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SECOND-ORDER ABILITY STRUCTURE REVEALED IN RIGHTS AND WRONGS SCORES[1]

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Using 14 ability tests to measure 12 primary factors in a sample of 106 adult males, 2nd-order factorial structures were determined separately for both rights scores and wrongs scores. In both analyses the results agreed well with previous analyses in showing intellective functions interpreted as fluid intelligence, crystallized intelligence, general visualization, and general fluency. Other factors were interpreted tentatively. The correlations between rights scores and wrongs scores for the same test were found to range from +.51 to −.98, with most being in a range indicating about 25% of variance in common for the 2 kinds of scores. The results were interpreted as indicating that whereas the same basic functions were evidenced by the interrelationships among the 2 kinds of scores, decisions about the level of function of a particular individual could be quite disparate for rights as compared with wrongs scores.

In recent studies by Cattell (1963), Horn (1965) and Horn and Cattell (1966a; 1966b; 1967) evidence was found to support a theory of fluid and crystallized intelligence. According to this theory the influences operating in development to produce what is generally called "intelligence" produce not one general intellectual dimension, but two. Both of these involve basic processes of intelligence (perception of relations, eduction of correlates, span of apprehension, concept formation, concept attainment, the use of generalized solution instruments, etc.) but in one, termed fluid intelligence (abbreviated Gf), the emphasis is on problem solving in the immediate situation with test materials that are largely culture fair whereas the other, labeled crystallized intelligence (abbreviated Gc), more nearly indicates the limits of acculturation.

The tasks which define the Gf factor definitely require "intellectual work," are not simply scanning and movement tasks, but the test materials are such that for most of the people tested, the fundaments (i.e., the contents between which relations can be said to exist) are about equally common or equally novel. In terms of the primary abilities analyzed in the Horn and Cattell studies with adults, the processes and materials are best represented by Induction (I-Letter Series[2]), Figural Relations (CFR-Matrices), Associative Memory (Ma) and Figural Classifications (CFR). Similarly, "intellectual work" is indicated by the tasks which define Gc, but in this case the work involved is not so much that of the immediate problem-solving task as it is that which would have occurred previously through intensive acculturation to which only some, not all, of the people tested would have been exposed. The primaries which particularly characterized this function in the Horn and Cattell studies were Verbal Comprehension (V), Mechanical Knowledge (Mk), Numerical Facility (N) and Experiential Evaluation

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[2] The primary factor abbreviations employed throughout this article were taken from French, Ekstrom, and Price (1963), if they list the factor, or otherwise from French (1951) or Guilford and Merrifield (1960).

(EMS). The fluid and crystallized dimensions were found to be highly cooperative in General Reasoning (R), Semantic Relations (CMR-Word Analogies) and Formal Reasoning (Rs-Syllogisms).

Both of the above factors were clearly distinguished from a general visualization function (abbreviated Gv) and a general fluency dimension (abbreviated F). Gv involved all tasks which required work with spatial materials. It included the I and CFR primaries of Gf, but was defined principally by Spatial Orientation (S), Visualization (Vz), Speed of Closure (Cs), Flexibility of Closure (Cf), and Perceptual Speed (P). The general fluency function was manifested in all tasks requiring scanning or production of conventional concept labels (words), as best represented by Associational Fluency (Fa), Ideational Fluency (Fi), and Word Fluency (Fw).

Horn (1965) and Horn and Cattell 1966b, 1967) reviewed cross-sectional studies and produced new data to show that when putative measures of intelligence are classified as "primarily fluid," "primarily crystallized," and "about evenly mixed with these two," much of the seemingly contradictory evidence on age differences in intelligence can be seen to be consistent. Younger adults were found to be superior in fluid functions; older adults were found to be superior in crystallized functions and no systematic age differences were found for primaries and omnibus tests which involved these two factors in about the same degree.

This line of research thus shows promise of integrating previously diverse bits of evidence in the field of human abilities. Perhaps more interesting, it shows some promise of furthering rapprochement among previously separate, if not antagonistic, subfields in which the principal concern is with questions about the processes of problem solving, perception, etc. But this promise can be realized only if the results obtained heretofore are not specific to either a particular sample of primary factors or a particular sample of subjects. It is of considerable interest, therefore, to test the generality of the above results. The main purpose of the present study is to do this.

Several years ago Fruchter (1950, 1953) effectively demonstrated that wrongs scores obtained from time-limit tests can have reliabilities and validities comparable to those for rights scores obtained from the same tests. More important, he showed that these results exist even when the correlations between rights and wrongs scores are quite low—some hovering near zero! In a factor analysis which included some rights-score and some wrongs-score variables he found a factor defined exclusively by wrongs scores. Horn (1965) replicated this finding and in his further analyses found that the carefulness factor (as Fruchter had labeled it) was largely independent of other factors even at the second and higher orders.

These findings thus suggest that quite different functions are measured with wrongs scores as compared with rights scores from timed ability tests.

But there are at least two ways in which to interpret this last statement. According to one interpretation it is evident from the fact of low correlation between comparable rights and wrongs scores that different functions are represented in these variables. For a low correlation means that a person who would be judged "good" in his performance on a given test when the rights score is used might be—that is, is nearly as likely to be—judged "poor" in his performance on this same test when the wrongs score is used.

Thus if "function" is thought of in terms of level of performance, it is indeed evident that different functions are tapped by rights and wrongs scores.

However, one might think of "function" not simply in terms of level of performance, but in terms of patterns of correlations of variables with all other variables with which they might be compared. It is possible for such patterns to be very similar for rights and wrongs scores even when the two kinds of scores correlate rather lowly. Horn and Cattell (1965) recently discussed this possibility under the heading of vehicles in measurement. A vehicle is an individual differences characteristic through which other characteristics are expressed. Thus carefulness may be systematically expressed in all of several kinds of abilities, but it need not be an influence which prevents the appearance of the factors representing these abilities when a properly rotated factor-analytic solution is obtained. In other words, if carefulness is a unitary function which affects all abilities in approximately the same way relative to a rights-score function, then it is possible for the same factors to appear in analyses of wrongs scores alone and in analyses of rights scores alone.

With respect to these considerations, then, the present investigation was designed to: (*a*) replicate previous findings with a new sample of subjects and a somewhat different sample of variables, and (*b*) see if the basic structure is found both among rights-score and among wrongs-score variables.

PROCEDURE

Selection of Variables

As in the previous studies of the fluid-crystallized concept, the French, Ekstrom, and Price (1963), French (1951), and Guilford and Merrifield (1960) collations of rotated factor results provided the basis for selection of primary factor variables. To ensure some continuity with previous studies, nine primary factors used in the Horn and Cattell (1966a) analyses were selected for the present investigation. These are the variables identified with asterisks in Table 1, which provides basic descriptive data on all variables. One test was used to measure each primary factor. For P, Vz, Cf, and I the tests were entirely different from those used in the Horn-Cattell studies, although they represented the same primaries. The other tests were different only in the sense that they involved somewhat different items.

Three new primary factors and two new tests, for which the factor structure was not known, were added to the core variables described above. The Aiming primary, (A), was included on the hypothesis that it would help define general visualization, while two memory primaries—Memory Span (Ms) and Memory for Designs (Md)—were included on hypotheses that a major portion of their variances would go to fluid intelligence. The Dominoes Test, originated by Anstey and developed by Vautrain (1954), was included on the assumption that it would have both general visualization and fluid intelligence variance, but mainly the latter. S. Mednick's (1962)[3] Remote Associations Test (RAT) was included as a measure of creativity. Burt (1962) has pointed out that useful creativity (in contrast to the originality of the insane, the delirious, the day-dreamer, etc.) involves the noegenetic processes of intelligence, as most fully described in Spearman's work. On this basis it was predicted that to the extent that the RAT indeed measures creativity, it should have some variance on the fluid-intelligence factor. Intuitively it would seem to involve some variance on the general-fluency factor as well.

Subjects and Administration

The subjects were 106 male inmates at the Colorado State Penitentiary.[4] The tests were administered in 10 parts on 10 occa-

[3] The authors are very grateful to S. A. Mednick for making the materials for this test available to us prior to the publication, by Science Research Associates, of a commercial version of the RAT.

[4] The authors thank George Levy, Senior Psychologist at the Colorado State Penitentiary, for his help in securing this sample. In this respect they are also grateful to Wayne Patterson, Warden, and Harry Tinsley, Director of Institutions.

TABLE 1

PRIMARY FACTOR VARIABLES, TESTS USED TO ESTIMATE THEM, AND MEANS AND STANDARD DEVIATIONS

Primary factor	Symbol	Test used	*M*	Sigma
1. Perceptual Speed	*P	Cancel Numbers	515.9	73.00
2. Aiming	A	Dot in Circle	1145	386.0
3. Visualization	*Vz	Follow the Line	425.6	102.8
4. Flexibility of Closure	*Cf	Copy the Pattern	110.5	50.41
5. Figural Relations	*CFR	Figure Series	37.49	7.429
6. Deduction	D	Dominoes	28.76	8.086
7. Memory Span	Ms	Forward-Backward Digits	434.6	188.6
8. Memory for Designs	Md	Redraw the Figure	4.868	5.206
9. Induction	*I	Letter Series	58.06	15.82
10. Semantic Relations	*CMR	Common Word Analogies	22.06	5.449
11. Creativity	RAT	Remote Associations	86.79	32.75
12. Associational Fluency	*Fa	Similar Words	66.74	29.63
13. Ideational Fluency	*Fi	Things	103.9	40.78
14. Verbal Comprehension	*V	General Information	36.12	7.683
		Word Variables		
15. Perceptual Speed	P	Cancel Numbers	21.67	16.48
16. Aiming	A	Dot in Circle	413.5	334.4
17. Visualization	Vz	Follow the Line	9.934	33.54
18. Flexibility of Closure	Cf	Copy the Pattern	56.32	43.84
19. Figural Relations	CFR	Figure Series	22.54	7.417
20. Deduction	D	Dominoes	24.09	8.238
21. Memory Span	Ms	Forward-Backward Digits	45.36	43.89
22. Induction	I	Letter Series	15.11	10.50
23. Semantic Relations	CMR	Common Word Analogies	18.38	5.141
24. Creativity	RAT	Remote Associations	19.65	14.57
25. Associational Fluency	Fa	Controlled Associations	41.65	22.81
26. Ideational Fluency	Fi	Things	5.207	6.334
27. Verbal Comprehension	V	General Information	22.31	6.709

sions spaced over 5 days. There was one testing session in the morning, one in the afternoon on each day. A testing session lasted about 1½ hours.[5] The men were volunteers. They were paid $2.00 each for their services.[6] The morale and motivation was judged to be very high throughout, although somewhat lower in the later sessions than in the earlier ones.

ANALYSES AND RESULTS

The number of correct answers and the number of incorrect answers was determined for each part-test. A variable was then obtained as a simple (unweighted) sum of the raw scores on the 10-part tests. All variables, both rights and wrongs scores, were intercorrelated by the product-moment formula. The resulting table of intercorrelations has been deposited with ADI.[7] In Table 2 are shown the correlations between rights and wrongs scores for the same test.

The submatrices involving rights

[5] Marlan Wilson, Jr. directed the administration and scoring of the tests. He was assisted by Messrs. Mathis and Conrad. The help of these men is very gratefully acknowledged.

[6] We were told that on the prison-to-street exchange, this is equivalent to about $10.00 to $15.00 outside the prison.

[7] Material supplementary to this article has been deposited with the American Documentation Institute. Order Document No. 9274 from ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D. C. 20540. Remit in advance $1.25 for microfilm or $1.25 for photocopies and make checks payable to: Chief, Photoduplication Service, Library of Congress.

TABLE 2

CORRELATIONS BETWEEN RIGHTS AND WRONGS SCORES FOR THE SAME TEST

	Variable symbol												
	P	A	Vz	Cf	CFR	D	Ms	I	CMR	RAT	Fa	Fi	V
r	.12	−.54	−.52	−.55	−.98	−.63	.30	−.64	−.98	−.40	.51	.46	−.92

scores alone and wrongs scores alone were abstracted and factored separately. Algebraic independence was maintained among the variables of these two sets. In each set the squared multiple correlation of a variable with the other variables of the set was determined and inserted in the principal diagonal as a communality estimate. Principal axes factors were then extracted until a latent root became zero. The number of factors to be rotated was further limited by selecting only those factors with roots larger than one which had at least one loading greater than .20 (absolute value)—that is, taking factors in order of size of latent roots, when a principal axis factor was encountered which did not have at least one loading greater than .20 (absolute value), that factor and all factors with smaller latent roots were eliminated in the rotations. The factors were rotated in accordance with the criteria of Kaiser's (1958) varimax procedure.

The rotated results from the factor analyses are shown in Tables 3 and 4. The squared multiple correlation communalities are shown in the next-to-last column in each of these tables; the communalities based on factor loadings are shown at the far right.

DISCUSSION

The correlations of Table 2 suggest that when a task is highly speeded but conceptually simple, (Canceling,

TABLE 3

ROTATED FACTORS FROM ANALYSIS OF RIGHTS SCORES

Symbols	Primary factors	Second order factors							
		I	II	III	IV	V	VI	SMR	h^2
P	Circle Numbers	11	01	59	12	09	−06	31	39
A	Aiming	18	11	50	18	16	41	45	52
Vz	Visualization	25	07	67	08	18	23	52	62
Cf	Flexibility of Closure	45	28	37	24	32	32	63	67
CFR	Figural Series	61	03	31	10	35	18	59	63
D	Deduction	69	38	25	10	13	04	65	71
Ms	Memory Span	12	04	12	11	49	01	23	28
Md	Memory Designs	22	09	18	11	53	39	45	53
I	Letter Series	57	42	22	27	40	09	74	79
CMR	Common Word Analogies	39	51	16	16	48	06	67	73
RAT	Creativity	25	61	−01	15	−06	10	42	48
Fa	Controlled Associations	16	41	15	70	35	−04	76	84
Fi	Things	13	28	24	74	09	18	69	74
V	General Information	05	75	07	28	18	01	63	69

Note.—Decimal points have been omitted in the factor coefficients in order to simplify the presentation.

TABLE 4

ROTATED FACTORS FROM ANALYSIS OF WRONGS SCORES

Symbols	Primary factors	Second order factors						
		I	II	III	IV	V	SMR	h^2
P	Circle Numbers	10	11	58	−06	39	45	52
A	Aiming	16	03	56	−02	−04	32	35
Vz	Visualization	14	04	65	−03	07	37	45
Cf	Flexibility of Closure	46	33	28	−04	43	55	59
CFR	Figural Series	55	−11	31	−18	25	45	51
D	Deduction	69	31	15	13	03	60	62
Ms	Memory Span	25	17	−11	22	03	15	15
I	Letter Series	73	22	32	05	−11	63	69
CMR	Common Word Analogies	61	49	19	−15	24	66	73
RAT	Creativity	14	32	−15	61	−26	47	58
Fa	Controlled Associations	07	−09	−05	72	07	45	54
Fi	Things	−13	−11	06	61	−04	33	40
V	General Information	26	70	10	−06	04	50	57

Note.—Decimal points have been omitted in the factor coefficients in order to simplify the presentation.

for example), the number of errors correlates positively with the number of correct responses.

In the Canceling test the wrongs score is literally an error score, but this is not quite true for the other three tests for which there is positive correlation between rights and wrongs scores. In Memory for Digits the subject was required to recall the digits in the order in which they were given (in half the items) or in reverse order. If any one of the digits given in an answer was incorrect or out of order, the response on that item was scored wrong. In fluency tests (Similar Words and Things) a response was judged "wrong" if it seemed to be a quite bizarre, irrelevant association. For example, the response "clip" to the key word "warm" was scored wrong. In some cases, of course, one could legitimately question the decision that a particular response was bizarre or irrelevant. In this connection it is perhaps worth pointing out that the correlations between RAT (a measure of creativity) and the wrongs scores for Fa and Fi were −.02 and −.06 respectively—that is, certainly low enough to suggest that relevant but remote associations were not being counted in the wrongs scores for Fa and Fi.

The correlations between wrongs and rights scores are negative and very high for Figure Series, Common Word Analogies, and General Information. This indicates that these measurements were obtained under almost power conditions, as was the aim in this research. Had all subjects attempted all items in these tests, the correlations between rights and wrongs scores for the same test would necessarily be −1.0. The departure of the correlations from −1.0 is thus a rough indication of the speededness of test administration.

The correlations between rights and wrongs scores for A, Vz, Cf, D, I, RAT, Fa, and Fi indicate that the two kinds of scores have about 25% of their variance in common. This is certainly low enough to suggest that rights and wrongs scores represent somewhat distinct functions. It is therefore particularly interesting that

the factorial results for the analyses of these two kinds of scores are so similar. Although there are differences in detail, the same essential functions, replicating previous results, can be identified in both analyses. The first four factors are easily identified as the fluid intelligence, crystallized intelligence, general visualization, and general fluency dimensions previously identified by Horn and Cattell (1966a).

The fifth factor in the rights score analysis is characterized mainly by the memory tests but involves substantial loadings on CMR, I, CFR, Fa, and Cf. Each of these tests requires that the subject hold a relation or several relations in immediate awareness in order to compare it with other relations. This suggests a link between this factor and the process which Spearman first described as span of apprehension or what has been discussed in recent years under the heading of temporal integration.

Factor VI in the rights score analysis represents a visualization function but in this case, in contrast to Factor III, Memory for Designs is involved. There is thus a suggestion that visual retention, as represented in Factor VI, is somewhat distinct from the visual "fluency" represented in Factor III.

The simple structure in the analysis of wrongs scores is somewhat better than in the rights score solution: yet the salient factor coefficients tend to be of about the same magnitude. The suggestion is that analysis of wrongs scores yields a "cleaner" structure than analysis of rights scores. It is of some interest to compare the differences in loadings for the comparable salient variables in the rights-score and wrongs-score analyses, but it is probably not worthwhile to occupy space with a detailed discussion of these at this time.

It is perhaps worth noting, however, that Remote Associations in the rights-score analysis has a loading of only .15 on the general-fluency dimension, whereas in the wrongs-score analysis the comparable loading is .61. General fluency is interpreted as a facility in bringing concept labels (words) from a long-term storage center into immediate awareness. The results obtained here suggest that this facility is not characterized by criticality. That is, the concepts implied need not be related in a precise and logical manner, for this is the requirement imposed on associations in the rights score for RAT and this variable does not help to define F, whereas this restriction is removed in the wrongs score for RAT and in this case the variable does help to define F.

Factor V in the wrongs-score analysis does not match any factor found in the rights-score analysis or in our previous work. It is not readily interpreted. We will therefore hold our attempt at identifying it until after we find that it can be replicated.

Something of a paradox is thus posed by these results. When a test is scored by the wrongs-score procedure, the person who adopts a strategy of avoiding errors has the advantage; when the same test is scored by the rights-score procedure, the person who adopts a strategy of getting as many right as possible, even at the cost of a few errors, has the advantage. The same basic intellectual functions are indicated by analyses of both kinds of scores. This means that regardless of strategy adopted, knowing how a subject performs on one kind of task in a factor pattern, we can predict with considerably better than chance accuracy how he will perform in all other tasks defining the pattern. Yet, and here is the paradox, persons who are rated high in a particular func-

tion when this is defined in terms of rights scores will rather frequently (as suggested by correlations of the order of .5) be rated comparatively low in this same function when it is defined in terms of wrongs scores (that is, a constant minus number of wrongs, to define the variable so that a high score indicates good performance).

This is indeed an interesting finding and one which has practical as well as theoretical implications. It is to be hoped that follow-up studies now in progress will shed additional light on the questions here provoked.[7]

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ERRATUM

On page 332 of "Structure of Intelligence in Negro and White Children" by Ira J. Semler and Ira Iscoe in the December 1966 issue the last sentence in the section WISC Structure: Multivariate Analysis of Variance and Exploratory Factor Analyses should read: Factor II for the white sample was a complex factor involving loadings of the WISC A, DS, BD, and C subtests.