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THE PERCEPTUAL FACTOR

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A further study of the perceptual factor, previously isolated in a factor analysis of a battery of fifty-six tests, is made in a manner designed also to determine whether the same seven primaries would be found in a different population of subjects and with another battery of tests. The tests are described, and the results of the analysis are given in detail. Much attention is given to the matter of the orthogonality of primary factors and to their psychological meaningfulness.

In a factorial study of fifty-six psychological tests* there were isolated seven primary factors whose interpretation seemed quite clear. The psychological interpretation of several other factors was not immediately evident. The clearest factors were the verbal factor *V*, the number factor *N*, the space factor *S*, and the memory factor *M*. The factors which were less clearly defined were the perceptual factor *P*, the word factor *W*, and the inductive factor *I*. The present study was undertaken in order to learn more about the nature of the perceptual factor *P*. This factor had appreciable saturations in the following tests: Verbal Classification, Word Grouping, Disarranged Sentences, Identical Forms, and Picture Recall. The saturations indicated that from one-fifth to one-third of the total variance of these tests was attributable to the factor *P*. Several other tests with factor loadings of .40 could be used in studying the psychological nature of the factor. The highest saturation was found in Identical Forms, since one-third of its variance was attributable to the perceptual factor.

A study of these tests indicated that the perceptual factor might consist in a facility to perceive detail even when it is buried among perceptual distractors. It might involve speed as an essential characteristic, but this impression may be due to the fact that the perceptual tests were simpler than the tests which were heavily saturated with other factors. The interpretation of primary factors is made largely in terms of the kind of thinking that is involved in doing the tasks.

The characteristic that seemed to be common to all of the tests that were heavily saturated with the perceptual factor *P* was the

*Thurstone, L. L., *Primary Mental Abilities*, to be published by The University of Chicago as the first number of the Psychometric Monograph Series, April, 1938.

readiness to discover and to identify perceptual detail. The present experiment was planned to investigate this hypothesis. If a perceptual factor of this general nature exists, then it should be possible to predict that certain new tests should be heavily saturated with this factor even though they might be otherwise disparate in superficial appearance.

A test battery was assembled which included two or three tests for each of seven primary abilities that were isolated in the previous study, and also nine new tests that were planned to be tests of the perceptual factor with some variation in immediate content. The total battery so selected was given to a new group of subjects for a new factorial analysis. The first question was then to ascertain whether the old seven primary factors would again make their appearance as they should, since each of them was here represented by two or three tests. Then there was the further question whether the new tests that were designed so as to be saturated with the perceptual factor would appear together with the previous best test for this factor, namely, Identical Forms. If they did not hang together, then their saturations with the other factors should enable us to identify the nature of the new tests, and we should be forced to guess again about the psychological nature of the factor *P*.

The new test battery was assembled partly from the previous battery of fifty-six tests. These tests were as follows:* Addition (31), Multiplication (33), and Division (34) for the Number Factor *N;* Anagrams (15) and Disarranged Words (12) for the Word Factor *W;* Areas (29) and Tabular Completion (35) for the Induction Factor *I*; Opposites (10), Completion (11), Verbal Analogies (41) and Word Grouping (7) for the Verbal Factor *V*; Flags (20) and Pursuit (27) for the Space Factor *S*; Word-Number (46) and Initials (47) for the Memory Factor *M;* Identical Forms (26) and nine new tests for the Perceptual Factor *P;* Arithmetical Reasoning (39) and Reasoning (40) for the tentative factor *R*. In the previous study the Word Grouping test had high saturation in both the perceptual and the verbal factors *P* and *V*.

The new tests that were specially designed for this experiment will be described here briefly.

Scattered X's. This test has one page of instructions and fore-exercise followed by ten letter-sized pages of pied letters. The subject is asked to ring every letter *x*. Each page has twenty rows with thirty pied characters in each row.

Letter A. The test has one page of instructions and fore-exercise

*The numbering of the tests corresponds to that of the previous battery of fifty-six tests.

followed by ten letter-sized pages. Each page has five columns with forty words in each column. The subject is asked to check every word that contains the letter *a*. The test contains fifty columns of words. This test and the previous test are essentially cancellation tests.

Identical Names. One page of instructions and fore-exercise followed by a test of ten letter-sized pages. Each page has four columns of names with initials. At the top of each column is a name with initials. The subject is asked to find the top name repeated somewhere in the column and to check it. The test contains forty columns of names. The names are not arranged in alphabetical order.

Identical Numbers. One page of instructions and fore-exercise and a test of ten letter-sized pages. Each page has eight columns of forty three-place numbers. Each column is headed by a number. The subject is asked to find the first number repeated somewhere in the column and to ring it. The test contains eighty such columns.

Highest number. One page of instructions and fore-exercise and a test of ten pages. Each page contains eight columns of forty three-place numbers. The subject is asked to ring the highest number in each column. The test contains eighty columns of numbers.

Verbal Enumeration. One page of instructions and fore-exercise and a test of ten letter-sized pages. Each page contains five columns of forty words. At the top of each column is the name of a category of things such as flowers, clothing, furniture, trees, grains, vehicles, spices, coins, furs, diseases, beverages, and so on. The subject is asked to check four words in each column that belong to the category indicated at the top of the column.

Concrete Association. One page of instructions and fore-exercise and a test of ten letter-sized pages. Each page contains five columns with forty words in each column. The subject checks four words in each column that are closely associated with the heading of the column. Examples of categories for the columns are *politics, garage, estate, student, farm, bank, radio, hospital, government, river, business, lawsuit, lake, winter,* and so on. In each column there are four words closely associated with the category for the column. For example, the column headed *river* contains the four words *canoe, rapids, levee,* and *current.* The subject checks four associations in each column of forty words. The test contains fifty such columns.

Abstract Classification. One page of instructions and fore-exercise and a test of ten letter-sized pages. Each page contains five columns with forty words in each column. Each column is headed by a word that designates a category. Examples of these are *up, front, lightness, within, again, angular, narrow, etc.* The subject is asked to check four words in each column that belong in the category for the

column. For example, in the column headed *angular* the four response words are *corner, jagged, notch,* and *gable.* This test and the two previous tests are of the same character, but they vary in degree of abstraction.

Designs. One page of instructions and fore-exercise and a test of eleven pages of designs. Ten of these designs are shown in *Figure 1.* Each page contains ten rows with ten designs in each row. The sub-

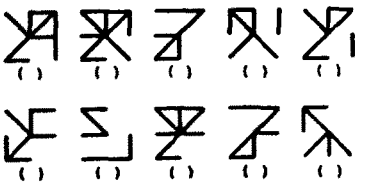

FIGURE 1

ject checks every design which contains the capital letter sigma, Σ. He is shown the letter Σ, which is called the "model," and he is asked to check every design which contains the model.

The object of the test was to determine whether the ability to extract a part of a design which is perceived as a whole is characteristic of the perceptual factor. The perception of the model within the design requires an act of abstraction which might, or might not, be involved in the perceptual factor. In the Scattered *X*'s test the subject looks for the letter x as in a cancellation test. But in that test the figure that he is looking for is presented as a whole. In the Designs test the figure that he looks for is presented as a part of a larger total figure. The task is tedious for most subjects, since the "model" must be extracted, or abstracted, as it were, from each design.

Before giving the new test battery we made some estimates of the relative degree of saturation of the perceptual factor which could be expected according to our tentative formulation of the nature of the perceptual factor. It seemed that Scattered *X*'s should have a high saturation with the perceptual factor. Verbal Enumeration and Concrete Association should have a higher saturation than Abstract Classification because the latter test involves clearly other intellectual factors besides speed of perception. The Identical Numbers and Identical Names should have high saturation and be comparable as to the perceptual loadings. The Letter *A* requires of the subject the abstraction of the letter a from meaningful words, and the Designs has the same characteristic, namely, that the "model" which the subject is looking for is imbedded in a design that is perceived as a unit. In both cases the subject must extract the object of his search from the larger unit

that he is perceiving, namely, the meaningful word in one case and the complete design in the other case. These two tests were questionable in relation to the nature of the perceptual factor. The Identical Forms should have an appreciable saturation in the perceptual factor since it had the highest saturation with the factor *P* in the previous experiment in which the factor was first tentatively recognized.

The new battery of twenty-seven time limit tests was given to a class of seniors at the Lane Technical High School in Chicago in the spring of 1936. The tests were given in five sessions to each group of about forty students. There were 215 subjects who completed the whole battery, and their test records were used in the correlational analysis. *Table 1* is a summary of the distributions of the twenty-seven tests in the present battery. The table shows the name of each test, its scoring formulae, the arithmetic mean, the upper and lower quartiles, the standard deviation, and the estimated reliability of each test by the Spearman-Brown correction formula. The tests were fairly reliable except for two tests, namely, Arithmetical Reasoning, which had twenty problems, and Reasoning, which had twenty syllogisms. Some of the distributions were skewed.

The scores were arranged to be less than 100 so that they could be tabulated in two columns of a Hollerith card for each test. Negative scores were avoided by adding an arbitrary constant as shown in the scoring formulae of *Table 1*. The cross products were obtained by a Hollerith multiplier, and the Pearson product-moment coefficients were determined from these products. The inter-test correlations are shown in *Table 2*. All the new tests for this battery have code numbers above 60. The tests from the previous battery of 56 tests have code numbers below 60.

The correlation table was factored by the centroid method,* and the resulting centroid matrix is shown in *Table 3*. The mean of the residuals was .00108, and the range was from +.07 to —.07. The standard deviation of the distribution of residuals was .0243.

The higher communalities are between .70 and .80. The communality represents the variance attributable to the common factors. The lowest communalities are about .40. The tests with low communalities are not satisfactory and need considerable improvement. Several of the new tests were scored not only for the number of right responses within the time limit, but also for the ratio of the correct responses to the total number of attempts. There are four such tests. In one of these, No. 74, the primary factors account for only 28 per cent of the total variance of the test.

*Thurstone, L. L., *The Vectors of Mind*, The University of Chicago Press, 1935, Chapter 3.

Improved rotational methods were used in determining the simple configuration of the present test battery. Each of the coordinate planes was determined independently. The new rotational methods will be described in a separate publication. They give the same final result as the older methods, but the new methods are more economical of time. The rotated configuration is represented in *Table 4*.

Table 5 shows the matrix of the transformation Λ from the centroid matrix F_0 in *Table 3* to the rotated matrix F_1 of *Table 4*. This relation can be stated in the matrix equation

$$F_0\Lambda = F_1. \tag{1}$$

The cosines of the angular separations of the unit reference vectors are their scalar products. These may be written in the form

$$\Lambda'\Lambda = N, \tag{2}$$

where N is a symmetric matrix of cosines or correlations between the reference vectors.

The direction cosines of the primary vectors are proportional to the rows of the inverse Λ^{-1}. * Hence the direction cosines may be expressed by $D\Lambda^{-1} = M$, where D is a premultiplying diagonal matrix. The entries in D are so chosen as to normalize the rows of the product matrix M.

The correlations between the primary traits in the experimental population are the cosines of the angular separations of the unit primary vectors. These correlations or scalar products are given by the equation

$$MM' = R_p, \tag{3}$$

where R_p is the matrix of the correlations. This can be written

$$(D\Lambda^{-1})\,(D\Lambda^{-1})' = R_p, \tag{4}$$

or

$$D\Lambda^{-1}\,\Lambda'^{-1}D = R_p. \tag{5}$$

By (2)

$$\Lambda^{-1}\,\Lambda'^{-1} = N^{-1}, \tag{6}$$

and hence

$$DN^{-1}D = R_p. \tag{7}$$

In order to obtain the correlations R_p between the primary traits, one computes the inverse of the symmetric matrix $\Lambda'\Lambda$. The diagonal matrix D is then written so as to reduce the diagonals of N^{-1} to unity. The matrix R_p is shown in *Table 6*.

*An economical method of computing the inverse has been devised by Mr. Ledyard Tucker, which he will describe in a forthcoming paper.

Table 4 is of interest in the present experiment. The coordinate planes have been so rotated as to maximize the number of nearly vanishing projections. There are no significant negative projections. It is of major interest to ascertain whether the same factors that were isolated as primary in the previous experiment can be identified in the present experiments. The test battery was considerably altered, and the tests were given to a new population for a new factorial analysis.

The order of the columns is of no significance. They are given here in the order in which they happened to appear in the computations. Each plane was set in accordance with the configuration revealed in the plots of pairs of columns. A comparison will be made between the factors previously determined and the factors in the present battery. Some of the factors are clearly the same, while several new factors appeared in the present experiment. The residual plane in *Table 4* is not identified.

Inspection of the first column *V* leaves little doubt that the first factor is verbal in character. The highest saturations are in Abstract Classification, Completion, Opposites, Verbal Analogies, Verbal Enumeration, and Word Grouping. These tests characterize the verbal factor *V*.

In the inspection and comparison of factor loadings of a test it must be recalled that the square of the saturation is the variance attributable to the factor. Loadings below .20 or .30 are unstable since they represent less than ten per cent of the variance of the test. A shift in saturation from .30 to .45 represents a shift of about ten per cent of the variance of a test. With the improvement of tests it should be possible to reduce their complexity so that a higher and higher proportion of their variance is attributable to a single primary factor. All that we can expect in the present state of knowledge is to identify the principal landmarks among the human abilities. There is some satisfaction in finding that the primary factors in a simple configuration determined by one population are essentially the same as those found in another population.

Column *N* has the highest saturations in Addition, Division, Highest Number, Identical Numbers, and Multiplication. This is evidently the number factor *N* that was found in the previous battery. Here, as before, the simple arithmetical processes carry the highest saturations in this factor.

Column *S* has the highest saturations in Areas, Designs, Flags, Identical Forms, and Pursuit. All but one of these tests were used in the previous battery, and they characterize the visual space factor *S*. It is not surprising to find the new test Designs in this list. We had expected to find a spatial component in this test, but it was inserted in

the new battery to determine whether it would also appear among the perceptual tests. The Identical Forms made a shift, a reduction of its perceptual component and an increase in its spatial component. It had an appreciable saturation in both of these factors in the previous study, and it retained both factors in the present battery. The angular displacement of the test vector for Identical Forms in the plane of these two factors, Perception and Space, is about 28 degrees. These shifts may be due in part to a shift in the abilities that the subjects used in doing the tasks. The former group was the most highly selected group of subjects that the author has ever worked with. The present group was a class of seniors in a vocational high school. Considerable work with individual subjects will be required to ascertain whether some of these tasks can be performed by the vicarious functioning of one ability for another ability that is normal for the task. In the present case the less gifted subjects relied more on visual imagery for a task that seemed to be more immediate and perceptual for the gifted subjects. This shift in the abilities used for any particular performance may be expected also within any group of subjects.

When a test shows saturation with two or more factors we have no means of knowing by factorial analysis whether the several abilities enter into the test for every subject, or whether some subjects use one ability and other subjects use other abilities for the same performance. A study with individual subjects could reveal these differences, especially when the subjects indicate how they solve each problem. One solution for this ambiguity is to develop tests which involve only one factor. Since it seems desirable to work toward tests which involve mostly one primary ability and very little of the others, we may eventually have test batteries that are highly specialized as to the functions involved so that individual differences will be conspicuous.

Column *M* has only two tests with appreciable saturations. These are the two memory tests so that the identification is evidently the memory factor *M*. No attempt was made in this study to analyze the memory factor. That will be reserved for future experiments.

Column *W* has appreciable saturations in three tests, namely, Anagrams, Disarranged Words, and Identical Names. Two of these tests were retained for this battery to represent the Word factor *W*, and the other test was one of the new ones. The new test fits the previous description of this factor. Both verbal factors appeared in this battery as in the previous one. The psychological differentiation between these two verbal factors needs considerable further study.

Column *I* does not have many large saturations, and it is necessary to consider tests with low saturations in this factor in order to

make a tentative interpretation. This includes the loadings higher than .30. The list then includes Areas, Arithmetical Reasoning, Reasoning, Tabular Completion, and Verbal Analogies. These tests all involve reasoning, and the common factor seems to be inductive. This factor has therefore been identified as Induction. An experiment is now in progress with ten new tests of induction in addition to those here listed. The resulting analysis should reveal with more certainty the nature of this primary mental ability.

Column *P* has high saturations for all of the new tests except Designs, and this factor has, therefore, been identified as perceptual. The new tests were designed for this battery in order to determine whether they would have appreciable projections on the same primary vector. This has happened so that the uniqueness of this perceptual factor seems quite certain. The previous battery did not have several good tests for this factor. The new tests were designed so as to accentuate the perceptual factor if it existed. The Identical Forms shifted toward the visual space factor, and the Word Grouping shifted toward the verbal factor *V*, so that the identification of the new perceptual factor with the one previously suspected is not clear. However, it does seem clear that the new tests introduced into this experiment are closely related by a conspicuous common factor. This factor we shall denote *P*.

The interpretation of the factor *P* will be aided by comparing the relative saturations of the nine new tests with this factor. The highest saturations are found in Concrete Association and in Verbal Enumeration. The third test in this sequence was Abstract Classification which was designed so as to be similar in character but with more abstract material. The ratings of these three tests as regards the perceptual factor is as we had expected. The two simple ones rank highest while the more abstract form of the test is less satisfactory as an index of this factor.

The three tests Highest Number, Identical Number, and Identical Names rank next, since about half of their common factor variance is accounted for by the perceptual factor. The fact that some of these tests are numerical and others verbal in immediate content does not affect their saturation with the perceptual factor. The Scattered *X*'s was thought to be a simple task which should have a high saturation with the factor *P*, but it does not rank so high in this factor as was anticipated. The common factor in these tests may be fluency of association with perceptual material. Visual acuity is probably not involved in this factor. It is probable that this factor is of considerable significance in determining the speed of reading, and it may be involved in reading disabilities. Further experimental study should be

made with the present battery augmented by new tests of visual discrimination with liminal and with supraliminal discriminations, various reading tests, communality of association, tests involving the identification of a designated object with varying degrees of perceptual distraction in the same modality and in different modalities, and tests of visual acuity. Such investigations will delimit each factor, and they will probably disclose new ones.

Another column shows only two significant saturations, namely, the two scores for the Designs test. This column is denoted "Doublet Designs." This is not sufficient variety of test material to identify this factor. The test does not belong with the factor *P*. It might be of some significance that the number of right responses in unit time and the ratio of the right responses to the total number of attempts both have high saturations on this factor. We regard this factor as a doublet of unknown psychological nature. One factorial column has appreciable saturations in the four ratio scores. It is denoted "Ratios." The factor involved may be concerned with Accuracy or Caution. More data should be available on a variety of test material for this factor before it can be identified. This finding does suggest, however, that the relative frequency of errors may represent a unique trait.

In the interpretation of factorial analyses the assumption of linearity is an important limitation. It is unlikely that the mental abilities combine linearly except as a first approximation. Consequently the large saturations of the tests should be studied for the purpose of discovering the principal landmarks among the mental abilities and not with the hope that the exact factor loadings will remain over different ages and selective conditions. The extension of factor analysis to second degree functions will remove this limitation. The psychological implications of second degree functions in factor theory will be discussed in a separate paper.

One of the criticisms of factor analysis is that if similar tasks are inserted into a test battery, they will identify a common factor and that new factors can, therefore, be manufactured indefinitely. This does not happen. Several failures to verify such postulated factors may serve to answer this form of criticism.

In preparing the present battery of tests it was postulated that quickness in perceiving detail among distractors was a factor. All but one of the new tests did define a common factor; but one of them, Designs, failed to join the others. We had guessed wrong, at least in part, about the nature of this factor. It now seems that the perceptual unity of the design from which the detail, the Σ, had to be extracted moved this test to some other categories. This will lead to separate experiments in which the degree of perceptual unity which

hides or inhibits the object of search can be varied. This may again be a false lead, but a factorial analysis can answer the question.

In preparing the fifty-six tests for a former test battery, the assumption was made tentatively that verbal reasoning, numerical reasoning, and space reasoning would be separate factors and that these would be different from verbal abstraction and visual imagery. Groups of tests were designed for these categories. The factor analysis demolished all of these predetermined groupings that had guided the test construction. The analysis cut across these anticipated groupings and revealed different factors. *But these new factors have reappeared in successive test batteries.* It was assumed that visualizing flat space, visualizing solid space, and visualizing movement in solid space were different abilities. Groups of tests were constructed for these groupings. Factor analysis again cut across these groupings. The factorial methods will be most useful when they are applied to experiments specially designed to test psychological hypotheses. Merely to apply factor analysis to any available correlation table is as fatuous as any other manipulation of scientific tools without a motivating idea. Under such conditions it can frequently be shown formally that the factor analysis even becomes indeterminate.

The problem of orthogonality is of peculiar psychological importance. Should we assume that the primary human abilities are uncorrelated (orthogonal), or should we assume that they are correlated (oblique)? We should do neither. When the correlational matrix has been factored by the centroid method, or by any other equivalent method, the relations between the tests are known within the restrictions of linearity in the smallest possible dimensionality. The coordinate axes constitute merely an arbitrary orthogonal reference frame. A new set of coordinate axes must then be found that is psychologically significant. Each axis should represent an ability or faculty. These may be determined if the tests show a simple configuration. Then we can ascertain the intercorrelations of the primary abilities. The analytical methods do not impose either restriction.

We might be tempted to take for granted that the primary abilities are and should be uncorrelated, but in the present state of knowledge such an assumption is not safe. So far we have found the primary abilities to be practically uncorrelated. When the primaries are determined independently, there seems to be a consistent tendency for them to be slightly positively correlated. But the results are not yet sufficiently conclusive to justify a declaration about slight positive correlation. In two experiments there was some indication that the number factor and the space factor were correlated to the extent of about .20, while the other primaries had correlations between zero

and .10. These are of the order of magnitude that would be expected by chance variation, so that the finding is not conclusive. We have made adjustment by choosing the nearest orthogonal reference frame.

It may be useful to consider a case, perhaps fictitious and perhaps real, in which the primaries could be correlated. Let it be assumed that there are individual differences, not only in the primary abilities of adults, but also in the rate of mental development in childhood. Two children of like age might then differ in mental development even if they were destined to attain the same mentality as adults. It is unlikely that all individuals develop mentally at the same rate relative to their adult levels. It might be found that ten-year-old children of accelerated mental growth have most of their mental abilities more developed than ten-year-old children of slower mental growth. Such a situation would result in positive correlation between the primary abilities, due to maturation, even though these abilities would be uncorrelated in the same population when the individuals become adults. The fact that the correlation could be explained would not make it spurious. The primary abilities could be redefined in terms of predicted adult performance if some independent measure of mental growth were available. If such a measure were not available, the rank of the system would be lower by one, and the primary factors would appear correlated. This problematic case is described here merely to show one of several situations in which primary mental abilities might be positively correlated even at point age.

In current discussion of factor analysis there is frequent reference to factors that are called "mathematical" as distinguished from factors that are "real" and psychologically meaningful. It should be clear that, as psychologists, we are not interested in mathematical artifacts. Factor analysis can justify itself in experimental psychology only in so far as it aids in the discovery of psychologically significant categories. It is a source of considerable satisfaction to discover that different test batteries with different populations reveal the same psychological factors. These are not artifacts. It is unlikely that the grouping of tasks involving numerical, visual, verbal, and inductive thinking and memory appears consistently as a mathematical artifact in different populations and in different test batteries. To see these same verbal, numerical, spatial, and memory factors roll out of successive test batteries, even when the tests are identified only by code numbers, leads to the conviction that they are basic mental abilities, human faculties, rather than artifacts.

TABLE 1
Distributions of Scores in Twenty-Seven Tests to 215 Seniors at Lane Technical High School

Code No.	Name of Test	Scoring Formula	Mean	Standard Deviation	Q_1	Med.	Q_3	Reliability Coefficient
61	Abstract Classification	$\frac{R}{2}$	44.80	11.53	37.26	44.64	53.29	.96
62	Abstract Classification	$\frac{0}{(R+0)}$	24.45	13.59	14.40	21.94	33.04	.94
31	Addition	R	9.70	3.72	7.65	9.89	12.54	.88
15	Anagrams	R	13.46	4.70	10.55	13.53	16.75	
29	Areas	R	19.33	5.18	15.65	20.46	23.85	.95
39	Arithmetical Reasoning	R	3.83	2.63	2.32	3.96	5.97	.61
11	Completion	R	21.32	6.46	17.34	21.22	26.47	.90
63	Concrete Association	$\frac{R}{2}$	59.76	9.69	53.42	61.47	66.98	.96
65	Designs	R	59.27	15.21	52.69	60.87	70.78	.88
66	Designs	$\frac{0}{(R+0)}$	17.30	14.64	8.09	14.02	22.21	.86
12	Disarranged Words	R	38.70	9.24	33.31	38.88	45.50	.86
34	Division	R	8.03	3.81	5.62	8.16	11.02	.87
20	Flags	$(R—W+3)$	25.23	11.60	18.30	25.67	33.33	.92
67	Highest Number	R	42.84	9.67	36.68	41.75	49.71	.94
68	Highest Number	$\frac{W}{(R+W)}$	20.94	13.69	11.04	17.84	30.07	.85
26	Identical Forms	R	32.88	4.82	29.98	33.56	36.11	.92
69	Identical Names	R	28.79	5.27	.....	.....	.....	.95
70	Identical Numbers	R	60.77	10.43	.....	.....	.....	.97
47	Initials	R	5.33	3.32	3.30	5.50	8.07	.84
10	Inventive Opposites	R	26.84	7.89	21.86	28.14	33.25	.92
71	Letter A	$\frac{R}{2}$	53.00	16.87	41.73	52.08	62.52	.95
33	Multiplication	R	12.04	4.98	8.70	12.23	16.36	.89
27	Pursuit	R	46.29	8.15	41.55	46.94	51.70	.99
40	Reasoning	$(R—W+10)$	13.60	6.49	9.37	13.74	17.96	.50
73	Scattered X's	$\frac{R}{3}$	49.64	9.93	42.29	49.93	56.42	.93
74	Scattered X's	$\frac{0}{(R+0)}$	7.08	5.22	3.91	6.75	9.35	.73
35	Tabular Completion	R	21.27	7.89	16.28	21.61	27.68	.98
41	Verbal Analogies	R	20.46	8.45	15.97	20.77	25.81	.91
75	Verbal Enumeration	$\frac{R}{2}$	45.00	7.53	41.47	45.91	50.66	.91
7	Word Grouping	R	36.88	8.58	31.28	36.87	43.04	.91
46	Word-Number	R	4.49	3.30	2.61	4.54	6.66	.80

TABLE 2
The Intertest Correlations

	61	62	31	15	29	39	11	63	65	66	12	34	20	67	68	26	69	70	47	10	71	33	27	40	73	74	35	41	75	7
62	−.567																													
31	.277	−.193																												
15	.442	−.411	.318																											
29	.108	−.164	.105	.028																										
39	.315	−.327	.252	.282	.269																									
11	.641	−.565	.015	.438	.107	.418																								
63	.670	−.302	.230	.292	.086	.190	.437																							
65	.229	−.157	.079	.103	.239	.201	.117	.132																						
66	−.078	.312	−.112	−.045	−.268	−.189	−.140	.054	−.575																					
12	.481	−.440	.200	.582	.034	.181	.522	.361	.143	−.092																				
34	.345	−.224	.612	.313	.235	.430	.166	.317	.173	−.115	.232																			
20	.342	−.266	.096	.193	.120	.384	.390	.180	.225	−.164	.233	.214																		
67	.271	−.120	.490	.283	.230	.206	−.047	.400	.135	.009	.150	.463	.056																	
68	−.025	.248	−.303	−.140	−.304	−.178	.030	−.074	.078	.175	−.028	−.303	−.094	−.515																
26	.242	−.093	.192	.089	.341	.176	.048	.220	.237	−.094	.114	.193	.186	.350	−.194															
69	.229	−.170	.372	.248	.167	.039	.151	.438	.039	−.009	.318	.307	−.020	.508	−.256	.255														
70	.263	−.215	.478	.288	.208	.139	.026	.374	.163	−.127	.227	.452	.044	.658	−.392	.302	.622													
47	.260	−.210	.193	.160	.152	.196	.176	.236	.103	−.058	.170	.198	.101	.192	−.154	.218	.294	.242												
10	.511	−.548	.081	.378	−.001	.347	.665	.350	.142	−.147	.355	.159	.316	−.051	.065	.016	.053	.018	.061											
71	.254	−.135	.317	.341	.157	.190	.092	.267	.127	−.019	.277	.336	.089	.414	−.235	.212	.388	.467	.279	.070										
33	.266	−.151	.637	.304	.100	.316	.076	.222	.097	−.039	.166	.646	.078	.420	−.214	.137	.261	.420	.217	.119	.319									
27	.113	.032	.138	.081	.321	.162	−.032	.124	.181	−.047	.157	.199	.275	.297	−.101	.423	.195	.241	.084	−.122	.179	.157								
40	.194	−.315	.065	.170	.194	.354	.322	.082	.141	−.214	.195	.136	.184	.008	−.073	.088	.031	.025	.106	.248	−.079	.044	.115							
73	.193	.020	.197	.075	.282	.027	−.007	.252	.199	.091	.147	.212	.046	.453	−.086	.332	.353	.351	.232	−.034	.331	.153	.332	.006						
74	−.072	.258	−.098	−.083	−.250	−.056	−.040	−.034	.006	.210	−.068	−.076	−.086	−.108	.321	−.110	−.216	−.201	−.145	−.049	−.150	−.060	−.048	−.025	−.093					
35	.380	−.320	.260	.284	.302	.565	.421	.340	.169	−.167	.254	.467	.354	.269	−.290	.231	.174	.206	.154	.312	.132	.366	.214	.271	.160	−.077				
41	.376	−.438	.034	.263	.176	.475	.520	.264	.177	−.232	.299	.217	.298	−.009	−.094	.083	.018	.031	.074	.485	−.015	.052	.041	.259	−.016	−.105	.433			
75	.550	−.331	.216	.331	.129	.198	.473	.676	.126	−.007	.411	.266	.157	.338	−.067	.233	.451	.395	.185	.254	.234	.159	.144	.105	.207	−.061	.356	.300		
7	.630	−.488	.096	.370	.182	.350	.636	.489	.153	−.123	.399	.232	.337	.097	−.063	.162	.151	.157	.266	.531	.084	.100	.068	.254	−.015	−.096	.460	.565	.454	
46	.172	−.207	.190	.134	.181	.284	.220	.134	.086	.005	.109	.187	.160	.151	−.078	.132	.189	.175	.358	.109	.188	.188	.143	.074	.050	−.020	.215	.250	.211	.211

TABLE 3

Centroid Matrix

	I	II	III	IV	V	VI	VII	VIII	IX	X
61	.69	.36	—.21	.17	—.03	—.04	—.16	.19	.17	.11
—62	.59	.33	.09	—.18	.16	.27	—.10	.06	.19	.08
31	.51	—.35	—.27	—.34	—.15	—.11	.09	.17	.12	.03
15	.55	.20	—.27	—.14	—.12	.21	.12	—.19	.14	—.15
29	.39	—.24	.39	.10	.05	—.07	—.06	—.11	—.10	.16
39	.56	.16	.28	—.22	—.16	—.19	.22	—.05	—.14	—.06
11	.57	.65	—.01	.08	.02	.10	.09	—.03	—.04	.08
63	.60	.18	—.37	.26	.16	—.18	—.18	.15	—.10	.03
65	.35	—.10	.26	.15	—.37	.25	—.28	.28	—.25	—.17
—66	.28	—.06	.46	—.17	—.10	.43	—.27	.23	—.18	—.10
12	.54	.28	—.24	.09	—.11	.28	.09	—.18	.06	—.08
34	.62	—.22	—.14	—.31	—.19	—.24	.10	.12	—.06	.04
20	.41	.21	.25	.06	—.23	—.08	—.04	—.08	.17	—.11
67	.55	—.48	—.26	—.07	.10	—.22	—.18	—.16	—.03	—.14
—68	.36	—.33	.14	—.35	.36	—.08	—.10	—.27	.12	—.04
26	.42	—.27	.16	.30	—.08	—.13	—.14	—.08	.10	.05
69	.51	—.32	—.33	.13	.29	.15	.06	—.11	—.15	.08
70	.58	—.46	—.27	—.04	.18	.07	—.10	—.02	—.06	—.09
47	.40	—.15	.05	.21	.13	.08	.32	.15	.15	.10
10	.46	.57	—.04	—.12	—.04	.14	—.07	.07	.03	.14
71	.45	—.30	—.25	.08	—.03	.12	.09	—.10	.11	—.13
33	.48	—.28	—.24	—.35	—.23	—.16	.21	.18	.06	.10
27	.33	—.26	.15	.29	—.25	—.22	—.06	—.27	.06	—.08
40	.31	.22	.26	—.10	—.06	.05	.06	—.10	—.16	.15
73	.34	—.35	—.14	.31	—.08	—.04	—.09	—.17	—.10	.18
—74	.24	—.16	.16	—.07	.25	.18	—.11	—.10	.18	.13
35	.62	.14	.18	—.13	—.07	—.33	.06	—.06	—.14	.03
41	.48	.43	.27	—.09	.08	—.09	—.02	.07	—.06	—.10
75	.59	.18	—.29	.26	.23	—.11	—.06	.05	—.20	—.09
7	.60	.48	.08	.09	.16	—.06	—.03	.13	.07	.03
46	.35	—.04	.12	.12	.14	—.06	.36	.16	.06	—.17

TABLE 4

Rotated Factorial Matrix

Name of Test	Code No.	V	I	P	Ratios	Doublet Designs	N	W	S	M	Residual	h^2
Abstract Classification	61	.674	−.116	.362	.015	−.059	.082	.014	.082	.024	.319	.78
" $\frac{R}{R+0}$	−62	.575	.059	−.034	.411	.094	.113	.087	−.016	.064	.231	.65
Addition	31	.001	−.037	.242	.054	−.004	.727	.016	−.033	.031	.077	.66
Anagrams	15	.473	−.008	.013	.021	.018	.305	.441	.020	−.008	−.010	.59
Areas	29	−.046	.283	.172	.324	.038	−.011	−.070	.421	.076	.022	.43
Arithmetical Reasoning	39	.360	.482	−.021	−.015	.053	.325	−.012	.271	.091	.011	.60
Completion	11	.754	.253	.037	−.016	−.038	−.094	.225	.063	.042	.296	.78
Concrete Association	63	.495	.002	.668	−.068	−.060	.019	.042	−.075	−.012	.161	.72
Designs	65	.087	−.041	.172	.004	.659	.016	−.052	.385	−.008	.168	.67
" $\frac{R}{R+0}$	−66	.069	.094	−.106	.382	.674	.042	−.084	.200	.012	.158	.69
Disarranged Words	12	.499	−.077	.081	−.013	.044	.071	.494	.101	.015	.102	.58
Division	34	.121	.197	.290	−.014	.007	.671	−.025	.088	−.015	.086	.67
Flags	20	.416	.033	−.057	.012	.033	.048	−.049	.398	.053	−.042	.39
Highest Number	67	−.002	.015	.583	.190	−.016	.413	.083	.074	.088	−.300	.74
" $\frac{R}{R+W}$	−68	−.008	.198	.108	.568	−.089	.303	−.024	−.008	−.006	−.331	.62
Identical Forms	26	.030	−.045	.320	.152	−.025	−.006	−.049	.472	.088	−.044	.43
Identical Names	69	−.011	.113	.526	.230	−.015	.138	.409	−.070	.100	.044	.64
Identical Numbers	70	−.009	−.002	.540	.279	.135	.362	.219	−.027	.052	−.103	.68
Initials	47	.063	.053	.158	.105	−.072	.089	.123	.126	.476	.190	.41
Inventive Opposites	10	.644	.110	−.045	.077	.024	.041	.093	−.029	−.107	.347	.60
Letter A	71	.046	−.113	.283	.070	.035	.275	.331	.107	.143	−.096	.42
Multiplication	33	−.009	.066	.172	−.055	−.056	.740	.020	−.001	.040	.161	.66
Pursuit	27	−.002	.000	.227	−.030	−.065	.018	.038	.576	.008	−.225	.48
Reasoning	40	.230	.363	−.103	.128	.047	.011	.068	.205	−.050	.173	.29
Scattered X's	73	−.145	−.009	.437	.061	−.075	.008	.206	.328	−.072	.055	.44
" $\frac{R}{R+0}$	−74	.010	.000	.006	.499	−.017	.016	.035	.067	.069	.006	.28
Tabular Completion	35	.393	.437	.199	.022	−.062	.272	−.093	.258	−.008	.017	.59
Verbal Analogies	41	.599	.318	.024	.093	.107	.016	−.124	.086	.092	.051	.53
Verbal Enumerations	75	.484	.148	.613	−.060	.009	−.034	.163	−.078	.076	.047	.65
Word Grouping	7	.708	.167	.185	.101	−.034	−.048	−.053	.045	.155	.205	.66
Word Number	46	.178	.170	.123	−.038	.019	.117	.025	.053	.512	−.028	.36

TABLE 5

The Transformation Matrix

	V	I	P	Ratios	Doublet	N	W	S	M	Res.
I	.518	.220	.414	.234	.092	.346	.174	.276	.135	.123
II	.773	.133	−.285	−.239	−.112	−.337	.002	−.174	−.131	.210
III	−.045	.353	−.408	.360	.230	−.254	−.398	.526	.259	−.061
IV	.019	−.237	.406	−.273	−.070	−.701	.134	.406	.317	.070
V	.080	.216	.265	.513	−.190	−.282	−.067	−.561	.305	−.181
VI	−.067	−.207	−.332	.362	.518	−.172	.587	−.072	.080	.291
VII	−.121	.391	−.296	−.359	−.271	.217	.362	−.061	.634	.095
VIII	.019	−.224	.180	−.185	.380	.187	−.531	−.325	.394	.488
IX	.184	−.664	−.336	.232	−.410	.124	−.173	.090	.271	−.113
X	−.267	.139	−.047	.268	−.481	−.062	−.034	.117	−.262	.744

TABLE 6

Correlations R_{pq} Between Primary Abilities in the Experimental Population

	V	I	P	Ratios	Doublet	N	W	S	M
V	1.000	—.025	—.038	.058	.004	.092	.014	.069	—.024
I	—.025	1.000	—.015	—.044	.039	—.051	—.108	—.036	—.030
P	—.038	—.015	1.000	.103	—.043	.052	.076	.092	.040
Ratios	.058	—.044	.103	1.000	—.053	.064	.060	.022	.062
Doublet	.004	.039	—.043	—.053	1.000	—.026	—.030	—.017	—.049
N	.092	—.051	.052	.064	—.026	1.000	.066	.176	.047
W	.014	—.108	.076	.060	—.030	.066	1.000	.003	.041
S	.069	—.036	.092	.022	—.017	.176	.003	1.000	.035
M	—.024	—.030	.040	.062	—.049	.047	.041	.035	1.000