post-multiplying the V -matrix by the inverse of the diagonal matrix, D . The values of D (Table 9) are also used in computing the matrix T (Table 10), which represents the eight direction cosines of each primary vector. Table 12 gives the correlations between each primary factor and each of the tests.

V. *The Second-Order Analysis*

The complete centroid method of factoring was used, and Table 8 was factored three times to stabilize the communalities. Table 13 shows the orthogonal matrix, F_2 ; the residuals after five factors had been extracted appear in Table 14. The centroid matrix was rotated to an oblique simple structure. The oblique matrix, V_2 , and the transformation matrix, Λ_2 , are shown in Tables 15 and 16, and the cosines between the reference vectors in Table 17. Because of the impossibility of determining the second-order primary vectors with much confidence, the correlations between the primaries were not calculated.

TABLE 2
Table of Intercorrelations, R^*

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
1																										
2	73																									
3	44	49																								
4	25	36	37																							
5	47	63	43	46																						
6	36	40	38	41	54																					
7	15	25	06	25	16	34																				
8	08	13	-03	33	04	22	56																			
9	-09	-05	-08	09	-01	08	20	11																		
10	19	12	-10	08	17	18	23	11	34																	
11	48	47	55	32	49	49	23	11	04	09																
12	17	19	18	41	26	44	26	23	19	10	35															
13	31	46	21	44	41	33	20	26	-03	03	27	2														
14	34	46	28	43	36	34	34	25	04	-07	39	41	54													
15	32	48	32	36	54	34	23	11	-10	-02	20	11	45	45												
16	08	19	15	23	36	10	11	04	-06	06	09	-07	40	25	41											
17	30	31	28	41	37	53	42	32	22	17	50	55	24	50	20	12										
18	21	31	13	59	34	33	41	42	12	12	17	39	47	48	33	27	46									
19	21	22	15	46	24	35	51	38	09	17	23	37	33	51	28	26	55	63								
20	25	29	03	44	24	31	52	42	00	12	17	39	45	49	27	21	52	63	73							
21	32	51	38	47	70	45	16	08	13	16	34	26	35	30	40	33	31	39	26	22						
22	20	28	23	49	44	51	48	25	20	24	38	44	25	39	20	12	64	49	52	49	34					
23	36	43	29	45	42	44	45	37	15	19	44	42	33	44	21	16	53	50	45	53	40					
24	24	21	39	23	36	24	00	-10	-07	09	27	16	10	08	08	-03	17	09	01	-03	35	21	18			
25	13	23	13	18	18	13	37	20	34	09	28	18	12	27	15	11	29	15	14	07	18	28	17	12		

*Decimal points omitted.

VI. *Interpretation of the First-Order Factors*

Primary factors are usually interpreted by referring to loadings on the reference vectors. However, the aim of factor analysis is to express the test vectors as linear combinations of the primary vectors, and this is given by the *A*-matrix (Table 11) rather than the *V*-matrix. In an oblique solution the *A*-matrix is proportional to the *V*-matrix by columns but not by rows. A test may have its highest loading on a given reference vector, but its highest component is not necessarily of the corresponding primary vector.

For example, Test 4 had its highest loading on reference vector *F*, and its second highest on *A*, whereas in the *A*-matrix its largest component is of the primary vector C_3 , which corresponds to the reference vector *A*. A more marked reversal occurs in the case of Test 14. Due to our inadequate knowledge of the significance of factor loadings, it is difficult to say whether these reversals are important. However, because it seems more logical to interpret from the *A*-matrix, this has been done. Since most people interpret from the *V*-matrix, the loadings on the reference vectors, as well as the components of the primary vectors, will be quoted.

TABLE 3
Distribution of Eighth-Factor Residuals*

Residual	Frequency
.08	4
.07	8
.06	19
.05	18
.04	27
.03	47
.02	80
.01	90
.00	104
-.01	59
-.02	52
-.03	52
-.04	26
-.05	22
-.06	9
-.07	0
-.08	4
-.09	0
-.10	4

N = 625

*This table includes the residuals from the diagonal entires.

1. Verbal Closure, C_3

Test	Loadings on A	Components of C_3
19. Anagrams	.54	.91
20. Scrambled Words	.52	.88
18. Incomplete Words	.48	.81
4. Mutilated Words	.38	.64
22. Four-Letter Words	.36	.61
17. Hidden Words	.34	.57
14. Letter Series	.33	.56
12. Identical Numbers	.32	.54

The tests which are high on this factor all require speed in organizing or reorganizing a set of highly practiced symbols, such as letters or numbers. No spatial or pictorial tests are represented. The factor has a high correlation (.70) with the word fluency factor, W , and the four tests with the highest loadings all require the subject to think of words rapidly, and to select the one which fits the restrictions set by the problem. However, these restrictions are more rigid than those for the W tests. In the tasks high on C_3 only *one* word

TABLE 4
Orthogonal Centroid Factor Matrix, F

Test	I	II	III	IV	V	VI	VII	VIII	h^2
1	.53	.38	.05	.11	.30	.16	.27	.21	.67
2	.66	.43	.13	-.06	.28	.06	.16	.15	.77
3	.46	.47	-.13	.28	.18	-.17	-.09	.05	.60
4	.68	-.03	.04	.10	-.21	-.27	-.15	.11	.63
5	.70	.48	-.12	-.14	-.23	.07	.10	-.09	.83
6	.65	.10	-.22	.07	-.06	.09	.07	-.20	.54
7	.55	-.41	.06	-.20	.18	-.08	.23	-.12	.62
8	.40	-.42	.21	-.10	.16	-.25	.10	-.21	.53
9	.17	-.37	-.39	-.39	.04	.05	-.19	.20	.55
10	.23	-.13	-.20	-.27	-.07	.18	.31	.18	.35
11	.59	.23	-.31	.18	.30	.06	-.04	-.11	.64
12	.53	-.25	-.25	.26	-.04	.03	-.10	-.11	.50
13	.57	.14	.39	-.02	-.09	.06	-.17	-.06	.54
14	.66	-.04	.25	.14	.15	.15	-.31	-.03	.66
15	.51	.32	.34	-.14	-.07	.08	-.16	-.08	.54
16	.32	.18	.29	-.24	-.20	.04	-.20	-.04	.36
17	.71	-.26	-.24	.21	.04	.19	-.01	-.08	.72
18	.67	-.28	.26	.04	-.22	-.12	-.02	.11	.67
19	.65	-.38	.23	.18	-.14	.07	.05	.07	.68
20	.63	-.39	.37	.20	-.13	.11	.20	-.03	.80
21	.62	.33	-.12	-.19	-.26	-.12	.02	.02	.63
22	.67	-.24	-.21	.07	-.17	.08	.03	.03	.59
23	.68	-.12	-.02	.10	.11	-.06	.12	-.11	.53
24	.27	.28	-.33	.14	-.11	-.27	.05	.09	.38
25	.36	-.11	-.15	-.30	.28	-.04	-.20	.05	.38

will fit the requirements, while in the W tests there are numerous acceptable words.

Mutilated Words and Incomplete Words both have loadings on C_1 (identified as speed of closure), as well as on C_3 . In these two tests the elements to be combined are given in their correct order, and the task can be performed by a rapid synthesis. However, while administering Mutilated Words as an individual test, the writer observed that many subjects did this task analytically, one subject remarking that he was "learning to break the code." Moreover, if the word is not seen quickly and correctly as a whole, then the ability to change one's set rapidly seems to be important. Due to inability to relinquish an incorrect response, many people perform inadequately.

In Anagrams the subject must abandon the Gestalt which is formed by one English word and reorganize the letters into another word. Similarly, in Scrambled Words some of the nonsense syllables are pronounceable and form Gestalts that must be broken. In Four-Letter Words, Hidden Words, and Identical Numbers, one must find or retain a Gestalt in a distracting field. All these tests therefore seem to require elements of what has previously been called "flexibility of closure." The factor itself correlates .54 with flexibility of closure, C_2 .

Letter Series has almost equal components of this factor and factor R. If R accounts for the synthetic aspects of reasoning, then one would expect Letter Series also to have a loading on a factor representing analytic ability. Similarly, Number Series, Figure Classification, and False Premises had loadings on R and C_2 . What differentiates C_3 from C_2 seems to be the more highly practiced nature of the symbols involved in the C_3 tasks.

There is a slight negative correlation between the C_3 factor and C_1 . This may be due to C_1 tests being more pictorial and spatial in nature, or it may be that the C_3 tests require flexibility of closure rather than speed of closure. Most of the tests do seem to require flexibility of closure; but in Botzum's study, Four-Letter Words and Scrambled Words had loadings of

TABLE 5
Final Transformation Matrix, A

	A	B	C	D	E	F	G	H
I	.266	.170	.215	.113	.166	.123	.112	.097
II	-.389	.359	.001	-.196	.302	.065	.104	-.227
III	.171	.028	-.447	.121	.053	-.076	.336	-.302
IV	.529	-.505	.316	-.397	.044	.167	-.159	-.543
V	-.355	-.477	.061	.430	.535	-.228	.198	.182
VI	.145	.157	.427	-.559	.109	-.809	.219	.103
VII	-.219	-.070	-.208	.180	.413	-.093	-.834	-.396
VIII	.523	-.574	-.652	-.497	.639	.481	-.237	.594

TABLE 6
Oblique Factor Matrix, $V = FA$

Test	A	B	C	D	E	F	G	H
1. Figures	.03	-.08	.02	-.07	.64	-.02	-.08	-.02
2. Cards	-.06	.08	-.03	.07	.56	.03	.07	.04
3. Gestalt Completion	.02	-.04	.17	-.04	.30	.27	.07	-.09
4. Mutilated Words	.38	.06	-.01	.02	-.02	.43	.07	.07
5. Concealed Figures	-.07	.53	.21	-.05	.12	.05	.01	-.06
6. Designs	.05	.26	.41	.03	.01	-.05	-.01	-.08
7. First Letter	.02	.00	.03	.45	.07	-.06	-.07	.10
8. Suffixes	.03	-.03	-.02	.53	-.09	.04	.03	-.03
9. Writing Phrase	.06	-.03	.02	.06	-.04	.02	.04	.64
10. X's	.01	.06	-.06	-.03	.20	-.08	-.29	.25
11. Identical Forms	-.04	.01	.44	.05	.24	-.02	.09	.01
12. Identical Numbers	.32	-.05	.41	-.02	-.12	.05	.01	.02
13. Number Series	.19	.27	.03	.02	.01	.00	.36	-.06
14. Letter Series	.33	.02	.23	-.01	.07	-.06	.46	.07
15. Figure Classification	.03	.39	.03	.03	.05	-.04	.38	-.04
16. False Premises	.04	.39	-.07	.01	-.10	.00	.33	.06
17. Hidden Words	.34	-.03	.46	-.03	.03	-.08	.01	.07
18. Incomplete Words	.48	.03	-.10	.06	-.03	.25	.05	.05
19. Anagrams	.54	-.08	.06	-.02	.01	.07	.01	-.01
20. Scrambled Words	.52	-.04	.05	.05	.02	-.04	-.04	-.18
21. Copying	.00	.41	.04	.01	.06	.24	-.01	.07
22. Four-Letter Words	.36	.06	.26	-.06	-.02	.08	-.08	.13
23. Backward Writing	.15	.01	.21	.21	.11	.05	-.02	-.05
24. Hidden Pictures	.02	.02	.06	-.05	.11	.39	-.22	-.03
25. Sentences	-.08	.01	.06	.25	.11	.01	.23	.42

.34 and .26 on C_1 , whereas in this study their loadings on C_1 are negligible. Due to this ambiguity the factor has merely been named *Verbal Closure*, although a more accurate descriptive phrase might be "flexibility of closure, utilizing highly practiced symbols."

2. *Flexibility of Closure, C_2*

Test	Loadings on B	Components of C_2
5. Concealed Figures	.53	.79
21. Copying	.41	.61
15. Figure Classification	.39	.58
16. False Premises	.39	.58
13. Number Series	.27	.40
6. Designs	.26	.39

This factor seems to correspond to Thurstone's flexibility of closure, C_2 (17). Three of the tests are ones which defined this factor in the mechanical aptitude study (19), namely Copying, Designs, and Concealed Figures (a

TABLE 7
Reference Vector Cosines, $C = A'A$

	A	B	C	D	E	F	G	H
A	1.000	-.450	-.108	-.616	.028	.318	-.017	.123
B	-.450	1.000	.291	.132	-.518	-.331	.280	-.183
C	-.108	.291	1.000	-.082	-.397	-.541	.257	-.265
D	-.616	.132	-.082	1.000	-.125	.025	.027	-.111
E	.028	-.518	-.397	-.125	1.000	.103	-.304	.232
F	.318	-.331	-.541	.025	.103	1.000	-.291	.127
G	-.017	.280	.257	.027	-.304	-.291	1.000	.221
H	.123	-.183	-.265	-.111	.232	.127	.221	1.000

longer form of Gottschaldt Figures). In addition to these three tests we find three reasoning tests with significant loadings.

The C_2 factor is interesting, since it cuts across the content of the test material. What seems to be required in all these tests is "freedom from *Gestaltbindung*," the Gestalt being formed either by the objective stimulus or by the mental set adopted by the subject. In Concealed Figures one must break the Gestalt formed by the large figure in order to find the small figure. One must also ignore the optical illusions which are often created by the large figure, distorting the appearance of the small figure.

It has been suggested that Copying has a loading on C_2 because the subject must not be influenced by the regularly spaced dots in reproducing the given figure (19). This hypothesis could be investigated by giving the test without the dotted background, as well as with a more highly distracting background. The former method of presentation should decrease its loading on C_2 , the latter increase it. However, it is possible that this test would have a high loading on C_2 , whatever the method of presentation. The C_2 factor seems to represent the ability to abstract. According to Goldstein and

TABLE 8
Correlations between Primary Factors, $R = D(A'A)^{-1}D$

	C_3	C_2	P	W	S_1	C_1	R	H
C_3	1.000	.538	.078	.704	.348	-.269	-.182	.052
C_2	.538	1.000	.117	.333	.540	.092	-.175	.095
P	.078	.117	1.000	.169	.353	.465	-.099	.204
W	.704	.333	.169	1.000	.304	-.180	-.155	.120
S_1	.348	.540	.353	.304	1.000	.190	.158	-.134
C_1	-.269	.092	.465	-.180	.190	1.000	.241	-.068
R	-.182	-.175	-.099	-.155	.158	.241	1.000	-.376
H	.052	.095	.204	.120	-.134	-.068	-.376	1.000

TABLE 9
Diagonal Entries of D_{ip}

	C_3	C_2	P	W	S_1	C_1	R	H
d	.594	.673	.740	.688	.706	.743	.834	.877

Scheerer (7) a characteristic of abstract behavior is "to grasp the essential of a given whole; to break up a given whole into parts, to isolate and to synthesize them." This seems a very good description of what one must do in Copying.

In Figure Classification the subject must not be bound by his first hypothesis concerning the classification of the figures into two groups, nor persevere with previous solutions inapplicable to subsequent items. In Number Series the subject must also test one hypothesis after another, applying the four arithmetical operations of addition, subtraction, multiplication, and division. In False Premises the subject must not be influenced by the surface absurdity of the statements. "A monkey hanging from the branch of a tree is a fruit," sounds a ridiculous assertion, but in terms of the given premises the conclusion is to be marked "True."

The Designs Test has a relatively low loading on this factor, its highest loading being on perceptual speed, P. Designs was devised as a perceptual speed test but in the first study in which it was used (15) it did not have a loading on P. Thurstone later hypothesized that Designs would have loading on C_2 , and this was borne out in the mechanical aptitude study (19). However, it now appears that for a more highly educated group of subjects the test is one which requires perceptual speed to a greater extent than flexibility of closure.

The two reasoning tests which have their highest loadings on C_2 are the two tests which had the highest loadings on Botzum's deduction factor. This tends to corroborate Thurstone's hypothesis that C_2 is associated with deduc-

TABLE 10
Direction Cosines of Primary Vectors, $T = DA^{-1}$

	I	II	III	IV	V	VI	VII	VIII
C_3	.741	-.480	.350	.078	-.219	.147	.131	.074
C_2	.711	.279	.112	-.386	-.452	.110	.187	-.060
P	.552	.092	-.678	.331	.202	.033	-.124	-.248
W	.647	-.504	.229	-.194	.256	-.249	.219	-.249
S_1	.717	.473	.129	.037	.339	.141	.259	.207
C_1	.243	.494	-.354	.227	-.098	-.634	-.312	.109
R	-.010	.380	.492	.122	.334	-.026	-.695	-.051
H	.144	-.386	-.581	-.579	.031	.127	-.201	.316

tive reasoning (19). The writer prefers to differentiate the reasoning factors in terms of the methods of problem-solving involved, rather than in the epistemological terms of induction and deduction. Yela (20) maintains that the solution to reasoning problems can occur either "by closure of the unfinished configuration, perceiving the lacking element as connected with the others, or by analytically reasoning out the principle connecting the items." This is not a new idea. Duncker (5) differentiates "analytic and synthetic evidence," which, he contends, are not opposites. Synthetic evidence can occur because "not all possible aspects of a thought object are necessary to its construction, just as little as all possible aspects of a visual object are necessary to the unambiguous comprehension of its structure." Duncker maintains, however, that the inspectional (synthetic) technique can become a handicap in problem solving, as the thought material is then more thoroughly imbued with perceptual functions, which may be a hindrance in restructuring problems. This observation is interesting in the light of the slight negative correlation of both C_2 and C_3 with synthetic reasoning, R.

TABLE 11
Test Vectors as Linear Combinations of Primary Vectors, $A = VD^{-1}$

	C_3	C_2	P	W	S_1	C_1	R	H
1. Figures	.05	-.12	.03	-.10	.91	-.03	-.10	-.02
2. Cards	-.10	.12	-.04	.10	.79	.04	.08	.05
3. Gestalt Completion	.03	-.06	.23	-.06	.43	.36	.08	-.10
4. Mutilated Words	.64	.09	-.01	.03	-.03	.58	.08	.08
5. Concealed Figures	.12	.79	.28	-.07	.17	.07	.01	-.07
6. Designs	.08	.39	.55	.04	.01	-.07	-.01	-.09
7. First Letter	.03	.00	.04	.65	.10	-.08	-.08	.11
8. Suffixes	.05	-.04	-.03	.77	-.13	.05	.04	-.03
9. Writing Phrase	.10	-.04	.03	.09	-.06	.03	.05	.73
10. X's	.02	.09	-.08	-.04	.28	-.11	-.35	.29
11. Identical Forms	-.07	.01	.59	.07	.34	-.03	.11	.01
12. Identical Numbers	.54	-.07	.55	-.03	-.17	.07	.01	.02
13. Number Series	.32	.40	.04	.03	.01	.00	.43	-.07
14. Letter Series	.56	.03	.31	-.01	.10	-.08	.55	.08
15. Figure Classification	.05	.58	.04	.04	.07	-.05	.46	-.05
16. False Premises	.07	.58	-.09	.01	-.14	.00	.40	.07
17. Hidden Words	.57	-.04	.62	-.04	.04	-.11	.01	.08
18. Incomplete Words	.81	.04	-.14	.09	-.04	.34	.06	.06
19. Anagrams	.91	-.12	.08	-.03	.01	.09	.01	-.01
20. Scrambled Words	.88	-.06	.07	.07	.03	-.05	-.05	-.21
21. Copying	.00	.61	.05	.01	.09	.32	-.01	.08
22. Four-Letter Words	.61	.09	.35	-.09	-.03	.11	-.10	.15
23. Backward Writing	.25	.01	.28	.31	.16	.07	-.02	-.06
24. Hidden Pictures	.03	.03	.08	-.07	.16	.52	-.26	-.03
25. Sentences	-.13	.01	.08	.36	.16	.01	.28	.48

Burt (3), in describing two types of problem-solvers, writes: “. . . the characteristic procedure of the former group is explicit or analytic reasoning, depending on a succession of logical steps; of the latter, an implicit or synthetic apperception of what later psychologists have called Gestalten or configurations, depending on an intuitive insight which embraces the component aspects almost simultaneously.”

It would seem that the C_2 factor is associated with the more analytical method of problem-solving, whereas factor R may represent the synthetic ability discussed by Burt, Duncker, and Yela, among others.

The C_2 factor seems to correspond closely with Meili's plasticity factor, where the Gottschaldt Figures, Kohs' Blocks, problems taken from Duncker, and Number Series all have significant loadings (10). It is also similar to Factor A in Rimoldi's study (11).

3. *Perceptual Speed, P*

Test	Loadings on C	Components of P
17. Hidden Words	.46	.62
11. Identical Forms	.44	.59
12. Identical Numbers	.41	.55
6. Designs	.41	.55
22. Four-Letter Words	.26	.35

This factor corresponds with Thurstone's perceptual speed factor, P (14, 15). The test with the highest loading on this factor is Hidden Words. This test was constructed in an effort to obtain a verbal equivalent of the Concealed Figures test. At the time of construction the writer hypothesized that the test might have its chief loading on perceptual speed, since it is difficult to write sentences with such strong Gestalt properties that one's perception of the embedded word is hindered. It was, in fact, gratifying to find that Hidden Words has almost equal components of C_2 and P.

The tests high on this factor require the subjects to find a given stimulus among distracters, which do not have strong Gestalt properties that distort or conceal the desired stimulus.

4. *Word Fluency, W*

Test	Loadings on D	Components of W
8. Suffixes	.53	.77
7. First Letter	.45	.65
25. Sentences	.25	.36

The three tests with loadings on this factor define one of the most clear-cut hyperplanes in the simple structure. The tests Suffixes and First Letter were introduced into the present battery as key tests for the word fluency factor, W. Sentences has an obvious "fluency-with-words" component, although it does not require the subject to think of isolated words fitting some restriction, which is how word fluency has been interpreted previously (14).

The test Sentences was introduced because Meili (10), using a similar test and a population of children, had obtained a loading on the globalization factor. In our case the highest loading for this test was on speed of handwriting, H. The test did have a small loading on R, which seems similar to Meili's globalization factor. For adults, well versed in using words, the ability to induce something common to the given words does not seem to be as important as in the case of children.

5. *Space, S₁*

Tests	Loadings on E	Components of S
1. Figures	.64	.91
2. Cards	.56	.79
3. Gestalt Completion	.30	.43
11. Identical Forms	.24	.34
10. X's	.20	.28

The highest loadings on this factor are the two key tests for the first space factor, *S₁*. Thurstone interprets this factor as "the ability to visualize

TABLE 12
Correlations between the Tests and Primary Factors, $R_{jp} = FT'$

	<i>C₃</i>	<i>C₂</i>	<i>P</i>	<i>W</i>	<i>S₁</i>	<i>C₁</i>	<i>R</i>	<i>H</i>
1. Figures	.24	.37	.31	.18	.81	.13	.07	-.12
2. Cards	.30	.53	.30	.31	.87	.21	.19	-.08
3. Gestalt Completion	.02	.22	.51	.03	.57	.58	.27	-.18
4. Mutilated Words	.53	.47	.32	.40	.36	.41	.05	.05
5. Concealed Figures	.30	.81	.43	.18	.65	.36	-.04	.02
6. Designs	.38	.50	.57	.33	.46	.21	-.13	.07
7. First Letter	.58	.32	.19	.76	.28	-.19	-.25	.23
8. Suffixes	.49	.16	.07	.72	.11	-.12	-.07	.04
9. Writing Phrase	.13	.06	.18	.19	-.10	-.05	-.25	.73
10. X's	.24	.31	.07	.18	.19	-.18	-.44	.37
11. Identical Forms	.16	.25	.71	.24	.58	.34	.08	.06
12. Identical Numbers	.44	.19	.56	.35	.18	.16	-.13	.16
13. Number Series	.49	.52	.08	.33	.45	.08	.33	-.17
14. Letter Series	.55	.32	.32	.42	.48	.07	.38	-.04
15. Figure Classification	.33	.56	.06	.22	.48	.13	.36	-.15
16. False Premises	.25	.46	-.09	.13	.22	.06	.25	-.03
17. Hidden Words	.60	.33	.64	.48	.38	.05	-.18	.22
18. Incomplete Words	.76	.49	.11	.57	.31	.06	-.04	.05
19. Anagrams	.81	.39	.18	.57	.31	-.11	-.10	.02
20. Scrambled Words	.87	.41	.09	.64	.34	-.25	-.13	-.13
21. Copying	.29	.70	.33	.21	.48	.41	-.06	.12
22. Four-Letter Words	.60	.45	.47	.43	.31	.09	-.28	.28
23. Backward Writing	.54	.38	.44	.57	.47	.11	-.09	.04
24. Hidden Pictures	-.04	.20	.38	-.04	.24	.53	-.11	.02
25. Sentences	.15	.15	.26	.34	.22	.08	.08	.42

a rigid configuration when it is moved into different positions" (19). This ability would certainly be called for by the tests Figures and Cards. In the case of Identical Forms and Gestalt Completion no movement from one position to another occurs. However, Identical Forms has consistently had a small loading on S_1 (1, 2, 14). The loading of Gestalt Completion on this factor has not occurred previously. It may be that in devising new forms of this test the factorial composition has been somewhat changed.

The small loading of X 's on S_1 may not have occurred by chance. Chapman (4) in analyzing the MacQuarrie test found a correlation of .68 between the factors he identifies as spatial and motor. The highest loading on the spatial factor was given by the test Dotting, which requires skill similar to that used in making X 's rapidly.

6. *Speed of Closure, C_1*

Tests	Loadings on F	Components of C_1
4. Mutilated Words	.43	.58
24. Hidden Pictures	.39	.52
3. Gestalt Completion	.27	.36
18. Incomplete Words	.25	.34
21. Copying	.24	.32

This is evidently the factor identified as speed of closure in previous studies (2, 19). Mutilated Words and Gestalt Completion were the two tests which defined this factor before, and Hidden Pictures and Incomplete Words had loadings on it. This factor is represented by tests in which the subject must unify a relatively unstructured perceptual field.

Scrambled Words and Four-Letter Words, which had small loadings on this factor in Botzum's study, do not appear on our C_1 factor. The difference between these two tests and Mutilated Words and Incomplete Words seems to be that in the latter no rearrangement of parts must take place, as in Scrambled Words, or extraction from a background, as in Four-Letter Words.

TABLE 13
Centroid Factor Matrix, F_2

	I	II	III	IV	V	h^2
C_3	.80	.15	-.43	.01	-.15	.87
C_2	.65	.28	.17	-.36	-.31	.76
P	.30	.31	.50	.40	.14	.62
W	.69	.12	-.34	.23	.08	.67
S_1	.45	.61	.09	-.27	.24	.71
C_1	-.17	.54	.50	.23	-.06	.63
R	-.41	.50	-.20	-.06	.11	.47
H	.32	-.40	.31	.17	.05	.39

TABLE 14
Fifth-Factor Residuals in Second Order

	C ₃	C ₂	P	W	S ₁	C ₁	R	H
C ₃		.00	.03	-.01	-.02	.00	.01	-.01
C ₂	.00		-.06	.01	.03	.02	-.01	.03
P	.03	-.06		-.01	.06	.02	-.03	-.02
W	-.01	.01	-.01		-.01	.00	.00	.02
S ₁	-.02	.03	.06	-.01		-.03	.01	-.02
C ₁	.00	.02	.02	.00	-.03		.02	.00
R	.01	-.01	-.03	.00	.01	.02		.01
H	-.01	.03	-.02	.02	-.02	.00	.01	

One might think that Hidden Pictures would require flexibility of closure rather than speed of closure, but this test has consistently had loadings on C₁. It seems that it is not breaking the Gestalt formed by the large picture, but the sudden combination of the correct elements, which accounts for individual differences in Hidden Pictures. In Concealed Figures and Designs the subject is shown the figure which he is to locate and can therefore perform the task analytically, whereas in Hidden Pictures he merely knows that he is looking for pictures of people. Reproductions of the faces to be found could be given preceding each large picture, to find out whether the test would then have a loading on C₂.

7. Reasoning, R

Tests	Loadings on C ₁	Components of R
14. Letter Series	.46	.55
15. Figure Classification	.38	.46
13. Number Series	.36	.43
16. False Premises	.33	.40
25. Sentences	.23	.28
24. Hidden Pictures	-.22	-.26
10. X's	-.29	-.35

The three tests with highest loadings on this factor are all classic tests of induction. As previously stated the writer prefers to differentiate reasoning factors in terms of the mental processes involved, rather than in terms of the types of problems involved. Figure Classification, Number Series, and False Premises each have higher components of the C₂ factor, and Letter Series a higher component of C₃, than they have of R. The abilities represented by C₂ and C₃ were interpreted as being analytical in nature. It may be that the synthetic apperception of the solution may be represented in the present factor. Letter Series would appear to be the test which lends itself most easily to this form of solution. Some subjects perform this test by singing the series sub-vocally, which is a method of solution that depends on

TABLE 15
Oblique Factor Matrix, V_2

	AA	BB	CC	DD	EE*
C_1	.15	.03	-.05	.57	-.13
C_2	.65	.00	.02	-.02	-.21
P	-.01	-.07	.61	.15	.09
W	-.05	-.01	.07	.59	.07
S_1	.55	.37	.02	.01	.25
C_1	.03	.30	.62	-.04	-.13
R	-.04	.65	.03	.03	.00
H	-.02	-.59	.09	-.05	.13

*Residual

perceiving the series as a Gestalt, rather than analyzing the principle upon which it was built. It has already been mentioned that some of the variance of Sentences is accounted for by the ability to induce the common elements which permit the given words to be combined into a meaningful sentence.

Our interpretation of this factor as reasoning "by closure of the unfinished configuration," to use Yela's words (20), seems to be justified by the inter-correlations of this factor with the others, since its highest positive correlation is with speed of closure (.24). This gives some corroborative evidence for

TABLE 16
Transformation Matrix, A_2

	AA	BB	CC	DD	EE
I	.34	-.33	.02	.33	.09
II	.22	.86	.41	.23	-.12
III	.33	-.39	.43	-.56	.08
IV	-.86	-.10	.75	.70	-.14
V	.00	.00	-.30	-.17	.98

TABLE 17
Reference Vector Cosines, C_1

	AA	BB	CC	DD	EE
AA	1.01	.03	-.41	-.62	.15
BB	.03	1.01	.10	.24	-.15
CC	-.41	.10	1.01	.44	-.41
DD	-.62	.24	.44	.99	-.31
EE	.15	-.15	-.41	-.31	1.01

Thurstone's supposition that "the first closure factor might be associated with inductive thinking," although the association is not as strong as the association of the second closure factor with analytical thinking.

This is the only factor in which there are negative loadings to account for. X's was the last test in the battery, and the subjects were aware of this. The brighter subjects may have been bored by this simple task, whereas the less apt subjects may have felt this was "positively their last chance" to make a good score. The negative loading of Hidden Pictures may also be due to motivational factors.

Factor R is similar to Factor G in Rimoldi's study (11), on which Figure Classification, Letter Series, and Number Series all had loadings. Referring to the tests with loadings on G, Rimoldi writes: ". . . In the easy items the analytical activity is limited and the grouping of the letters and numbers is rather obvious. As soon as the problems increase in difficulty the grouping becomes less obvious, and the subject has to discover the structure characterizing each item." It is probably the easier items which account for the loadings of these tests on R, but further research is needed to verify this.

8. *Speed of Handwriting, H*

Tests	Loadings on H (Reference Vector)	Components of H (Factor)
9. Writing Phrase	.64	.73
25. Sentences	.42	.48
10. X's	.25	.29

This seems to be the same factor as Bechtoldt's (1) factor H, which he identifies as "the ability to operate hand-finger neuromuscular response mechanisms at a high rate of speed for short periods of time." As we only have tests involving handwriting represented here, we have identified the factor by the simpler term "speed of handwriting." It was gratifying to find that this factor had only one positive correlation of any size (.20), this being with perceptual speed. Individual differences in ability to write quickly therefore did not appear to affect the scores of most of the paper-and-pencil tests used in this battery.

VII. *Interpretation of the Second-Order Factors*

With only eight primary factors it is not possible to determine the second-order factors with much confidence. The interpretation of these factors is therefore highly tentative.

Factor AA

Flexibility of Closure	.65
Space	.55

It must be remembered that our flexibility of closure factor had three reasoning tests with high loadings on it. Bearing this in mind, it becomes apparent that this factor is very similar to the second-order factor α , isolated by Botzum, on which space, deduction, induction, and flexibility of closure all had loadings over .60.

In the mechanical aptitude study, although a second-order analysis was not performed, it seems likely that inductive reasoning, space, and flexibility of closure would come up on one second-order factor, since I and C_2 correlate .63; I and S_1 , .38; and S_1 and C_2 , .53.

The space primary has loadings on both this factor and factor BB. This also occurred in Botzum's study. He points out that one can perform the space tests in two ways. One can either analytically reason out which way the figure would be facing, when turned in the plane of the paper; or one can actually imagine the rotation of the figure. It is our hypothesis that the analytic procedure in problem-solving is what is represented in Factor AA.

Factor BB

Reasoning	.65
Space	.37
Speed of Closure	.30
Speed of Handwriting	- .59

This factor seems to represent the ability to solve problems synthetically. We tentatively interpreted our reasoning factor R as the ability to synthesize, and above we mentioned the possibility of solving the space tests by two different methods, one analytic, the other synthetic. The synthetic element in speed of closure is obvious. Both Jones (8) and Rimoldi (11) have reported second-order factors which they interpret as synthetic ability.

The negative loading of speed of handwriting on this factor is difficult to account for. In Bechtoldt's study speed of handwriting correlated $-.22$ with Factor G, which seems to correspond to our C_1 factor. Future studies will establish whether this negative relationship occurs consistently.

Factor CC

Speed of Closure	.62
Perceptual Speed	.61

This factor seems to be speed of perception. It is similar to factor F found in the factorial study of perception (17). It is not a more generalized speed factor, since speed of handwriting does not have a loading on it. Similarly, Thurstone's F was clearly differentiated from reaction time. Dispersed attention seems to be required in both the speed of closure and perceptual speed tests.

Factor DD

Word Fluency	.59
Verbal Closure	.57

This factor is interpreted as the ability to think of words rapidly which fit certain formal requirements, such as beginning with a certain letter, ending with a certain suffix, containing certain given letters, etc. If our interpretation of the verbal closure factor was correct, one would expect that C_3 would have a loading on the second-order factor AA as well as DD. However, its loading on AA is only .15. Due to the indeterminacy of the second-order structure, this does not necessarily invalidate our interpretation of C_3 . Furthermore, the cosine of the angle between AA and DD is $-.62$; therefore a positive correlation between the second-order primaries corresponding to AA and DD could be predicted. This is probably due to the flexibility required by all the tests with loadings on these two second-order factors.

Factor EE has been left without interpretation as it appears to be a residual factor.

VIII. Discussion

The aim of this study was to ascertain whether abilities on speed of closure and flexibility of closure tests would generalize to tasks requiring higher cognitive functions. It yielded strong evidence that flexibility of closure is associated with analytical reasoning. The association is so close that only one factor appears on which all the flexibility of closure tests and three of the reasoning tests have loadings. Correlated .54 with this factor is one in which rearrangement of letters or numbers to form new Gestalts, or the picking out of Gestalts from a distracting background, is required in all the tests. This indicates that there is a connection between perceptual flexibility, the flexibility required to solve analytical reasoning problems, and the flexibility needed to solve problems utilizing highly practiced symbols but where meaning is not important.

The evidence for the generalization of speed of closure in the higher cognitive domain is not so clear-cut as for flexibility of closure. It is only in the second order that we find a coming together of the speed of closure and reasoning tests, but this second-order factor does seem to indicate that both perceptual and reasoning problems can be solved by a rapid synthetic process.

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