# STUDY OF SOME FACTORS RELATED TO INTELLIGENCE* 

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#### Abstract

From the factorial analysis of a battery of nineteen individual tests seven factors were obtained, six of them being interpreted. Some of these factors are probably related to similar ones isolated in different batteries and by different authors. The test Progressive Matrices here analyzed is loaded in some of the factors, as was expected according to the nature of its problems and to previous experience with the test. The loadings of the tests and of the factors in the second-order factors are given. It is expected that this extension of factor theory will prove useful for the isolation of more fundamental psychological parameters, although at present it is difficult to evaluate precisely the theoretical and practical implications of this aspect of factor analysis.


## Introduction

The present paper deals with the factorial study of a battery of individual tests, among which were included several of the performance type, Raven's Progressive Matrices and some others. None of the tests, as far as inspection can determine, has any verbal component, and it is expected that such a battery will give information about the space and perceptual factors. Raven's Progressive Matrices, insofar as previous experience with the test seems to indicate, appears saturated in these same factors, and consequently its inclusion in the present battery may give clues about its factorial complexity.

Since the basic works of Thurstone (21, 18), the space factor has been found in different batteries of the usual paper-and-pencil type. The suggestion has been advanced that there are several kinds of space factors or space-perception factors (5). The Minnesota form boards have been reported as saturated only in space (4). Moreover, the space factor has become of considerable interest, not only from the theoretical point of view but also as a component of the so-called mechanical ability, in the sense that it has been found repeatedly in studies of this kind (4, 5, 24, 23).

Perceptual factors have been discovered in most of the studies

[^0]mentioned above, and their nature seems to be rather complex and their relation with the space factor somewhat complicated. The development of tests of spatial and perceptual abilities has been strongly recommended for the purposes of occupational analysis (17).

Performance tests have not been employed for factorial studies as frequently as the usual verbal paper-and-pencil type. F. Gaw (2) says that "Performance tests as above mentioned should be considered as essentially measurements of intelligence," where the manual response "is but a means to an end." Since the materials commonly employed for testing intelligence are usually strongly dependent on scholastic achievement, information, the use of words, and so on, it would be interesting to know whether the same fundamental activities which play a part in the performance of paper-and-pencil tests are found in a battery of this kind, where these influences are apparently absent.

Moreover, group testing techniques have led psychologists to stress the importance of the end results of the activity called forth in testing. Few of the common paper-and-pencil tests give much information about the way in which psychological activity develops during the actual performance of the problems. Therefore the conclusions are based more in terms of the final results than in terms of the evolution of the process.

It is known that the same end results can be obtained by different means. Each individual has a personal way of apprehending the problems presented to him and solves them accordingly. How are these different ways of approach effective for the solution of the problem? What is the rigidity or plasticity that a person possesses enabling him to use different patterns of action? To what extent can the knowledge of these facts be employed for the improvement of the individual? These and similar questions are of theoretical and practical importance, and it is expected that careful observation of each subject during the performance of the tests will contribute to their understanding.

Raven's Progressive Matrices have been considered by Raven and Spearman as loaded in " $g$ " (16). The analysis of the items and the reports of the subjects seem to indicate that the test was devised according to noegenetic principles. But the question is: Can its variance be attributable to " $g$ " only? Previous experience with over 2000 cases $(12,13)$ seemingly reveals that different mental functions are active in the solution of the items. Moreover, the results obtained by means of the Raven tests are indicative of the fact that these have good discriminative power, can be used in a wide age range, and fulfill most of the requirements needed for testing normal and handi-
capped subjects (3, 11, 12, 13). Therefore the knowledge of its factorial composition is of theoretical and practical value.

Furthermore, the study of the second-order domain $(12,20)$ may contribute to the understanding of many of the problems here underlined, since it appears to refer to more fundamental components.

## Description of Tests, Population, and Method

The Seguin-Goddard form board (No. 1) ;* the Healy Construction Test A (No. 2) ; the Diagonal test (No. 12) ; and the Triangle test (No. 13) are all scored in time, following the indications given by F. Gaw (2). In test 1 the score is the time taken in the shortest of three trials. Tests 2,12 , and 13 consist of rectangular wooden frames and a certain number of wooden blocks to fit in them. Test 2 has six rectangular blocks; Test 12 has five wooden blocks, three triangular, one rectangular, and one trapezoidal; and test 13 has a square and a rectangular frame, both cut in the same wooden board where four identical triangles have to be fitted exactly. Each test has a time limit and the score is the number of seconds that the subjects employ for the solution of the test.

In the Cube Imitation test, No. 11 (2), the experimenter places in a row in front of the subject four black cubes, all of the same size; then with a fifth cube he taps them according to a special pattern of varying difficulty, which the subject must then repeat. There are twelve different patterns, and the score is the number correctly reproduced. If the subject fails in three successive trials, the test is terminated.

The Porteus Maze test (No. 8) has been scored and administered according to the instructions given by F. Gaw (2).

The Object Assembly test was scored as three separate tests: the Mannequin (No. 3), the Face (No. 4), and the Hand (No. 5). The number of pieces placed correctly within a certain time limit gives the score. For more details, the reader is referred to $D$. Wechsler (22).

Cancellation of letters, test 9 , is similar to the current tests of the same name.

In Cancellation of Figures, test 10 , the subject has to cancel all the squares which have an extra line placed inside and in the same position as the one in the square drawn at the top of the page.

Digits forward, test 6, and Digits backward, test 7, are similar to the current tests of the same name.

Drawing of a man, test 19 , was scored according to the instruc-

[^1]tions given by C. Burt (1). Three different judges scored each subject. Tests 8 and 9 are both scored in terms of mental age, showing a very close agreement in the results. The means and standard deviations for test 19 and test 8 are, respectively: means 9.406 and 9.554 , with standard deviations of 1.564 and 1.681 . These values are not statistically different.

Raven's Progressive Matrices-Sets A, B, C, D, and E-were scored as separate tests: Nos. 14, 15, 16, 17, and 18, respectively. The whole series consists of 60 patterns in which a part is missing, and at the bottom of each pattern there are six or eight different parts that fit in the missing part, only one of which the subject has to select to complete it. There is no time limit and the score is the number of items answered correctly. In the present population the mean and standard deviation are 27.48 and 9.29.

In the present study all the tests were given individually in different sessions, none of them lasting for more than an hour. The whole period of testing was completed in no more than three sessions, each session in different days.

The age of the 138 subjects of both sexes ranges between eight and fifteen years, with a mean age of 11 years 4 months, and a standard deviation of 1.860 . The subjects were randomly selected from different schools.

The correlations between the variables are given in Table 1. The centroid matrix in Table 2 was obtained after three successive factorizations, that is, to the point where the communalities were stable. The residuals were negligible after seven factors were removed. Their root mean square deviation has a value of .023 .

The centroid matrix was rotated by different methods according to the development of the problem. In all, 23 radial rotations were made, plus several trials with the single plane method and other devices. The final matrix of transformation is given in Table 3. Table 4 is the rotated factorial matrix, and Table 5 gives the cosines of the angles between the reference vectors.

As was expected from the inspection of the correlations (see below), the tests cluster along the reference axes after a few rotations, although a satisfactory positive manifold was secured only after several rotations.

The factorization of the correlations between the primaries, Table 6, led to the expression of the primary factors in terms of three centroids. This factorization was repeated several times in order to fix the communalities. The results are shown in Table 7. From here it is easy to express the loadings of the tests in the centroids of the second-order domain. These values are given in Table 8.

Tables 9 and 10 express the matrix of transformation (obtained by plotting directly on the sphere the normalized values of the factors and the tests in the second-order centroids) and the cosines between second-order reference vectors. Tables 11 and 12 give the saturations of the primary factors and of the tests in the new reference vectors of the second-order domain.

## Discussion and Results

The inspection of the table of correlations shows several interesting facts. There is quite a wide range in the magnitudes of the coefficients and a number of them are negative, mainly, those corresponding to variables $1,12,13$, and 2 , although these four variables are positively correlated among themselves.*

All these four tests were scored in time, and consequently better performance means less time. Reversing their signs, only fifteen negative correlations remain for the whole table of 171 coefficients. Only one of these negative correlations may be considered as significant at a 5 per cent level of confidence. The other ones have no statistical significance. In batteries of this kind, negative correlations have been reported by different authors. From this analysis one may conclude that probably one of the parameters will refer to speed and will involve the four tests mentioned above.

The five sets of the Raven test are highly correlated among themselves, and in general, have higher correlations than the other tests with the rest of the variables. In consequence, it is expected that their variance will be split into several different factors, one of them common to all and the others common with several other tests of the battery. As stated in the introduction, this was strongly suspected from a simple analysis of the tests and a rather extensive experience with it.

The three variables of the Object Assembly test are not highly correlated among themselves. Only test 4 shows somewhat larger correlations with the rest of the variables, mainly with the sets of the Raven test.

Tests 9,10 , and 7 have very low correlation with the rest of the variables; and the fact that tests 9 and 10 have a very low correlation between themselves is contrary to expectations, a point that may be further clarified in terms of the factorial composition of the tests.

Comparison of the loadings of the tests in " $g$," using Spearman formula $\# 21$ (16), and their saturation in the first centroid shows a very close agreement. Nevertheless the residuals after this factor has been taken out are still quite large.

[^2]FACTOR B
14. Raven A ..... 50
4. Face ..... 45
2. Healy A ..... 41
13. Triangle ..... 34
8. Porteus ..... 33
15. Raven B .....  31
19. Drawing of a Man .....  30
17. Raven D .....  28
12. Diagonal ..... 23
16. Raven $\mathbf{C}$ ..... 23
(Here as in the other factors we shall list all loadings above .20)
The fact that Raven A and tests 2, 4, and 13 have high loadings in this factor seems to indicate that it deals with the perception of spatial relations. The impossibility of perceiving the relations between the different blocks is a cause of frequent failure in tests 2, 12, and 13. The subjects usually report that they do not know how the different blocks go together, although they perceive their shapes correctly. The subjects usually inspect the blocks carefully, handling them in different ways, following their contours with their fingers and trying to work partial solutions outside the corresponding frames. In spite of this exploration, they fail to bring them together in their correct relations.

Many subjects change from a systematically planned way of attacking the problem to a random activity; others seem to go through the problems by means of a methodical exploration of the parts; and still others, who begin to explore the parts in a random way, reach a certain moment in which the problem clarifies itself (insight?), and from there on go smoothly to the solution. The experience of seeing into the problem is accompanied by an evident relaxation of the subject. Some individuals who verbalize their actions seem to indicate by their expression that they are seeking essentially for the relationships between the parts in order to obtain the required results.

In test 4, the perception that the blocks should make a profile seems to help the subjects, but the position of some of the pieces does not become in any way obvious, for instance those of the ear. In test 8 , it is also clear that the subjects have to perceive the relations between the different walls of the maze to find their way through.

In the drawing of a man, success depends not only upon the number of the parts drawn correctly but also upon the relations of these parts. The older children draw the parts better and relate them more logically. The whole problem, at least in so-called normal children, is pervaded by the influence of better articulation of the parts toward obtaining a better "gestalt."

The different sets of the Progressive Matrices, with the exception of $E$, have saturations in this factor. Set $A$ requires a perception of the spatial relations of the different parts. The subject sees a whole which has some part missing. The analysis of the most frequent mistakes seems to indicate that the "figure-ground" relationship in many cases may be characterized as "inverted."

In Set B, most of the figures imply the play of strong gestalt forces. The wrong answers suggest that the subjects do not see the interrelationship of the parts in terms of the total, but merely give as a solution one of the figures adjacent to the missing part in the main pattern. In Sets $C$ and $D$, there are other forces involved and the ones mentioned above are less obvious, although present.

Summarizing, we could say that factor B seems to imply the perception of relations in space necessary for the construction of a whole. Since the tests loaded in factor $B$ are different, this factor probably portrays some fundamental central component of psychological activity.


Test 11 requires the subjects to reproduce a certain temporal and spatial pattern. The subject has in front of him a series of cubes ordered in a row. He has to keep in mind the pattern (factor E ) and reproduce it against the strong configuration given by the linear arrangement of the cubes. It is interesting to observe that some persons do not even attempt to solve the problem in the way they are asked but tap the cubes in succession. That this is not a simple misunderstanding of the order is evidenced by the fact that some of them, after having completed some patterns satisfactorily, fail in the more complex ones-either by reproducing some previous pattern (some of them are strongly preferred by the subjects) or, strangely enough, by tapping the cubes in succession. It seems that the subjects have to resist strong forces which try to destroy the requested "gestalt."

In Cancellation of Figures the subject has a given percept which must be individualized among other somewhat similar percepts. Cancellation of letters represents a different problem, since the materials employed are heavily loaded with other components, although the task is based on an apparently similar performance.

In Digits Backwards the subject has to repeat the numbers in inverse order, therefore making a configuration while keeping in mind
the one previously given. The task is carried on with strong field and figures forces working against its completion.

Apart from Raven B, where a similar sort of activity is required, the remaining tests, other than those listed, are not loaded in factor $C$.

It is obvious that other tests such as 1,19 , and 6 do not require the sort of activity implied by factor C. In tests 2,12 , and 13 , although there is a conflicting force it works in a different way, since the situation deals with the spatial relations needed to build a certain pattern without conflicting with any outside çonfiguration. Factor C, on the other hand, refers to the conflict between two or more configurations, the elements inside them being given. The more plastic the subject, the more he is likely to be "gestalt free" and the better will be the solution.

This factor is probably in close connection with Thurstone's $E$ (19) and Meili's "plasticité" (6), while factor $B$ seems to be related to Thurstone's $A$ (19). New experiments might be devised to further clarify this point, probably throwing light upon the dynamics of perception and the nature of certain "gestalt" forces. Since all the tests in Factor $C$ are rather dissimilar, this factor probably transcends the content of immediate perception or performance.

| $F A C T O R D$ |  |  |
| :---: | :---: | :---: |
| 3 | Mannequin | . 51 |
| 11 | Cube Imitation | . 46 |
| 6 | Digits Forwards | . 34 |
| 5 | Hand | . 28 |
| 8 | Porteus Maze | . 24 |
| 18 | Raven E | . 21 |

The interpretation of this factor will require further confirmation, since it seems to refer to a rather fundamental variable which has not been isolated factorially up to the present.

The problem presented by test 3 was usually solved rather easily. The most frequent mistakes are concerned with the wrong location of the limbs, in the sense that the right and left sides are frequently interchanged. The subject explores both lower and upper limbs, handling them for a certain while until they perceive the side to which they belong. Some others believe that they have solved the problem satisfactorily, but after a while change the position of the limbs to their correct sides. Those who do not solve the problem usually have no difficulty with the head piece, but locate the legs or arms in the non-correspondent side of the body, this being by far the most frequent mistake. Therefore it seems that for the solution of this test the distinction of left and right side is necessary.

In Cube Imitation the subject has to remember a pattern and fight against distracting forces, but he also has to recognize that the different cubes are placed in certain relations of left and right among themselves and with reference to the subject's own body.

In test 6 the same activity may be needed inasmuch as most of the subjects represent the numbers spatially. In test 5 (the Hand) the most frequent mistakes consist of transposing the order of the fingers, especially those which require a finer discrimination of left and right-2nd, 3rd, and 4th fingers.

The same sort of relationship seems to be clear in test 8 where the subject is instructed like this: "Suppose this is the plan of the paths in a garden. These lines are the walls which you can not get over. . . ." It is evident that in such a situation the subject tends to think in terms of right and left turns.

In Raven E a study of the most frequent mistakes indicates that many subjects work the problems going in a left-right direction. It is interesting in this respect that Raven and Miller (7) have reached a similar conclusion.

It is probable that this factor is connected with other space factors, but its significance seems to be different in the sense that the distinction of right and left refers to a fundamental bodily dimension. The factor is probably connected with the body scheme (Schilder). Nielsen (8) says: "It is on the basis of handedness and language that the major and minor sides are differentiated and the disturbances of the body scheme are readily separable into those of the major and those of the minor sides." There are cases of amnesia for the limbs of only one side, and Gerstman's syndrome includes, among other symptoms, confusion of right and left side, not only in the patient's own body but also in that of others. Sometimes the agnosia extends to all animate and inanimate objects.

The existence and significance of this factor needs better testing by means of carefully planned experiments in connection with the integration and disintegration of the body scheme. Probably it would be more easily isolated in younger ages among normals.

## FACTOR E

6. Digits Forwards ................. . 55
7. Digits Backwards ............... . 35
8. Raven E ............................... . 32
9. Cube Imitation .................... . 23

This seems to be a memory factor. Digits Backwards and Digits Forwards have evidently a memory component. Cube Imitation requires memory inasmuch as the retention of the pattern given is nec-
essary for the fulfillment of the task. It is dificult to explain why Set E is loaded in this factor. Nevertheless, this set is much more complex than the others, and it is not unlikely that immediate retention would play an important part. Other tests of this battery do not seem to require memory, in the sense here understood.
FACTOR F
1 Seguin-Goddard .................... . 51
12 Diagonal .................................. . 40
19 Drawing of a Man ................ . 34
13 Triangle ................................... . 32
10 Cancellation of Figures ...... . 31
18 Raven E ................................. . 29
17 Raven D ................................. . 28
5 Hand ............................................ . 26
2 Healy Construction A ........... . 23

Tests 1, 2, 12, and 13, are scored as a function of time. Better solution means shorter time. In Cancellation of Figures the subjects have to perceive a certain given figure while working at the highest possible speed. In Sets E and D of the Progressive Matrices, the speed with which certain details are perceived seems to be important for the solution. In many cases the subjects fail to perceive certain characteristics of the items and the solution is wrong. The problems in these sets seem to be different from the problems in the other sets, although some items of Set C seem to require the same sort of activity. It is our experience that many subjects can not solve these problems, although they devote a long time to them. A quick perception of details is apparently missing, as can be corroborated by going over the test following its completion. Once the perceptual detail has been pointed out to the subject, the solution comes easily.

It is interesting to observe that tests 2,12 , and 13 appear in factor $B$ as well, but with this characteristic: the more they are saturated in $B$ the less saturated in $F$ and, vice versa. In test 2 the perception of relations has a large share in the variance of the test. In test 12 it is usually observed that the more rapid the perception of the trapezoidal piece the easier the solution. In test 13 the problem is facilitated once the identity of the pieces is apprehended. The average times for the solution of these three tests are the following: No. 2, $92^{\prime \prime}$; No. 13, $83^{\prime \prime}$; No. 12, $33^{\prime \prime}$-thus adding further weight to the interpretation that this factor probably deals with speed of perception.

Thurstone (19) has described a factor similar to the one here mentioned. He says that in this factor "the subject has no real per-
ceptual problem," adding later on "but we have no evidence that the factor is quite so simple." This factor seems to agree with what we have expected from the analysis of our correlation table.

> | FACTOR G |  |  |  |
| :--- | :--- | :--- | :--- |
| 1 | Seguin-Goddard | ....................... | .35 |
| 8 | Porteus Mazes ................. | .31 |  |
| 9 | Cancellation of Letters ........ | .30 |  |

It is not advisable to attempt an interpretation of this factor, since it appears in only three tests and has small loadings in all of them.

> FACTOR A

| 17 | Raven 1 | . 68 |
| :---: | :---: | :---: |
| 16 | Raven C | . 55 |
| 15 | Raven B | . 54 |
| 18 | Raven E | . 45 |
| 14 | Raven A | . 34 |

All the other variables apart from the five sets of the Raven test have practically zero loadings in this factor, which therefore appears as common to all the sets of this test and corroborates the analysis of the correlations.

This series of matrices has been considered by its author and by Spearman as highly saturated in " $g$."

The careful analysis of the 60 items will convince the reader that the noegenetic principles have been taken into consideration in its construction. According to Spearman (15), " $g$ " is a combination of both noegenesis and abstraction; and, referring to the test under consideration, he says that it can be solved in two ways, viz, analytically and synthetically, and that the former procedure, not the latter, tends to load noegenetic processes with " $g$."

The contribution of other factors to the variance of the Raven test has already being studied. Some of them may refer to what Spearman calls synthetic activity.

One possible way of interpreting factor $A$ would be to regard it as similar to Thurstone's $I$, since the subjects' reports indicate that they try to find a "rule or principle for each item of the test." In this sense the test will probably require the analytical activity mentioned by Spearman. Owing to the design of the present battery, these points can not be tested in this study and moreover would lead to the analysis of the complex relations between " $g$," $I, R$, and $D$, which have been dealt with elsewhere (14). Accordingly, it is preferable at this time to refrain from giving any interpretation of Factor $A$.

## Study of the Second Order

Some of the correlations between the primaries are negative-a fact which may depend upon the selection of the subjects (20). Nevertheless, it is difficult to explain how factors can be negatively correlated in the field of intelligence. By studying Table 6 one realizes that most of the negative values correspond to factor $F$, which according to the previous interpretation seems to be connected with speed of perception. Since this is the only factor apparently related to speed, it would not be too difficult to explain the values of the correlations between this primary and the rest of them.

There are few studies with reports about the second order. In a study not yet published (14), the writer employed the technique of expressing the original variables in terms of the second-order factors. The experience seems to indicate some interesting findings, although one must be cautious about giving premature judgment in the matter.

The basic logic underlying this extension of factor theory resides in the fact that if the primaries are correlated, their variance should, theoretically at least, be explainable in terms of common variance, plus specific and error variance. The factors thus obtained probably refer to more fundamental psychological variables, since they are the expression of the correlations between the primaries. Expressing the tests in these new factors, one may get an idea of how their performance clusters around these second-order factors. This technique raises many questions not yet thoroughly answered in factorial theory.

Since part of the variance of the primaries (uniqueness) has been lost in the process of factorization, it is not to be expected that the original correlations will be exactly reproduced by means of the loadings in the second-order centroids. In the process of rotation it is advisable to look for the best fit for both factors and tests, without expecting a perfect agreement, for the reasons mentioned above.

FACTOR $\gamma$


Factor $B$ was interpreted as perceiving relations in space necessary for the construction of a whole, which suggests that this factor is of a rather complex nature.

As can be seen by studying Table 12, all the tests saturated in $B$ have loadings in the second order factors $\gamma$ and $\alpha$.

Although Factor $A$ was not interpreted, it seems to be related
to some sort of analytical activity, and all tests loaded in it have saturation in $\gamma$.

Factor $D$ may well require a discriminative analysis, judging from the subjects' reports. Factor $G$ was not interpreted.

Very likely factor $\gamma$ represents a sort of analytical activity. According to the figures in Table 12, it is rather general in scope and, by the part it plays in both factors and tests, seems to be present in different kinds of psychological performances. Probably it is related to what has been called abstraction and indicates a rather fundamental dimension of psychological dynamics, such as capacity to abstract from complex presentations.

If this interpretation is correct, it is understandable why factors $F, E$, and $C$ are not loaded in $\gamma$.

## FACTOR a

| Factor $B$ | . 57 |
| :---: | :---: |
| Factor $F$ | . 26 |
| actor | -. 22 |

It is not possible to interpret this factor with certainty. According to the previous description, Factor $B$ seems to be related to Thurstone's $A$ (19) and probably to the factors described by Mieli (6) as "complexité" and "globalization," which the author defines as: "capacité de réaliser clairement et avec précision des structures complexes" and ". . la facilité avec laquelle des données relativement separées s'unissent pour former un tout." These two factors, according to Mieli, may go together but may also be opposed to his factor "plasticité," which is similar to our Factor C.
$F$ has a small loading in $\alpha$, and probably for the perfect operation of this factor a certain amount of syntheses of the given percept may be needed.

Therefore, and only as a tentative explanation, this factor could be considered as expressing a certain synthetic (syncretic) activity.


This is a bipolar factor, one of whose extremes is represented by the memory factor. At the present moment it is not possible to interpret it.

## Summary

From 19 variables we have extracted seven factors and interpreted only five of them.

Factor $D$, which seems to be concerned with the relation of right and left, needs further investigation.

Factor $A$ can not be easily identified in terms of the present battery.

Factor $B$ seems to imply the perception of relations in space necessary for the construction of a whole and is probably related to Thurstone's $A$ and to Mieli's "complexite"" and "globalization." It is a rather complex component and its significance seems to transcend the actual test context.

Factor $C$ is probably related to the difficulties encountered in the construction of a gestalt, when there are disturbing forces. It is clearly differentiated from factor $B$ and seems to be related to Thurstone's $E$ and to Mieli's "plasticité."

Factor $F$ has been interpreted as speed of perception and is probably similar to Thurstone's factor of the same name. Since the last three mentioned factors, or their counterparts, have appeared in different studies employing different tests and populations, it would seem that they represent certain basic psychological concepts.

Factor $E$ has been considered as an inmediate memory factor. Factor $G$ has not been interpreted.

Of the second-order domain factors, none has been definitely identified. It would seem that factor a represents some sort of synthetic activity, while factor $\gamma$ represents the capacity to abstract from complex presentations. Probably these two processes play a part in all intellectual activities, although their relative weight may vary according to the circumstances.

Factor $\beta$ has not been interpreted.
It is evident that the factorial composition of this battery reveals the existence of space and perceptual factors plus one of speed. The interrelations between space and perceptual factors is complex and probably the stress should be laid more on the understanding of their interplay.

Some of the names usually given to factors are related more to the special make-up of the tests than to the psychological activity going on in the subjects. Very probably factors will have to be classified in terms of their psychological nature. There is a suspicion that some of them, such as the verbal and numerical ones, would represent the ability to deal with symbols. This ability would possibly be different from the one represented by the other factors and would be quite general in scope.

The Progressive Matrices are loaded in several factors, plus a factor common to all of them. In terms of multiple factor analysis their composition seems to be complex.

|  | Product-Moment Correlations Between the Tests* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 187 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -012 | 081 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 161 | 255 | 109 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 216 | 188 | 201 | 227 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 177 | 168 | 127 | 267 | 222 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 050 | 144 | -168 | 195 | 050 | 211 |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 197 | 261 | 241 | 366 | 210 | 244 | 108 |  |  |  |  |  |  |  |  |  |  |  |
|  | 105 | 033 | -022 | 123 | 035 | -082 | 114 | 165 |  |  |  |  |  |  |  |  |  |  |
|  | 158 | 039 | -059 | -048 | -024 | -041 | 239 | -011 | 048 |  |  |  |  |  |  |  |  |  |
|  | 091 | 063 | 296 | 168 | 150 | 182 | 260 | 311 | 115 | 170 |  |  |  |  |  |  |  |  |
|  | 220 | 241 | -028 | 157 | 187 | 147 | 082 | -012 | -003 | 064 | -113 |  |  |  |  |  |  |  |
|  | 230 | 308 | 091 | 195 | 081 | 112 | 089 | 261 | 045 | 170 | 086 | 175 |  |  |  |  |  |  |
|  | 063 | 240 | 150 | 503 | 157 | 304 | 139 | 361 | 064 | 038 | 257 | 126 | 252 |  |  |  |  |  |
|  | 078 | 172 | 157 | 341 | 168 | 151 | 136 | 353 | 110 | 056 | 293 | 066 | 230 | 592 |  |  |  |  |
|  | 100 | 154 | 125 | 356 | 178 | 261 | 248 | 357 | 165 | -024 | 214 | 085 | 198 | 540 | 645 |  |  |  |
|  | 222 | 243 | 217 | 333 | 206 | 192 | 084 | 484 | 090 | 028 | 164 | 196 | 236 | 539 | 656 | 669 |  |  |
|  | 206 | 111 | 177 | 217 | 214 | 235 | 159 | 219 | 069 | -025 | 133 | 184 | 134 | 257 | 328 | 417 | 552 |  |
|  | 283 | 202 | 148 | 303 | 301 | 015 | 115 | 238 | 003 | 083 | 124 | 157 | 254 | 252 | 251 | 244 | 351 |  |

TABLE 2
The Centroid Factorial Matrix*

| Test |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| No. | I | II | III | IV | V | VI | VII | $h^{2}$ |
| 1 | 370 | 349 | 204 | -122 | -024 | -187 | -150 | 373 |
| 2 | 404 | 240 | 128 | 067 | 114 | 161 | 093 | 289 |
| 3 | 268 | -297 | 295 | -349 | -032 | 118 | 160 | 409 |
| 4 | 559 | 034 | -065 | 115 | 146 | 288 | -093 | 444 |
| 5 | 390 | 099 | 231 | -064 | -105 | 091 | -053 | 241 |
| 6 | 391 | -083 | 102 | 170 | -351 | 313 | -155 | 444 |
| 7 | 320 | 194 | -411 | 110 | -279 | 032 | -081 | 406 |
| 8 | 578 | -114 | 057 | -196 | 202 | 179 | -146 | 483 |
| 9 | 161 | 055 | -209 | -110 | 077 | -073 | -249 | 158 |
| 10 | 135 | 264 | -229 | -173 | -142 | -201 | 217 | 278 |
| 11 | 410 | -157 | -237 | -424 | -263 | 159 | 058 | 526 |
| 12 | 270 | 334 | 194 | 300 | -071 | -054 | 110 | 332 |
| 13 | 411 | 205 | 058 | -038 | 165 | -028 | 170 | 273 |
| 14 | 653 | -220 | -192 | 189 | 170 | 135 | 169 | 623 |
| 15 | 650 | -332 | -211 | 074 | 192 | -160 | 132 | 663 |
| 16 | 671 | -317 | -166 | 216 | 094 | -153 | -114 | 670 |
| 17 | 751 | -313 | 148 | 151 | 179 | -322 | 034 | 843 |
| 18 | 512 | -157 | 199 | 166 | -197 | -243 | -129 | 468 |
| 19 | 461 | 214 | 093 | -088 | 117 | -026 | 038 | 290 |

* The decimal point has been omitted for all the entries.

TABLE 3
The Final Transformation Matrix

|  | A | B | C | D | E | F | G |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| I | .32 | .41 | .20 | .25 | .23 | .36 | .18 |
| II | -.56 | .19 | .12 | -.26 | -.01 | .60 | .31 |
| III | -.02 | .01 | -.64 | .43 | .05 | .37 | .00 |
| IV | .35 | .28 | -.25 | -.56 | .27 | -.08 | -.39 |
| V | .00 | .65 | -.45 | -.56 | -.88 | -.16 | .32 |
| VI | -.66 | .44 | -.09 | .26 | .24 | -.55 | -.09 |
| VII | .14 | .32 | .50 | .02 | -.18 | .19 | -.77 |

TABLE 4
The Rotated Factorial Matrix*

| Test No. | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -01 | 04 | -03 | 11 | 06 | 51 | 35 |
| 2 | -08 | 41 | -01 | 03 | 04 | 23 | 07 |
| 3 | 06 | 05 | 00 | 51 | 01 | 03 | -05 |
| 4 | 03 | 45 | -01 | 05 | 11 | -04 | 14 |
| 5 | -02 | 12 | 03 | 28 | 20 | 26 | 12 |
| 6 | 00 | 05 | 01 | 34 | 55 | -02 | -04 |
| 7 | 01 | 00 | 41 | -01 | 35 | 08 | 04 |
| 8 | 03 | 33 | -07 | 24 | -02 | 02 | 31 |
| 9 | 00 | -01 | 04 | -06 | -04 | 00 | 30 |
| 10 | 00 | -05 | 44 | -01 | 00 | 31 | -02 |
| 11 | -02 | -06 | 45 | 46 | 23 | -04 | 04 |
| 12 | 04 | 23 | -01 | -07 | 18 | 40 | -07 |
| 13 | 04 | 34 | 09 | 00 | -09 | 32 | 07 |
| 14 | 34 | 50 | 17 | -03 | 04 | -05 | -11 |
| 15 | 54 | 31 | 20 | -03 | -07 | 03 | -03 |
| 16 | 55 | 23 | 07 | -04 | 11 | 01 | 06 |
| 17 | 68 | 28 | -06 | 06 | -02 | 28 | 04 |
| 18 | 45 | -05 | -04 | 21 | 32 | 29 | 03 |
| 19 | 04 | 30 | 03 | 07 | -08 | 34 | 19 |

* The decimal point has been omitted for all entries.

TABLE 5
Cosines of Angles Between Reference Vectors
A B $\quad \mathbf{C}$
D E
F $\quad \mathbf{G}$

A .99
B $\quad-.12 \quad 1.00$
C $\quad .05 \quad-.14 \quad .99$
$\begin{array}{lllll}\mathrm{D} & -.15 & -.34 & .12 & 1.01\end{array}$
$\begin{array}{llllll}\mathrm{E} & -.01 & -.36 & .23 & .48 & .99\end{array}$
F $\quad .13-.04 \quad .14 \quad .09 \quad .05 \quad 1.00$
G $\quad-.30 \quad-.06 \quad-.35$

TABLE 6
Correlations Between the Primary Vectors
A B C
D $\quad \mathbf{E} \quad \mathbf{F}$
G
A. 1.00

B $\quad .24 \quad 1.00$
$\begin{array}{llll}\text { C } & .11 & .14 & 1.00\end{array}$
$\begin{array}{lllll}\mathrm{D} & .21 & .24 & .02 & 1.00\end{array}$
$\begin{array}{lllllll}\mathrm{E} & .07 & .27 & -.06 & -.37 & 1.00\end{array}$
F $\quad-.24 \quad-.09 \quad-.20 \quad-.10 \quad-.06 \quad 1.00$
$\begin{array}{llllllll}\mathrm{G} & .38 & .27 & .37 & .04 & .24 & -.27 & 1.00\end{array}$

TABLE 7
Loadings of the Primaries in the Centroids of the Second Order

|  | $\mathrm{I}^{\prime}$ | $\mathrm{II}^{\prime}$ | $\mathrm{III}^{\prime}$ |
| :--- | ---: | ---: | ---: |
| A | .49 | -.12 | -.10 |
| B | .66 | .24 | .52 |
| C | .29 | .02 | -.14 |
| D | .31 | -.70 | .36 |
| E | .14 | .44 | -.04 |
| F | -.38 | .08 | .25 |
| G | .75 | .22 | -.37 |

TABLE 8
Loadings of the Tests in the Centroids of the Second Order

$$
\mathrm{I}^{\prime} \quad \mathrm{II}^{\prime} \quad \mathrm{III}^{\prime}
$$

| 1 | .19 | .09 | .05 |
| ---: | ---: | ---: | ---: |
| 2 | .26 | .16 | .29 |
| 3 | .21 | -.43 | .27 |
| 4 | .68 | .17 | .21 |
| 5 | .23 | -.03 | .19 |
| 6 | .23 | .02 | .16 |
| 7 | .20 | .22 | -.09 |
| 8 | .61 | -.04 | .17 |
| 9 | .25 | .11 | -.17 |
| 10 | -.04 | .02 | -.01 |
| 11 | .35 | -.26 | .05 |
| 12 | -.03 | .22 | .23 |
| 13 | .23 | .09 | .25 |
| 14 | .53 | .11 | .25 |
| 15 | .53 | .00 | .11 |
| 16 | .55 | .10 | .03 |
| 17 | .51 | -.04 | .18 |
| 18 | .24 | -.05 | .06 |
| 19 | .31 | .08 | .20 |

TABLE 9
Final Matrix of Transformation for the Second Order

|  |  | $\boldsymbol{\alpha}$ | $\boldsymbol{\gamma}$ |
| ---: | ---: | ---: | ---: |
| $\mathrm{I}^{\prime}$ | .00 | .15 | .70 |
| II $^{\prime}$ | .48 | .85 | -.23 |
| III $^{\prime}$ | .88 | -.50 | .68 |

TABLE 10
Cosines of Angles Between Preference Vectors in the Second Order
$\alpha \quad \beta \quad \gamma$
$\alpha \quad 1.00$
$\beta \quad-.03 \quad .99$
$\begin{array}{llll}\gamma & .49 & -.42 & 1.00\end{array}$

TABLE 11
Rotated Factorial Matrix for the Primaries in the Second Order

|  | $\alpha$ | $\beta$ | $\gamma$ |
| :--- | ---: | ---: | ---: |
| $\mathbf{A}$ | -.15 | .02 | .30 |
| B | .57 | .04 | .76 |
| C | -.11 | .13 | .10 |
| D | -.02 | -.73 | .62 |
| E | .18 | .42 | -.03 |
| F | .26 | -.11 | -.11 |
| G | -.22 | .48 | .22 |

TABLE 12
Rotated Factorial Matrix for the Tests in the Second Order

|  | $\alpha$ | $\beta$ | $\gamma$ |
| ---: | ---: | ---: | ---: |
| 1 | .08 | .08 | .15 |
| 2 | .33 | .03 | .34 |
| 3 | .03 | -.47 | .43 |
| 4 | .27 | .12 | .47 |
| 5 | .15 | -.09 | .30 |
| 6 | .15 | -.03 | .26 |
| 7 | .03 | .26 | .03 |
| 8 | .13 | -.03 | .55 |
| 9 | -.10 | .22 | .03 |
| 10 | .00 | .02 | -.04 |
| 11 | -.08 | -.19 | .34 |
| 12 | .31 | .07 | .08 |
| 13 | .26 | -.01 | .31 |
| 14 | .27 | .05 | .52 |
| 15 | .10 | .02 | .45 |
| 16 | .07 | .15 | .38 |
| 17 | .14 | -.05 | .49 |
| 18 | .03 | -.04 | .22 |
| 19 | .21 | .01 | .33 |

TABLE 13
Loadings of the Tests as Determined by Using Spearman's Formula

| 1 | .351 |
| ---: | ---: |
| 2 | .402 |
| 3 | .224 |
| 4 | .559 |
| 5 | .386 |
| 6 | .371 |
| 7 | .285 |
| 8 | .576 |
| 9 | .144 |
| 10 | .097 |
| 11 | .382 |
| 12 | .241 |
| 13 | .408 |
| 14 | .643 |
| 15 | .633 |
| 16 | .656 |
| 17 | .736 |
| 18 | .495 |
| 19 | .460 |

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[^1]:    * The numbers following the names of the tests refer to the code number employed in the present article.

[^2]:    * It will be noted in Table 1 that the signs of variables $1,12,13$, and 2 have been changed.

